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Radial Hilbert Transform in terms of the Fourier Transform applied to Image Encryption

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1. **Abstract.** In the present investigation, a mathematical algorithm under Matlab platform using Radial Hilbert Transform and Random Phase Mask for encrypting digital images is implemented. The algorithm is based on the use of the conventional Fourier transform and two random phase masks, which provide security and robustness to the system implemented. Random phase masks used during encryption and decryption are the keys to improve security and make the system immune to attacks by program generation phase masks.

1. Introduction

The threat of illegal data access has made the data encryption an important subject [1]. Several methods have been proposed [2–8] for optical image encryption including the most popular double random phase encoding (DRPE)[9]. This technique uses two random phase masks and the Fourier planes to encrypt the input image. Optical encryption offers several image parameters, e.g. phase, amplitude, color, spatial frequency, polarization, etc., that can be exploited to perform a robust and highly secure encryption. In this paper, we propose a technique based in the Fourier domain using the properties of Radial Hilbert Transform (RHT). The frequency spectrum of the image is then multiplied with two different random phase function and two Fourier Transform for getting the encrypted images. In the decrypt processing is required a random phase function additional and two fractional order which make this technique safer and more robust compared to the conventional technique of double random phase encoding in the Fourier domain.

2. Methodology

Hilbert Transform in terms of the Fourier

The Hilbert Transform for a function $g(x)$ is defined [10].

$$g_{HT}(x) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{g(x)}{x-\tau}d\tau \tag{1}$$

Where $g_{HT}(x)$ Hilbert Transform's signal, $g(x)$ is input signal.

Considering the theory of linear systems such as optical systems can show that the output signal for this type of system can be expressed as the convolution of the input signal with the impulse response.

$$g_{HT} = g(x) * h(x) \tag{2}$$

Assuming that the impulse response $h(x)$ is this form

$$h(x) = \frac{1}{i\pi x} \tag{3}$$

Equation 2 can be written

$$g_{HT} = g(x) * \frac{1}{i\pi x}. \tag{4}$$

As what is sought is to write the Hilbert transform in the terminus Fourier transform proceed to obtain the equation 4

$$F\{g_{HT}\} = F\left\{g(x) * \frac{1}{i\pi x}\right\}. \tag{5}$$

By properties of the TF have

$$F\{g_{HT}\} = F\{g(x)\} * F\left\{\frac{1}{i\pi x}\right\}. \tag{6}$$

Of the table Fourier Transform we have that the TF of impulse response is the sign function, and this correspond to the system transfer function.

$$F\left\{\frac{1}{i\pi x}\right\} = H_P\{\omega_x\} = -i\,\mathrm{sgn}(w_x). \tag{7}$$

Where the function $-i\,\mathrm{sgn}(w_x)$ is defined:

$$-i\,\mathrm{sgn}(w_x) = \begin{Bmatrix} i & if & w_x < 0 \\ 0 & if & w_x = 0 \\ -i & if & w_x > 0 \end{Bmatrix}. \tag{8}$$

This can be expressed in terms of Euler

$$-i\,\mathrm{sgn}(w_x) = e^{iP\theta}. \tag{9}$$

Where

$$e^{iP\theta} = \begin{Bmatrix} i \quad if \quad P = \dfrac{1}{2} \\ \\ -i \quad if \quad P = -\dfrac{1}{2} \end{Bmatrix}, \tag{10}$$

P = 0 indicates a phase mask that does no processing to the image, so it is not taken into consideration.
Therefore, the transfer function can be written

$$H_P\{\omega_x\} = e^{iP\theta}. \tag{11}$$

Then equation 6 can be written

$$F\{g_{HT}\} = F\{g(x)\}.H_P\{\omega_x\}. \tag{12}$$

Now as what we want is to get the output signal g_{HT} by properties of the TF we have:

$$F^{-}\{F\{g_{HT}(x)\}\} = F^{-}\{H_P\{\omega_x\}F\{g(x)\}\}. \tag{13}$$

Substituting 11 in 13 and solving we obtain The P-order Hilbert transform can be written as:

$$g_{HTP}(x) = F^{-}\{e^{iP\theta} F\{g(x)\}\}. \tag{14}$$

Where equation (14) represents the equation of the Hilbert transform of order P in terms of the Fourier Transform and the exponential function represents optically uniform random phase mask and is a technique used for the encrypted image.

(a) (b)

Figure 1. a) Original Image. b) Hilbert Transform.

So far we have found the one-dimensional Hilbert transform for signals, but our case study is two-dimensional signals (images). Then the generalized Hilbert transform in term of the Fourier transform to encrypt an image can be written:

$$g_{HTP}(x, y) = F^{-}\{e^{iP\theta} F(g(x, y))\}, \tag{15}$$

$$g'(x, y) = \{e^{iP\theta} F(g(x, y))\}. \tag{16}$$

Where $g'(x,y)$ is the image encrypted with only one random phase mask. Now for added system security multiplied by another phase mask

$$g_{HTP}(x,y)_{(encripted\ )} = F^{-}\{g'(x,y)F(e^{im\theta})\}. \tag{17}$$

Where P and m are different from zero, if they are zero we would have the same object image.

Now that we have clear the terms in equation 17, we do a block diagram showing the design of the system image encryption.

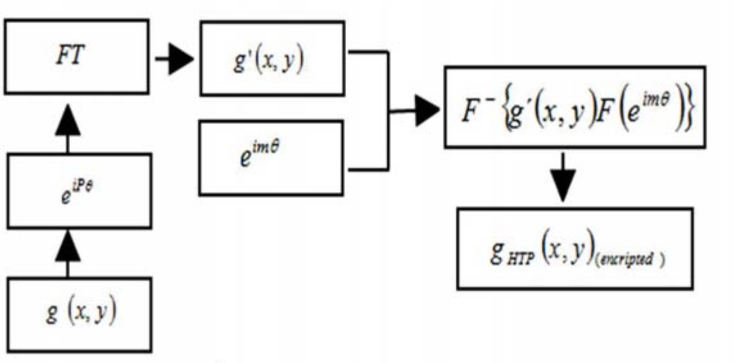

Figure 2. Block Diagram of the system design.

The software takes the original image $g\,(x,y)$ and multiplied by a mask of uniform random phase $e^{iP\theta}$, we find its Fourier transform and get $g'(x,y)$. For added security is again multiplied by another uniform random phase mask $e^{im\theta}$ and then apply the inverse Fourier transform, and finally obtain the encrypted image $g\ (x,y)_{encripted}$.

3. Results

To investigate the quality of encryption, digital simulations are performed on MATLABs 7.6 platform. The input image chosen for encryption is a cameraman of size 256 256 pixels.

Encryption

The input image is multiplied by a random phase mask and he finds the TF. Then for safety multiply it by another random phase mask and finally we calculate the TFI resulting in the encrypted image. Then do the reverse and get the decrypted image.

Decrypted

The image to be encrypted is changed to noise-like patterns and exact recovery of the image is possible only when the correct random phase functions and other key parameters are correctly used during the decryption.

Then to see if the encrypted image is fully recovered do the correlation of the original image with the encrypted image and got this gave us one, ie its maximum value which means that the algorithm manages to get the same image that is sent through the system.

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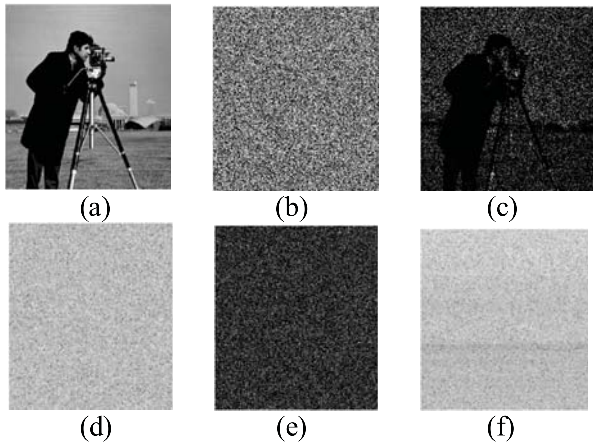

Figure 3. a) Original image. b) Randon phase mask. c) Multiplication phase mask for the original image. d) TF of the multiplication phase mask for the original image. e) Second Randon phase mask. f) Encrypted Image.

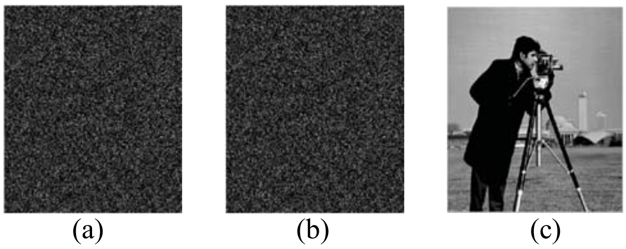

Figure 4. a) Second Randon phase mask. b) First Randon phase mask. c) Decrypted Image.

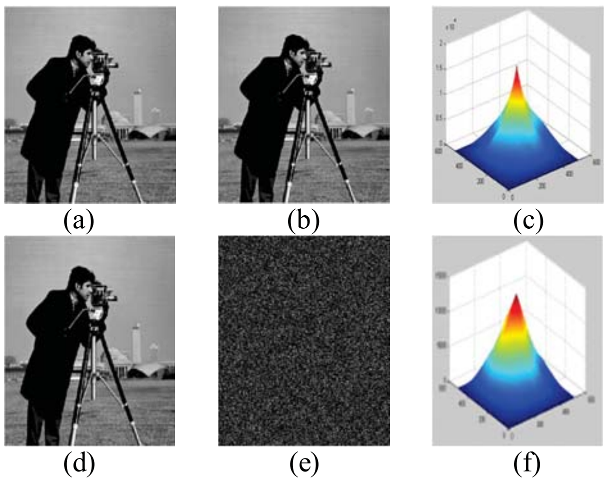

Figure 5. Correlation of the input image with the decrypted image. a) Original Image. b) Image decrypted with the correct key. c) Correlation peak of the original image with the decrypted image. d) Original image. e) Image decrypted with the incorrect key. f) Correlation peak of the original image with the decrypted image with incorrect key.

Conclusion

Based on conventional Hilbert transform found the Radial Transform Generalized Gilbert order P in terms of the Fourier Transform, which makes possible a visual and computational implementation of a cipher images or data without loss of information.

A double random-phase encoding-based technique for image encryption using the radial Hilbert has been proposed in the terminus of Fourier transform domain. Digital simulations are presented to demonstrate the robustness of the technique against the variation in the key parameters

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