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Signal differentiation in position tracking control of dc motors

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Abstract. An asymptotic differentiation approach with respect to time is used for on-line estimation of velocity and acceleration signals in controlled dc motors. The attractive feature of this differentiator of signals is that it does not require any system mathematical model, which allows its use in engineering systems that require the signal differentiation for its control, identification, fault detection, among other applications. Moreover, it is shown that the differentiation approach can be applied for output signals showing a chaotic behavior. In addition a differential flatness control scheme with additional integral compensation of the output error is proposed for tracking tasks of position reference trajectories for direct current electric motors using angular position measurements only.

1. Introduction

Control of electric machines is a research subject quite active due to these electric actuators are employed in many practical engineering systems as water pumping systems, electric vehicles, robots and tool machines. Some control schemes for DC electric motors have been reported in the literature (see, e.g., [1-7] and references therein). However, position trajectory tracking controllers for motors based on an efficient motion planning could demand measurements of several state variables increasing the controller implementation costs. Thus, the efficient control problem of electric motors using a minimum number of sensors is challenging, relevant and pertinent, and its solutions admit a wide variety of real applications in the development of engineering products and systems.

In this paper, a differential flatness control scheme with additional integral compensation of the output error is proposed for tracking tasks of position reference trajectories for direct current electric motors using angular position measurements only. The integral compensation is used to improve the robustness property of the output feedback dynamic controller with respect to constant load torque perturbations. An asymptotic differentiation approach with respect to time is proposed for on-line estimation of angular velocity and acceleration. The novel feature of this differentiator of signals is that it does not require any system mathematical model, which allows its use in any engineering system that requires the signal differentiation for its control, identification, fault detection, among other applications. Moreover, the satisfactory performance of the differentiation algorithm is verified for time-varying output signals generated by an uncertain chaotic nonlinear dynamical system of Lorenz. It is widely known that uncertain chaotic systems exhibit complex and unpredictable behavior and hence the state estimation is also an open and challenging research problem. The interested reader about chaotic systems and their control is referred to the works [8-10] and references therein. Some computer simulation results are provided to describe the satisfactory performance of the control scheme proposed in this paper as an alternative solution to the efficient angular position control problem of direct current electric motors.

2. Differential flatness-based control

Consider the mathematical model (1) of a DC permanent magnet motor with gearhead. Here L and R represent the inductance and the resistance of the armature circuit, respectively, k_e is the back electromotive force constant, k_m is the motor torque constant, n is the speed reduction ratio of the gearhead, J_m and J_0 are the inertia moments of the rotors of the motor and the gearhead, respectively, b_m and b_0 are the viscous damping coefficients of the motor and the gearhead, respectively.

$$\begin{aligned} L\frac{d}{dt}i &= -Ri - k_e n\dot{\theta} + u \\ \left(J_0 + n^2 J_m\right)\ddot{\theta} &= -\left(b_0 + n^2 b_m\right)\dot{\theta} + nk_m i - \tau_L \\ y &= \theta \end{aligned} \quad (1)$$

In this system, θ is the angular position of the output shaft of the gearhead, i is the electric current, u is the control voltage applied to the input terminals of the armature circuit, and τ_L denotes the constant load torque.

DC motor with gearhead is a differential flat system, with flat output given by the angular position of the output shaft of the gearhead, $y=\theta$. Hence all the state variables and the control input can be expressed in terms of the flat output and a finite number. System (1) then satisfies the input-output differential equation

$$y^{(3)} + \gamma_2\ddot{y} + \gamma_1\dot{y} = \gamma u - \frac{R}{J_{eq}L}\tau_L, \quad (2)$$

with

$$\gamma_1 = \frac{n^2 k_m k_e + Rb_{eq}}{J_{eq}L}, \quad \gamma_2 = \frac{b_{eq}}{J_{eq}} + \frac{R}{L}, \quad \gamma = \frac{nk_m}{J_{eq}L},$$

where J_{eq} and b_{eq} denote the total inertia moment and the equivalent rotational viscous damping coefficient of the system, respectively, both reflected to output shaft of the gearhead.

From equation (2), one obtains the following differential flatness-based controller with integral compensation for tracking tasks of some angular position reference trajectory $y^*(t)$:

$$
\begin{aligned}
u &= \frac{1}{\gamma}\left(v + \gamma_1 \dot{y} + \gamma_2 \ddot{y}\right) \\
v &= y^{*(3)} - \beta_3 \ddot{e} - \beta_2 \dot{e} - \beta_1 e - \beta_0 \int_0^t e(\tau)d\tau
\end{aligned} \tag{3}
$$

By replacing the control law (3) into the input-output model (2), and differentiating the resulting expression once with respect to time, it is obtained the closed-loop tracking error dynamics, $e = y - y^*$,

$$
e^{(4)} + \beta_3 e^{(3)} + \beta_2 \ddot{e} + \beta_1 \dot{e} + \beta_0 e = 0 \tag{4}
$$

Therefore, the asymptotic convergence of the tracking error to zero can then be achieved selecting the design parameters β_i, i = 0, 1, 2, 3, such as the characteristic polynomial associated to the closed-loop tracking error dynamics (4) is a *Hurwitz* polynomial.

Since the controller requires information of the angular acceleration and velocity signals which are assumed to be unavailable due to considerations of cost reduction in the controller implementation, in the next section is applied an approach to differentiate signals with respect to time.

3. Differentiation of signals

In the synthesis of a real-time signal differentiation scheme with respect to time, it is assumed that the position output signal $y(t)$ can be locally approximated by a forth degree Taylor polynomial family as

$$
y(t) = p_0 + p_1 t + p_2 t^2 + p_3 t^3 + p_3 t^4 , \tag{5}
$$

where all the coefficients p_i , i = 0, 1, …, 4, are completely unknown. Then, the position signal can be locally described as

$$
\begin{aligned}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= y_3 \\
\dot{y}_3 &= y_4 \\
\dot{y}_4 &= y_5 \\
\dot{y}_5 &= 0
\end{aligned} \tag{6}
$$

where $y_1 = y$, $y_2 = \dot{y}$, $y_3 = \ddot{y}$, $y_4 = y^{(3)}$, $y_5 = y^{(4)}$.

Hence, from (6) we propose the following state observer for asymptotic estimation of the acceleration and velocity signals:

$$
\begin{aligned}
\dot{\hat{y}}_1 &= \hat{y}_2 + \beta_4\left(y_1 - \hat{y}_1\right) \\
\dot{\hat{y}}_2 &= \hat{y}_3 + \beta_3\left(y_1 - \hat{y}_1\right) \\
\dot{\hat{y}}_3 &= \hat{y}_4 + \beta_2\left(y_1 - \hat{y}_1\right) \\
\dot{\hat{y}}_4 &= \hat{y}_5 + \beta_1\left(y_1 - \hat{y}_1\right) \\
\dot{\hat{y}}_5 &= \beta_0\left(y_1 - \hat{y}_1\right)
\end{aligned}
\quad . \tag{7}
$$

Then, the estimation error dynamics is governed by

$$
\begin{aligned}
\dot{e}_1 &= -\beta_4 e_1 + e_2 \\
\dot{e}_2 &= -\beta_3 e_1 + e_3 \\
\dot{e}_3 &= -\beta_2 e_1 + e_4 \\
\dot{e}_4 &= -\beta_1 e_1 + e_5 \\
\dot{e}_5 &= -\beta_0 e_1
\end{aligned}
\quad , \tag{8}
$$

where $e_i = y_i - \hat{y}_i$, $i = 1, 2, \ldots, 4$.

The characteristic polynomial of the observation error dynamics is then given by

$$
P_O\left(s\right) = s^5 + \beta_4 s^4 + \beta_3 s^3 + \beta_2 s^2 + \beta_1 s + \beta_0 \, , \tag{9}
$$

which is independent of the coefficients p_i of the Taylor polynomial expansion for *y(t)*. In this work, the following *Hurwitz* polynomial is proposed to select the observer gains:

$$
P_{DO}\left(s\right) = \left(s + p_o\right)\left(s^2 + 2\zeta_o \omega_{no} s + \omega_{no}^2\right)^2 \, , \tag{10}
$$

with p_o, ω_{no}, $\zeta_o > 0$.

The performance of the asymptotic observation scheme (7) was initially tested for the on-line differentiation of the time-varying output signal $y\left(t\right)$ generated by the uncertain chaotic system of Lorenz described by

$$
\begin{aligned}
\dot{\eta}_1 &= a\left(\eta_2 - \eta_1\right) \\
\dot{\eta}_2 &= b\eta_1 - \eta_1\eta_3 - \eta_2 \\
\dot{\eta}_3 &= \eta_1\eta_2 - c\eta_3 \\
y &= \eta_1
\end{aligned}
\quad , \tag{11}
$$

with $a = 10$, $b = 30$, $c = 1$, and initial conditions $\eta_1(0) = -10$, $\eta_2(0) = -10$, $\eta_3(0) = 20$.

Fig. 1 displays the time-varying output signal of the uncertain chaotic system of Lorenz (11). In Fig. 2 is shown the satisfactory differentiation with respect to time of the output signal. Therefore, the signal differentiation scheme (7) represents an alternative approach to get estimates of some derivatives requested for implementation of the position tracking controller (3).

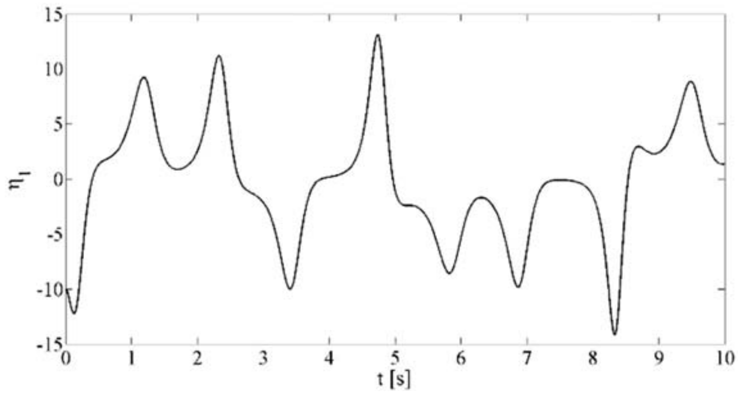

Figure 1. Output signal generated by an uncertain chaotic system of Lorenz.

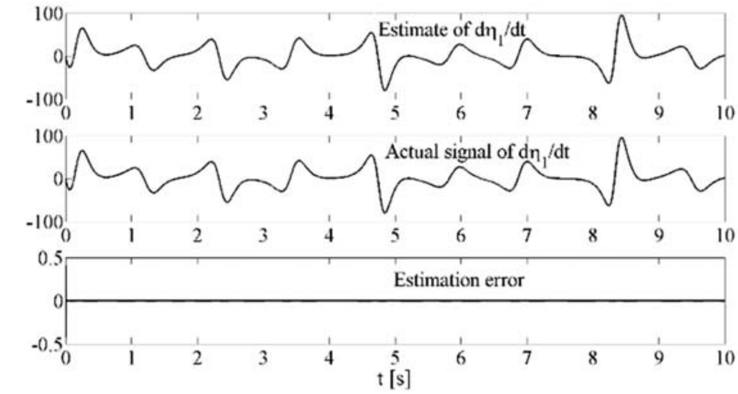

Figure 2. Estimated velocity of the output signal of an uncertain chaotic system of Lorenz.

4. Simulation results

Some computer simulations were performed to verify the performance of the angular position controller (3) using the differentiator of signals (7) for a DC motor with planetary gearhead characterized by the set of parameters: R = 2.5 Ω, L=0.612 H, k_m = 82.2 mNm/A, k_e = 82.3215 mV/rad/s, J_e = 82.3215 mV/rad/s, n = 81. The reference trajectory was specified to smoothly displace the output shaft of the gearhead from the rest position to the nominal operation position of $\pi/4$.

Figs. 3, 4 and 5 describe the closed-loop responses of the output angular position, electric current, and control voltage, when the system is subjected to the load perturbation torque given by $\tau_L = 2\left(1-e^{-5t}\right)$ Nm. The satisfactory tracking of the reference position trajectory can be observed in Fig. 3 in spite of load perturbation torque. Figs. 6 and 7 show the effective estimation of the velocity and acceleration signals using the differentiation scheme (7).

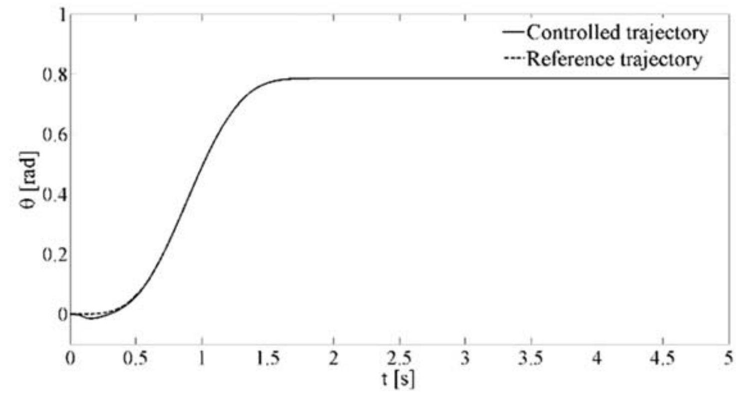

Figure 3. Closed-loop reference position trajectory tracking.

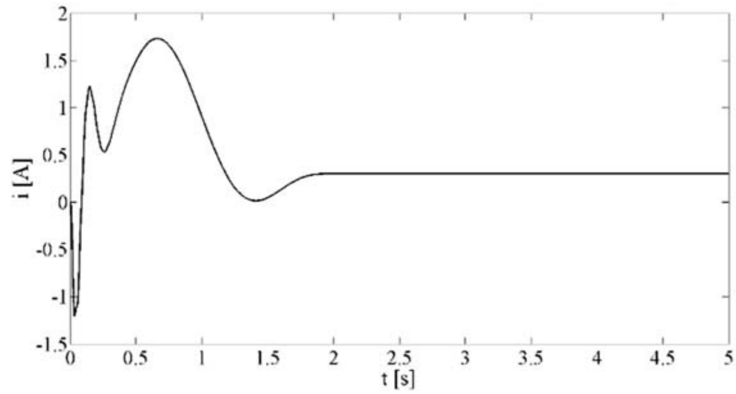

Figure 4. Closed-loop electric current signal.

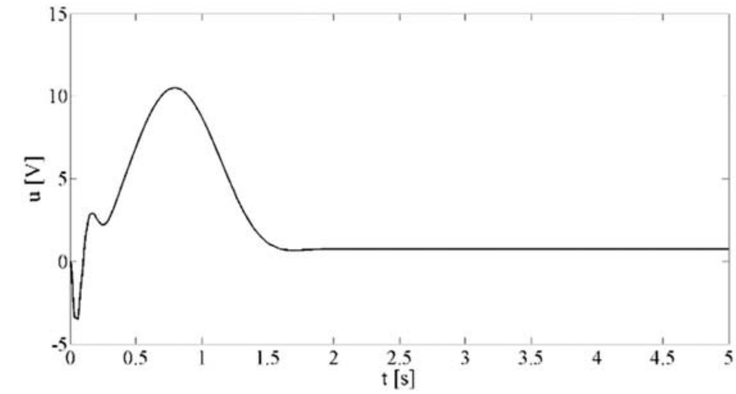

Figure 5. Control voltage applied to motor terminals.

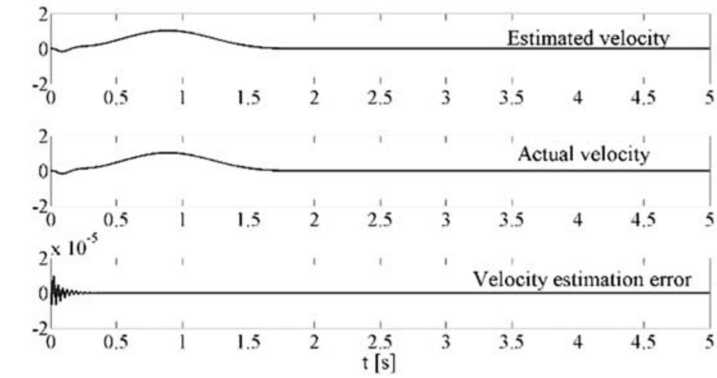

Figure 6. Estimation of the velocity signal.

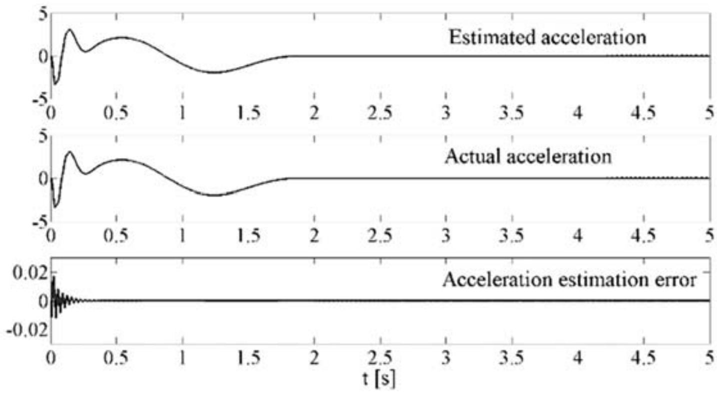

Figure 7. Estimation of the acceleration signal.

5. Conclusions

In this paper, a scheme to get estimates of time derivatives of some signal has been proposed for position tracking control tasks for DC electric motors using measurements of the output variable to be controlled. The main feature of the differentiation approach is that a mathematical model describing the system dynamics is not required. Hence, the real-time signal differentiation algorithm can be easily implemented to other applications of engineering system where unavailable time derivatives are requested. Moreover, the performance of the differentiator was also verified for time-varying output signals generated by an uncertain chaotic nonlinear dynamical system of Lorenz showing satisfactory results. For the synthesis of the signal differentiator, the bounded output signal was locally approximated by a family of forth order Taylor polynomials with unknown coefficients. In addition, the proposed output feedback tracking controller exploits the differential property exhibited by the system. Additional integral compensation of the output error was used in the controller to improve its robustness property against load constant torque perturbations. The computer simulations results manifest a good controller performance using the differentiation of signals. Thence, one can conclude that the tracking control scheme using the presented signal differentiator represents a very good option for position control tasks planned for a DC motor.

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