

Using the Finite Elements Method (FEM) for Nanotechnology Education. A rectangular cantilever as a mass sensor

H. Aya Baquero

Facultad de Ingeniería
Universidad Distrital Francisco José de Caldas
Bogotá, Colombia

E-mail: haya@udistrital.edu.co

Abstract. The Finite Element Method FEM can be used in the context of physics engineering education, particularly in nanotechnology training. Cantilevers and cantilevers arrays have been implemented as sensors within lots of applications. In the present paper, FEM was used to assess validity of basic models where cantilevers are used as mass sensors. Resonance frequency of a cantilever transversal vibration was found; this was a silicon one-side clamped cantilever. A number of minor mass elements Δm was added on the cantilever's free side. Then in each case, a new resonance frequency was found; this led to obtain the Δm values from shifts of resonance frequencies. Finally, those values were compared with CAD model values.

1. Introduction

The Finite Element Method (FEM) is a numerical method, intended to solve mathematics, physics and engineering problems (structural analysis, heat transfer, fluids, mass transport, electromagnetism, and more). Nowadays, FEM is applied in several areas such as biomechanics, aeronautics, geology, electronics, and micro technology. Basically, FEM is a method that allows reaching a numerical approximation of differential equations solutions and partial derivatives solutions [1]. It is known that except for special cases, reaching an analytical solution to those equations is impossible, and the only remaining possibility is to get estimated answers. FEM is maybe used as an educational tool in Micro and Nano technologies. Mostly, it allows checking the validity of cantilevers as mass sensors [2,3]. These micromechanical sensors can be operated in both static and dynamic mode. In the static mode, bending of the cantilever occurs due to changes in surface stress. In the dynamic mode, resonance frequency shift occurs due to changes in mass or/and spring constant of the cantilever. In the last decade, researchers have proved its discrimination sensitivity of DNA mismatches [4,5] in static mode. In dynamic mode cantilevers have been used in many kinds of applications such as sensors to detect the mass of a single virus particle [6,7,8]. FEM has been used as an active virtual learning tool that allows students to explore a range of concepts and detect techniques in cantilevers eliminating the use of tests on real devices.

2. Theory

The resonance frequency of a cantilever sensor vibrating in flexural mode in vacuum is stated as [7]



$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}, \quad (1)$$

where m is the cantilever mass and k is the spring constant, which for a rectangular cantilever of length l , thickness d and width w is calculated as

$$k = \frac{E w d^3}{4 l^3}, \quad (2)$$

where E is the Young's modulus.

In contrast, mass changes can be accurately determined by tracking the resonance frequency of the cantilever during mass adsorption or desorption. The cantilever elastic properties remain unchanged during the molecule adsorption/desorption process and damping effects are minor. This operation mode is called dynamic mode. The new resonance frequency may be stated as

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{k}{m+\Delta m}}, \quad (3)$$

where Δm is the mass change due to adsorption.

Mass change on a rectangular cantilever is calculated according to

$$\Delta m = \frac{k}{4\pi^2} \left(\frac{1}{f_1^2} - \frac{1}{f_0^2} \right) \quad (4)$$

Frequency f_1 may be written in terms of f_0 , as follows

$$f_1 = \frac{f_0}{\sqrt{1 + \frac{\Delta m}{m}}} \quad (5)$$

A first-order Taylor series taking into account $f_1 = f_0 + \Delta f$ results in:

$$f_1 = f_0 \left(1 - \frac{\Delta m}{2m} \right), \quad (6)$$

then

$$\sigma = \frac{\Delta m}{\Delta f} = \frac{2m}{f_0}, \quad (7)$$

where the term σ represents mass change sensitivity of cantilever and Δf is the frequency change.

3. Finite Element Method to measure the mass change

A silicon micro cantilever CAD model was created (Young's Module: $E=131$ Gpa, density: $\rho=2330$ Kg/m³) with the following dimensions: length $l=600$ μm , wide: $w=50$ μm and height: $d=18$ μm . See figure 1.

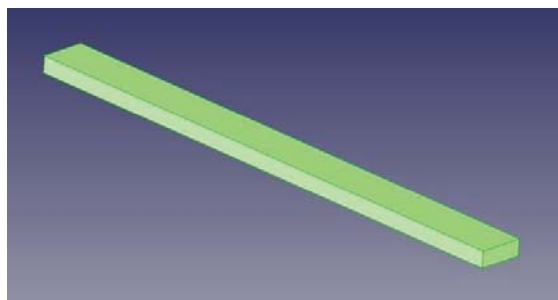


Figure 1. CAD model of silicon micro cantilever with $l=600$ μm , $w=50$ μm and $d=18$ μm

By FEM, resonance frequency f_0 of the first mode of transverse vibration of the micro cantilever clamped at one end was obtained as shown in figure 2. Being the micro cantilever resonant frequency $f_0=60794.47$ Hz and its mass $m=1.258$ μg , then the cantilever mass change sensitivity is $\sigma=0.041$ ng/Hz. A number of CAD models were created; each model has its own Δm value, see figure 3. Then, for each case, a new resonant frequency value f_1 was obtained by FEM and Δm^* values were taken out from (4), and finally, error was evaluated. Results are featured in table 1.

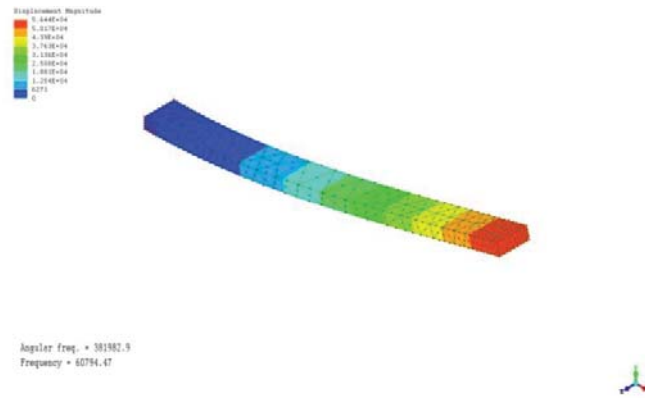


Figure 2. FEM resonance frequency of silicon micro cantilever

Free-software was used to do this work; this means that some limitations appeared mostly in the meshing step. The reason for this limitation was the restriction of node number and elements. However, the data given in table 1 show satisfactory results. Using FEM commercial packages (ANSYS, COMSOL, etc.) better performance micro sensors with greater geometric complexity can be analyzed and designed.

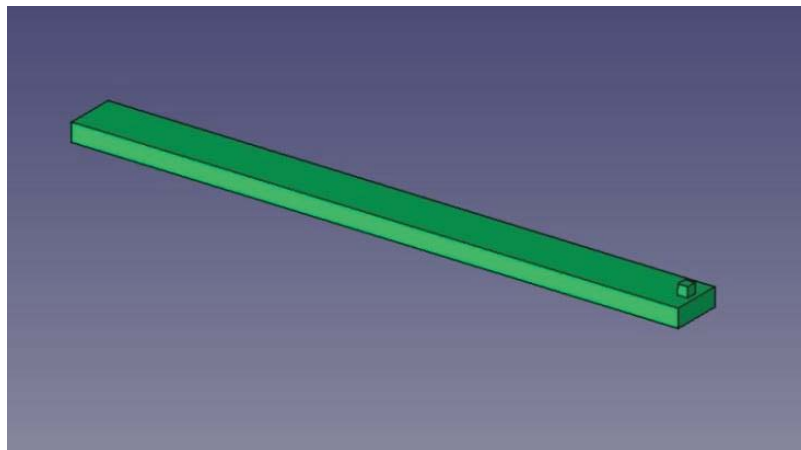


Figure 3. CAD model of micro cantilever with added mass Δm .

Table 1 Results obtained from FEM implementation.

Δm [ng]	f_0 [Hz]	f_1 [Hz]	Δm^* [ng]	$\left \frac{\Delta m^* - \Delta m}{\Delta m} \right 100$
2.097	60794.47	60589.14	2.055	2.03
4.194	60794.47	60388.93	4.079	2.73
6.291	60794.47	60185.49	6.013	2.13

8.388	60794.47	59985.66	7.493	2.02
10.485	60794.47	59789.41	10.262	2.12

Δm^* calculated using FEM and (4).

4. Conclusions

Results show a high coincidence between the theoretical model and the virtual solution to the problem. The comparison of mass values Δm between CAD model and FEM simulations, using the expression (4), shows an error close to 2%. Using FEM in the design and application of MEMS sensors has great potential of future development of new learning strategies of new technologies. 3D CAD modeling FreeCAD 0.13 and FEM LISA 8.0.0 free-software were used. H. A. acknowledges partial financial support from Facultad de Ingeniería at Universidad Distrital through Proyecto Curricular de Ingeniería Electrónica.

5. References

- [1] Aya B H, Cano M R and Zhevandrov B P 2011 *Ingeniería e investigación*, **31** 1 7-15
- [2] Gimzewski J K, Gerber Ch, Meyer E and Schlittler R R 1994 *Chem. Phys. Lett.* **217** 589
- [3] Thundat T, Warmack R J, Chen G Y and Allison D P 1994 *Appl. Phys. Lett.* **64** 2894
- [4] Fritz J, Baller M K, Lang H P, Rothuizen H, Vettiger P, Meyer E, Güntherodt J G, Gerber Ch and Gimzewski J K 2000 *Science* **288** 316
- [5] Hansen K M, Hai-Feng Ji, Wu G, Datar R, Cote R, Majumdar A and Thundat T 2001 *Anal. Chem.* **73** 1567-1571
- [6] Gupta A, Akin D and Bashir R 2004 *Appl. Phys. Lett.* **84** 1976
- [7] Chen G Y, Warmack R J, Thundat T, Allison D P and Huang A 1994 *Rev. Sci. Instrum.* **65** 2532
- [8] Subhashini S and Vimala Juliet A 2013 *Computer Networks & Communications (NetCom). Lecture Notes in Electrical Engineering* **131** 75-80.