Optimization of an irreversible Stirling regenerative cycle

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Abstract. In this work a Stirling regenerative cycle with some irreversibilities is analyzed. The analyzed irreversibilities are located at the heat exchangers. They receive a finite amount of heat and heat leakage occurs between both reservoirs. Using this model, power and the efficiency at maximum power are obtained. Some optimal design parameters for the exchanger heat areas and thermal conductances are presented. The relation between the power, efficiency and the results obtained are shown graphically.

1. Introduction

At recent years the efficient use of energy has acquired a greater importance, one of the main points is the reuse of thermal energy wasted at many devices. All the mechanical devices like engines and turbines capable of produce electrical energy and power, waste important quantities of heat. Stirling engines are one of the few devices capable of reusing the wasted energy efficiently. Theoretical models [1-5] have been used to show how a regenerative engine can upgrade the overall efficiency of a thermal system. The engine model considered in this work has a regenerator coupled to improve the use of energy.

The Stirling engines can have several internal configurations, commonly known as alpha, beta and gamma. Throughout the years, these devices have been studied and their optimization at maximum power output has been developed [1]. Other approaches utilize some real design parameters as a non-ideal regenerator or dead volume to deal with its design analysis [2].

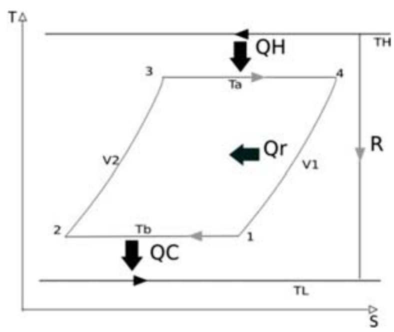


Figure 1. Stirling cycle with irreversibilities due to heat transfer between the hot and cold reservoirs and with ideal regenerator.

Latest developed models have contributed with formal expressions for maximum power output conditions and maximum efficiency. The use of the expressions has contributed to optimize these engines [1-2] and [5-6]. Some of these ideas are developed here and some other irreversibilities are added. The irreversibilites included here are finite heat transfer between the hot and cold reservoirs and a heat leak between both of them. Adding up these design parameters, some useful expressions can be obtained at maximum power output and maximum efficiency. The obtained expressions relate power and efficiency of the engine with some real design parameters like the thermal conductance or the heat exchangers area. Expressions of this type have been done before using Carnot cycle models [3-5] and [7]. This method delivered relevant results including invariant optimal expressions for both heat exchangers [4].

2. Methodology

The modeled cycle in this power comprises two isothermal and two isochoric processes (Figure 1). A heat leak occurs at the engine and this energy cannot be used to perform power. Both heat exchangers can communicate a finite amount of heat. The engine uses an ideal regenerator to reuse his own thermal energy and improve his efficiency. According to the First Law of the Thermodynamics the power can be obtained by the heat flows incoming and leaving the engine:

$$\dot{P} = \dot{Q}_{2-1} + \dot{Q}_{3-2} + \dot{Q}_{4-3} + \dot{Q}_{1-4}, \tag{1}$$

$$\dot{Q}_H = \dot{Q}_{4-3}; \dot{Q}_L = \dot{Q}_{2-1}. \tag{2}$$

The heat flow from the hot heat exchanger is $\dot{Q}_{4-3}$, meanwhile the heat flow leaving the engine from the cold heat exchanger is $\dot{Q}_{4-3}$.

$$\dot{Q}_{1-2} = \beta(T_2 - T_L), \tag{3}$$

$$\dot{Q}_{3-4} = \alpha(T_H - T_1), \tag{4}$$

where α and β are the thermal conductances of the heat and cold heat exchangers. Thermal conductances are the heat transfer coefficient times the area of each heat exchanger [4]:

$$\beta = U_L A_L, \tag{5}$$

$$\alpha = U_H A_H. \tag{6}$$

The heat flow at each isochoric process is:

$$\dot{Q}_{2-3} = mC_V(T_a - T_b), \tag{7}$$

$$\dot{Q}_{4-1} = -mC_V(T_a - T_b). \tag{8}$$

Substituting all the heat flows at equation (1):

$$P = -\beta(T_b - T_L) + \alpha(T_H - T_a) + mC_V(T_a - T_b) - mC_V(T_a - T_b), \tag{9}$$

but, both isochoric processes are equal, then:

$$P = \dot{Q}_H + \dot{Q}_L \, . \tag{10}$$

Because of there is not production of entropy inside the engine,

$$0 = \frac{\dot{Q}_L}{T_L} + \frac{\dot{Q}_H}{T_H} \, . \tag{11}$$

Eliminating $\dot{Q}_L$ and substituting into the equation (10):

$$P = \dot{Q}_H \left(1 - x\right) . \tag{12}$$

If $x = T_b / T_a$, the power can be expressed as:

$$P = \alpha \left(T_H - T_a\right)\left(1 - x\right) , \tag{13}$$

from the equation (11):

$$x = \frac{\dot{Q}_L}{\dot{Q}_H} \, . \tag{14}$$

Rewriting the heat flows at this equation:

$$x = \frac{\beta(T_b - T_L)}{\alpha(T_H - T_a)} , \tag{15}$$

but $T_b = xT_a$. So,

$$T_a = \frac{x\alpha T_H - \beta T_L}{x(\alpha + \beta)} \, . \tag{16}$$

Substituting (16) at (13) equation:

$$P = \alpha\left(T_H - \frac{x\alpha T_H - \beta T_L}{x(\alpha + \beta)} \right) . \tag{17}$$

Rewriting,

$$P = \frac{\alpha^2 \beta T_H \left(1 - \frac{\mu}{x}\right)(1 - \mathrm{x})}{\alpha + \beta} , \tag{18}$$

where $\mu = T_L / T_H$. The efficiency is definite by:

$$\eta = P\left(\dot{Q}_E\right)^{-1}, \tag{19}$$

$$\dot{Q}_E = \dot{Q}_H + \dot{Q}_r + \dot{Q}_f , \tag{20}$$

where $\dot{Q}_r$ is the heat absorbed by the regenerator in the process 4-1. The same amount of heat is delivered in the process 2- 3. $\dot{Q}_f$ is the heat leak between both heat exchangers and Ω is the thermal conductivity per unit area of the regenerator. If the average to obtain the heat at the regenerator is applied:

$$\dot{Q}_r = \frac{\Omega}{2}\left(T_b - T_a\right), \tag{21}$$

and the heat leak,

$$\dot{Q}_f = k\left(T_H - T_L\right). \tag{22}$$

Substituting (2), (4), (21) and (22) in (19), the efficiency is obtained:

$$\eta = \frac{P}{T_a\left(-\alpha + \frac{\Omega}{2}\left(x-1\right)\right) + T_H\left(k\left(1-\mu\right)+\alpha\right)}. \tag{23}$$

If the heat exchangers between side hot is modeled by:

$$\alpha\left(T_H - T_a\right) = \dot{m} T_a C_V \ln\lambda . \tag{24}$$

The mass flow is $\dot{m}$ and the volume ratio is λ . Resolving T_a in (24) and substituting in (23):

$$\eta = \frac{\left(1-\frac{\mu}{x}\right)\left(1-x\right)\frac{\alpha\beta}{\alpha+\beta}}{\frac{\alpha}{g}\left(\Omega\left(\frac{1-x}{x}\right)-\alpha\right)+\left(k\left(1-\mu\right)+\alpha\right)}. \tag{25}$$

where $g = \dot{m} C_v \ln\gamma$. Factorizing α and with $G = 1/g$, finally is obtained:

$$\eta = \frac{\left(1-\frac{\mu}{x}\right)\left(1-x\right)\frac{\beta}{\alpha+\beta}}{G\left(\Omega\left(\frac{1-x}{x}\right)-\alpha\right)+\frac{\left(k\left(1-\mu\right)+\alpha\right)}{\alpha}}. \tag{26}$$

If $\Omega = cte$ and fixing the thermal conductances and areas of each heat exchanger (design rules [4,8]):

$$\alpha + \beta = \gamma, \tag{27}$$

$$A_H + A_L = A. \tag{28}$$

Let $\gamma = UA$, where U is the overall heat transfer coefficient and A the overall area of both heat exchangers. Now, equations (27-28) are parameterized of the following way:

$$\phi_1 = \frac{\alpha}{UA}; 1 - \phi_1 = \frac{\beta}{UA}, \tag{29}$$

$$\phi_2 = \frac{\alpha}{U_H A}; 1 - \phi_2 = \frac{\beta}{U_L A}. \tag{30}$$

Now, optimizing the power (equation (18)) with respect to *x,* ϕ_1 and ϕ_2. The same result for the areas and conductances are obtained as in [4].

$$\phi_1 = \frac{1}{2}, \tag{31}$$

$$\phi_2 = \sqrt{R}\left(1 + \sqrt{R}\right)\left(R - 1\right)^{-1}. \tag{32}$$

Moreover; $x_{opt} = \sqrt{\mu}$ and $R = U_L / U_H$. Substituting these values in (31), (32)

$$P_{\phi_1}^{\max} = \frac{1}{4}\left(1 - \sqrt{\mu}\right)^2, \tag{33}$$

$$P_{\phi_2}^{\max} = R\left(-1 + \sqrt{\mu}\right)^2 \left(-1 + \sqrt{R}\right)^{-2}. \tag{34}$$

Both expressions relate the maximum power at the engine with their heat exchangers. Now, substituting the same values at the efficiency:

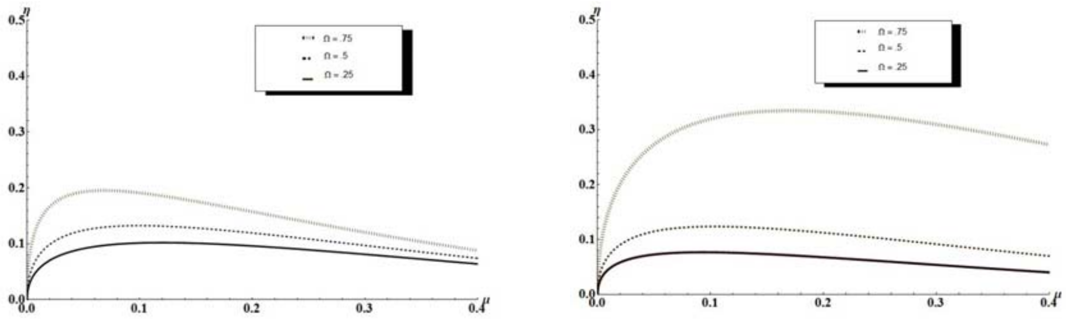
$$\eta_{P_{\phi_1}} = -\frac{\left(-1 + \sqrt{\mu}\right)^2 \sqrt{\mu}}{8k\mu^{3/2} - 4\Omega G + 2\sqrt{\mu}\left(-2 + G - 4K + 2\Omega G\right)}, \tag{35}$$

$$\eta_{P_{\phi_1}} = -\frac{R^{3/2}\left(1 + \sqrt{R}\right)\left(1 - \sqrt{\mu}\right)^2 \sqrt{\mu}}{\left(1 - \sqrt{R}\right)^2 k\left(1 + \sqrt{\mu}\right)^2 \left(1 - \mu\right)\sqrt{\mu} + \sqrt{R}\left(-G\Omega - \sqrt{\mu}\left(-1 + G\Omega\right)\right)}. \tag{36}$$

The equations (35-36) describe the behavior of the efficiency at maximum power output.

3. Results and Conclusions

Plotting efficiency at maximum power, the effects due to the regenerator are appreciated. The model contains more irreversibilities and if the regenerator efficiency can be improve, then the power and the efficiency increase (see figures 2 and 3, with $U_r A_r = C_r \varepsilon_r$; ε is la effectiveness of the regenerator). As consequence the model discussed in [7] is obtained, if $\Omega = 0$.



Figures 2 and 3: To the left is the behavior of the efficiency optimizing the heat area and to the right the behavior of the efficiency optimizing the heat conductances.

Solving equations (27) and (28) using the optimal values found at (31) and 32 can be obtained useful relationships for the heat exchangers areas.

$$A_H = \frac{1}{2} A \, , \tag{37}$$

$$A_L = \frac{1}{2} A \, . \tag{38}$$

Then if the total area is divided by half at every heat exchanger, best results for the performance of the engine are obtained. The power function has optimal values according to the design parameters chosen. Finally, both criteria are independent from the heat transfer law applied. This is relevant because independently of the law of heat transfer the same results can be obtained. Optimize the equation (26) for the efficiency is important to see if the equations (37-38) are satisfied. Further work is underway.

5. References

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