First-order irreversible thermodynamic approach to a non-steady RLC circuit as an energy converter

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Abstract. In this work we show a RLC-circuit as energy converter within the context of first-order irreversible thermodynamics (FOIT). For our analysis, we propose an isothermic model with transient elements and passive elements. With the help of the dynamic equations, the Kirchhoff equations, we found the generalized fluxes and forces of the circuit, the equation system shows symmetry of the cross terms, this property is characteristic of the steady state linear systems, but in this case phenomenological coefficients are function of time. Then, we can use these relations, similar to the linear Onsager relations, to construct the characteristic functions of the RLC energy converter: the power output, efficiency, dissipation and ecological function, and study its energetic performance. The study of performance of the converter is based on two parameters, the coupling parameter and the "forces ratio" parameter, in this case as functions of time. We find that the behavior of the non-steady state converter is similar to the behavior of steady state energy converter. We will explain the linear and symmetric behavior of the converter in the frequencies space rather than in the time space. Finally, we establish optimal operation regimes of economic degree of coupling for this energy converter.

1. Introduction

It is well-known that in actual dissipative heat engines, the maximum power output and maximum efficiency points do not coincide and their separation can be managed by some phenomenological parameter which depend on the engine's design and the material employed in its construction [1,2].

In 1931 [3], Lars Onsager published his linear theory of irreversible processes, also known as first-order irreversible thermodynamics, this theory is able to explain various phenomena that occur out of equilibrium within a framework of flows, forces and generalized coefficients. Onsager relations are symmetric with regard to the cross coefficients corresponding to the contributions of generalized forces to generalized flows. The most basic version of this converter posses two generalized fluxes and two forces in steady state, besides it is in contact with an ideal bath at fixed temperature. The converter can operate in several working regimes, so we can study the energetic performance of the converter.

This paper is organized as follows: in section 2 we obtain the relation between fluxes and forces, as in the Onsager theory, by using the Kirchhoff laws [4]. In section 3, we will construct the characteristic functions and will make the energetic performance analysis of the electromagnetic converter. Finally, we write some conclusions about of our results.

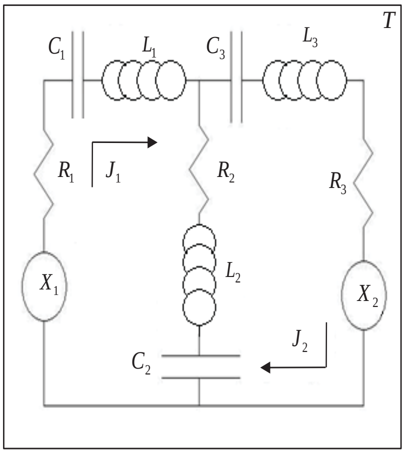

Figure 1. RLC-circuit of two meshes as non-steady state electromagnetic energy converter.

2. FOIT fundamentals

In this section, we first review the assumptions from which the linear Onsager relations for a power converter are built. Next, we will apply the Kirchhoff law of meshes to the RLC-circuit, in order to find the mesh currents depending on the resistances, capacitances, inductances and voltages of direct current sources of the circuit. With these equations we will try to write a kind of Onsager relations, identifying the generalized forces and the phenomenological coefficients of the system.

2.1. Constitutive equations

If we consider the simplest version of an energy converter, let J_1 and J_2 be two coupled generalized fluxes (J_1 being the driven flux and J_2 the driver flux); X_1 and X_2 are the conjugate generalized potentials associated with the fluxes. For the linear case, fluxes and potentials are given by the equation (1):

$$J_1 = \sqrt{L_{11}}\left(\sqrt{L_{11}}X_1 + q\sqrt{L_{22}}X_2\right) \tag{1}$$

$$J_2 = \sqrt{L_{22}}\left(q\sqrt{L_{11}}X_1 + \sqrt{L_{22}}X_2\right) \tag{2}$$

with $L_{ij} = L_{ji}$, the Onsager symmetry relation between crossed coefficients. In these equations we use the coupling coefficient defined by

$$q(t)^2 = \frac{L_{12}^2(t)}{L_{11}(t)L_{22}(t)} \geq 0\,, \tag{3}$$

This parameter gives us a measure of the coupling of spontaneous and nonspontaneous fluxes [5]. On the other hand, we can define a parameter of operation in terms of the forces, as follows

$$x(t) = \sqrt{\frac{L_{11}(t)}{L_{22}(t)}}\,\frac{X_1}{X_2} = \sqrt{\frac{L_{11}(t)}{L_{22}(t)}}\,x'\,, \tag{4}$$

where x' is the ratio between the forces (where L_{ij} are the FOIT-phenomenological coefficients).

2.2. RLC circuit as energy converter

In the RLC-circuit (see figure. 1) there are two fluxes and two forces, we denote by J_1 and J_2 the coupled fluxes (currents), as well as X_1 and X_2 the forces (voltages). To resolve this electric circuit we use the Kirchhoff laws [4]; if we apply the corresponding Kirchhoff equation to the mesh 1 (figure.1), we obtain the following

$$X_1 = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int J_1 dt + (L_1 + L_2)\frac{dJ_1}{dt} + (R_1 + R_2)J_1 - \frac{1}{C_2}\int J_2 dt - L_2\frac{dJ_2}{dt} - R_2 J_2, \tag{5}$$

while for the mesh 2 the equation is

$$X_2 = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)\int J_2 dt + (L_2 + L_3)\frac{dJ_2}{dt} + (R_2 + R_3)J_1 - \frac{1}{C_2}\int J_1 dt - L_2\frac{dJ_1}{dt} - R_2 J_1. \tag{6}$$

It should be noted that (5) and (6) are integral equations, then if we apply a linear transformation such that these equations are sent to the frequency space (Laplace transformation). The new equations system can be resolved easily, if we consider constant in time voltage sources in time, the new equations have the following algebraic form:

$$\frac{X_1}{s} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{I_1(s)}{s} + s(L_1 + L_2)I_1(s) + \frac{q_o}{sC_1} + (R_1 + R_2)I_1(s) - \frac{I_2(s)}{sC_2} - sL_2 I_2(s) - R_2 I_2(s), \tag{7}$$

and

$$\frac{X_2}{s} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)\frac{I_2(s)}{s} + s(L_2 + L_3)I_2(s) + \frac{q_o}{sC_3} + (R_2 + R_3)I_2(s) - \frac{I_1(s)}{sC_2} - sL_2 I_1(s) - R_2 I_1(s), \tag{8}$$

where R_1, R_2 and R_3 are the resistive elements, C_1, C_2 and C_3 are the capacitive elements, L_1, L_2 and L_3 are the inductive elements and q_0 is the initial charge of the capacitors. We note that (7) and (8) can be inverted, such that the fluxes $I_1(s)$ and $I_2(s)$ are expressed in terms of the forces X_1 and X_2. In order to obtain the fluxes J_1 and J_2 in the real solutions space, to invert the Laplace transformations shown in (7) y (8). In (9) the parameters $a_k = a_k(R_1, R_2, R_3, L_1, L_2, L_3, C_1, C_2, C_3)$ are the roots of Q(s). In this case the fluxes can be expressed by using a matrix as follows:

$$\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \begin{pmatrix} (L_2 + L_3)\displaystyle\sum_{k=1}^{2}\frac{\left[a_k^2 + \frac{R_2 + R_3}{L_2 + L_3}a_k + \frac{1}{L_2 + L_3}\left(\frac{1}{C_2} + \frac{1}{C_3}\right)\right]e^{a_k t}}{Q'(a_k)} & L_2\displaystyle\sum_{k=1}^{2}\frac{\left(a_k^2 + \frac{R_2}{L_2}a_k + \frac{1}{L_2 C_2}\right)e^{a_k t}}{Q'(a_k)} \\ L_2\displaystyle\sum_{k=1}^{2}\frac{\left(a_k^2 + \frac{R_2}{L_2}a_k + \frac{1}{L_2 C_2}\right)e^{a_k t}}{Q'(a_k)} & (L_1 + L_2)\displaystyle\sum_{k=1}^{2}\frac{\left[a_k^2 + \frac{R_1 + R_2}{L_1 + L_2}a_k + \frac{1}{L_1 + L_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\right]e^{a_k t}}{Q'(a_k)} \end{pmatrix} \begin{pmatrix} X_1 - \frac{q_o}{C_1} \\ X_2 - \frac{q_o}{C_3} \end{pmatrix}, \tag{9}$$

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with

$$
\begin{aligned}
Q'(a_k) = {} & 4\left[L_1(L_2+L_3)+L_2L_3\right]a_k^3+3\left[R_1(L_2+L_3)+L_1(R_2+R_3)+L_3R_2+L_2R_3\right]a_k^2 \\
& +2\left[\frac{(L_2+L_3)}{C_1}+L_1\left(\frac{1}{C_1}+\frac{1}{C_2}\right)+R_1(R_2+R_3)+R_2R_3+\frac{L_3}{C_2}+\frac{L_2}{C_3}\right]a_k \\
& +\left[R_1\left(\frac{1}{C_1}+\frac{1}{C_2}\right)+\frac{R_2+R_3}{C_1}+\frac{R_2}{C_3}+\frac{R_3}{C_2}\right].
\end{aligned} \tag{10}
$$

From the matrix (9) we can see that the Onsager relations for the RLC energy converter also exhibit symmetry of the coefficients associated with cross-flow contributions, if the generalized forces are taken as

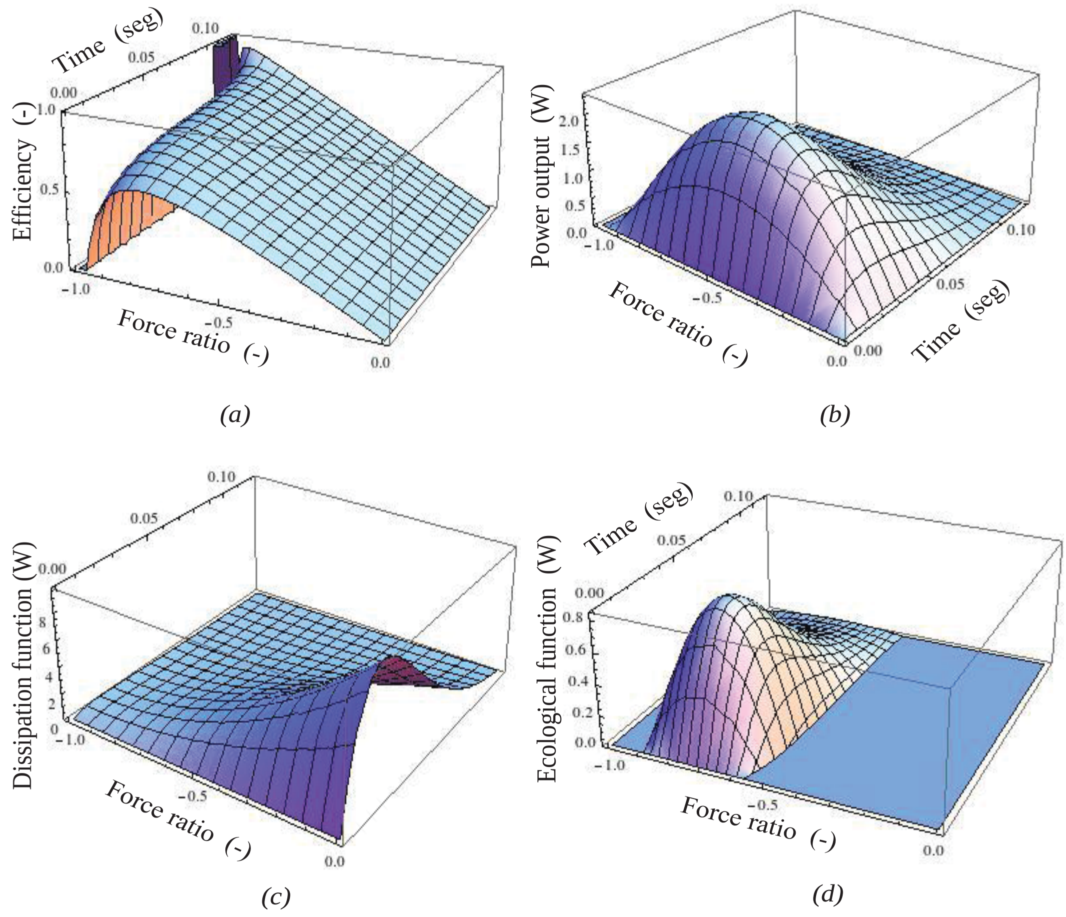

Figure 2. Characteristic functions in terms of the forces ratio and the time, for a given coupling parameter, a) efficiency and b) power output. Also c) dissipation function in terms of the forces ratio and the time, for a fixed coupling parameter and d) ecological function.

3. Performance of the RLC-converter

The energy analysis of the converter can start associating the summands of the entropy production with input power and output power. First, we construct the dissipation function multiplying the entropy production by the temperature T of the converter, which is constant. The dissipation function for an isothermal system can be expressed as [6]

$$\Phi(t) = T\sigma(t) = TJ_1(t)X_1 + TJ_2(t)X_2 = \left[(x')^2 L_{11}(t) + 2x'L_{12}(t) + L_{22}(t)\right]X_2. \tag{11}$$

From above equation the power output of the RLC-converter is given by

$$P(t) = -TJ_1(t)X_1 = -T\left[(x')^2 L_{11}(t) + x'L_{12}(t)\right]X_2^2, \tag{12}$$

this function measures the energy interchange between the system and the surroundings.

We can define also the efficiency following Caplan and Essig, the converter using the coupling parameter and the forces ratio we have

$$\eta(t) = \frac{power\ output}{power\ input} = -\frac{TJ_1(t)X_1}{TJ_2(t)X_2} = \frac{-x(t)\left[q(t)+x(t)\right]}{x(t)q(t)+1} = -\frac{(x')^2 L_{11}(t) + x'L_{12}(t)}{x'L_{12}(t) + L_{22}(t)}. \tag{13}$$

This function shows the influence of parameter design and operating parameter in the energy behavior of the converter.

Some authors [7] have found that the graphic output power versus efficiency of real machines draw loops, these loops have two conspicuous points, a maximum power output and a maximum efficiency, between these two operating points can be found a region. The most commonly used objective function to find the operating points that are in the region of compromise is called ecological function, whose definition we will write down, [8]

$$Ec(t) = P(t) - \Phi(t) = -T\left[2(x')^2 L_{11}(t) + 3x'L_{12}(t) + L_{22}(t)\right]X_2^2. \tag{14}$$

In Fig. 2 we show the time evolution of these two parameters, considering the conventional resistances and capacitances values reported in the literature about electric circuits; in this case we take the followings values: $R_1=10\Omega$, $R_2=680\Omega$, $R_3=12\Omega$, $L_1=0.2H$, $L_2=4H$, $L_3=0.1H$ $C_1=3\mu F$, $C_2=500\mu F$, $C_3=5\mu F$. From these values, we can see that the graphs in Fig. 2 show the temporal behavior well known, and the same behavior as for steady states.

In 1980 Stucki [9] proposed a thermodynamic model for the oxidative production of ATP in the mitochondria. He found that this process could be carried out under different operating modes, among them those that correspond to maximum efficiency so as to some other condition of optimization, he called this modes economic degrees of coupling. We can find the operation modes that correspond to economic degree of coupling for the RLC-converter.

4. Conclusions

This RLC circuit was taken as an example, because it is one of the simplest energy converters in its construction and exhibits dependence over time. We can see the same symmetry of the cross coefficients as in the steady state, perhaps the Curie theorem could be valid in the space of frequencies.

On the other hand, the converter time evolution is dominated for the coupling parameter, and its behavior depends of the transient elements. With these considerations, we can minimize the entropy production in electric power generating plants or reduce power dissipation in transmission networks.

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