

# Experimental First Order Pairing Phase Transition in Atomic Nuclei

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**Abstract.** The natural log of experimental nuclear level densities at low energy is linear with energy. This can be interpreted in terms of a nearly 1st order phase transition from a superfluid to an ideal gas of quasi particles. The transition temperature coincides with the BCS critical temperature and yields gap parameters in good agreement with the values extracted from even-odd mass differences from rotational states. This converging evidence supports the relevance of the BCS theory to atomic nuclei.

## 1. Introduction

For conventional superconductors, the standard Bardeen-Cooper-Schrieffer (BCS) theory [1] predicts a critical temperature/angular momentum at which the superconducting phase reverts to the normal one through a second order phase transition. However, for atomic nuclei, this second-order phase transition has never been truly verified experimentally, in spite of long and intense efforts. On the other hand, first-order phase transitions can also arise from the BCS Hamiltonian, as demonstrated in reference. [2]. In the present paper, we show that a first-order, rather than a second-order, phase transition is dramatically evident in experimental nuclear level densities below neutron threshold, and that this first-order transition is indisputably related to the presence of an energy gap in the quasiparticle spectrum. A large body of high-quality nuclear level-density data exemplified in figure 1. are now available in the literature [3–5]. The stunning, common feature of the level densities, particularly evident for deformed, mid shell nuclei, is the linear dependence of their logarithm with excitation energy. Above  $\approx 2\Delta$ , where  $\Delta$  is the pair-gap parameter, and up to about the neutron separation energy, they are well described by the constant temperature expression proposed by Gilbert and Cameron [6]:

$$\rho(E) \approx \exp\left(\frac{E}{T}\right) \quad (1)$$

where  $E$  is the excitation energy and  $T$  is the constant nuclear temperature. They found this expression to be in good agreement with the cumulative number of levels at low excitation energy, but did not provide any fundamental, quantitative explanation for this relation.

## 2. The Phase Transition

This experimental linear dependence of the entropy  $S(E) \approx \ln \rho(E)$  as given by equation (1) and



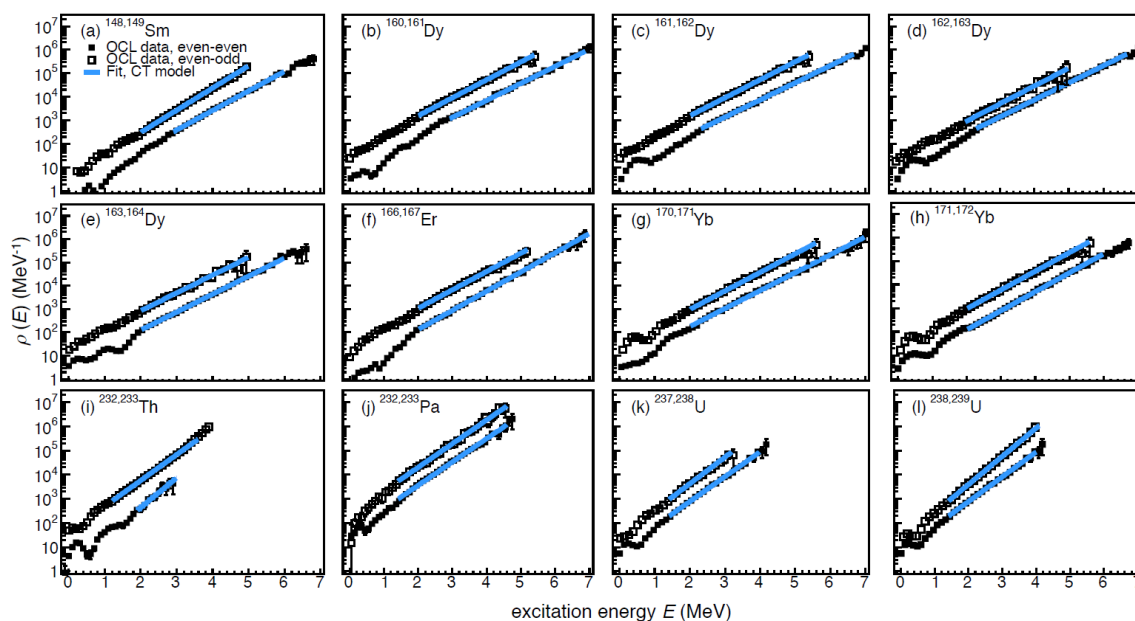
shown in figure 1 is the micro-canonical hallmark of first-order phase transitions. Surprisingly, we may have been staring at the biggest signal yet of such a transition without seeing it. This transition is, at least for nuclei well away from closed shells, clearly related to pairing. If we, provisionally, take the constant temperature of the experimental level density spectrum to be the BCS critical temperature, then, according to the well-known BCS relation

$$T_{cr} = \frac{2\Delta_0}{3.53} \quad (2)$$

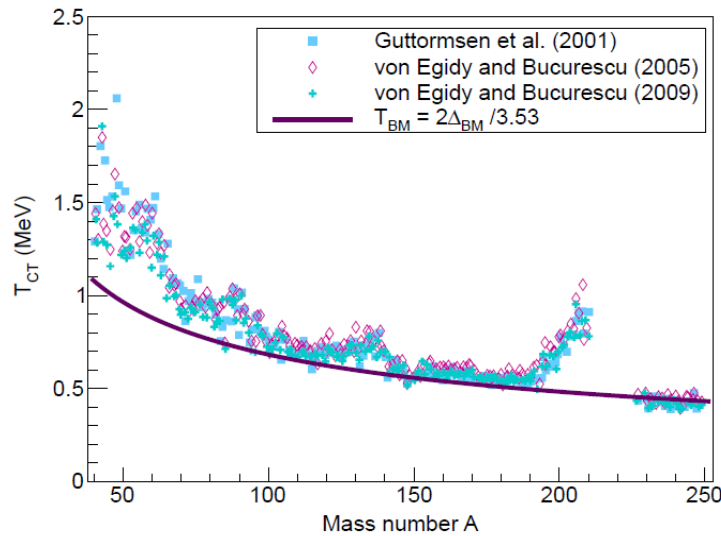
we can extract the gap parameter  $\Delta_0$  and compare it directly with that obtained from even-odd mass differences represented in the liquid-drop term as described e.g. by Bohr and Mottelson [7], and viceversa:

$$\Delta_{BM} \approx 12A^{-1/2}. \quad (3)$$

For a wide range of mass number  $A$ , the resulting relationship between mass number and temperature using equation (2) is shown in figure. 2, where the experimental constant temperatures  $T_{CT}$  are taken from references. [8–10]. The close agreement in magnitude and trend is remarkable for  $A > 100$  and away from closed shells, although the assimilation of the constant level density temperature characteristic of a first-order transition to a critical temperature associated with a second-order transition remains to be explained. As a consequence of this observation, given the even-odd mass difference, we can predict the low-energy nuclear level densities throughout the nuclear chart for regions away from magic proton/neutron numbers. Before we embark on the explanation of this remarkable feature, let us consider another striking experimental observation: the level densities of neighbouring even-even and odd- $A$  nuclei have nearly identical slopes, as seen in figure 1 showing data from the rare-earth region [11–15], and several actinides [16–17]. Therefore, the level densities of neighbouring isotopes can be made to overlap by means of a horizontal shift along the excitation-energy axis reference [18].



**Figure 1.** (Color online) Experimental level densities for rare-earth and actinide nuclei measured at the Oslo Cyclotron Laboratory (OCL), with a fit of the constant-temperature model (blue line) for excitation energies above  $\approx 2$  MeV.



**Figure 2.** (Color online) Relation between mass number  $A$  and constant temperature  $T_{CT}$  taken from references [8–10] for  $A = 40$ –250, compared to  $T_{cr}$  from equation (3) utilizing equation (2) (dark purple line).

The resulting shift is, not surprisingly, in very good agreement with the even-odd mass difference; see table 1. As a consequence, locally, for any given pair of even-even and odd- $A$  nuclei, we can calculate the common slope of the two level densities directly from the observed excitation-energy shift. Equally intriguing is the vertical shift between the even-even-odd- $A$  nuclear level densities, bringing the lower even-even level density on top of the higher odd- $A$  one (see figure 1). This difference in entropy, approximately constant throughout the energy range  $[2\Delta; S_n - 1 \text{ MeV}]$ , can be interpreted as the entropy carried by the extra quasiparticle. The experimental evidence thus suggests that as the system is excited; quasiparticles are created with a constant energy cost and carrying a constant amount of entropy, see table 1. This is a clear signature of a first-order phase transition, from a superfluid phase to an ideal gas of quasiparticles.

Nuclide	$T_{BM}$ (MeV)	$T_{CT}$ (MeV)	$T_{eo}$ (MeV)	$\Delta_{BM}$ (MeV)	$\Delta_{CT}$ (MeV)	$\Delta_{eo}$ (MeV)	$\Delta S$ ( $k_B$ )
$^{148}\text{Sm}$	0.56	0.51(1)	—	0.99	0.90(2)	—	—
$^{149}\text{Sm}$	0.56	0.46(1)	0.51(6)	0.98	0.81(2)	0.9(1)	2.0(2)
$^{160}\text{Dy}$	0.54	0.60(1)	—	0.95	1.05(1)	—	—
$^{161}\text{Dy}$	0.54	0.58(2)	0.57(6)	0.95	1.01(3)	1.0(1)	1.9(2)
$^{162}\text{Dy}$	0.54	0.60(1)	—	0.94	1.05(2)	—	—
$^{163}\text{Dy}$	0.53	0.57(2)	0.51(6)	0.94	1.00(4)	0.9(1)	1.9(2)
$^{164}\text{Dy}$	0.53	0.56(1)	—	0.94	0.99(1)	—	—
$^{166}\text{Er}$	0.53	0.52(1)	—	0.93	0.92(1)	—	—
$^{167}\text{Er}$	0.53	0.56(2)	0.51(6)	0.93	0.99(4)	0.9(1)	2.0(2)
$^{170}\text{Yb}$	0.52	0.57(1)	—	0.92	1.00(1)	—	—
$^{171}\text{Yb}$	0.52	0.55(1)	0.51(6)	0.92	0.96(1)	0.9(1)	1.8(2)
$^{172}\text{Yb}$	0.52	0.54(1)	—	0.91	0.95(1)	—	—
$^{231}\text{Th}$	0.45	0.41(1)	0.51(11)	0.79	0.72(2)	0.9(2)	2.4(4)
$^{232}\text{Th}$	0.45	0.34(1)	—	0.79	0.60(2)	—	—
$^{233}\text{Th}$	0.45	0.40(2)	0.51(11)	0.79	0.70(2)	0.9(2)	2.3(4)
$^{232}\text{Pa}$	0.45	0.44(1)	0.40(6)	0.79	0.77(2)	0.7(1)	1.7(2)
$^{233}\text{Pa}$	0.45	0.45(1)	—	0.79	0.79(2)	—	—
$^{237}\text{U}$	0.44	0.40(1)	0.40(6)	0.78	0.70(2)	0.7(1)	1.9(2)
$^{238}\text{U}$	0.44	0.42(1)	—	0.78	0.74(2)	—	—
$^{239}\text{U}$	0.44	0.37(1)	0.37(3)	0.78	0.65(1)	0.65(5)	2.5(5)

**Table 1.** Extracted temperatures  $T_{CT}$  from fitting the CT-model expression to the level-density data of rare-earth and actinide nuclei, and the corresponding pair-gap parameters  $\Delta_{CT}$  calculated from equation (2). These are compared to the global formula for  $\Delta_{BM}$  [equation (3)], for which the temperature is deduced using equation (2). Also, the even-odd experimental shift,  $\Delta_{eo}$  is given, and the corresponding temperature  $T_{eo}$  is estimated from this shift for the odd nucleus. The experimental entropy excess  $\Delta S$  is also given.

### 3. Level Density with Pairing

The presence of pairing in nuclei away from closed shells dominates the low energy level density and its energy dependence. For the uniform model in the case of an even-even nucleus, the ground state is shifted downward by an amount  $E_{cond.} = \frac{1}{2}g\Delta_0^2$ , where  $\Delta_0$  is the ground state gap parameter and  $g$  is the doubly degenerate single particle level density, which is related to the single particle level density parameter  $a$  according to  $a = \frac{\pi^2}{3}g$ . As the temperature increases, quasi particle excitations are produced until their blocking effect leads to a decrease and eventual breakdown of the pairing correlation at the critical temperature  $T_{cr} = \frac{2\Delta_0}{3.53}$ .

At this temperature the nucleus reverts to a non-interacting Fermi gas with its ground state shifted by an amount equal to the condensation energy. Therefore, at  $T_{cr}$  the excitation energy is

$$E_{cr} = \frac{1}{2}g\Delta_0^2 + \frac{\pi^2}{3}gT_{cr}^2 = \frac{1}{2}g\Delta_0^2 \left(1 + \frac{8}{3} \frac{\pi^2}{(3.53)^2}\right). \quad (4)$$

We can also evaluate the mean number of quasi particles  $Q_{cr}$  at the critical point [2]

$$Q_{cr} = 4gT_{cr} \ln 2.$$

We can now calculate the mean energy cost per quasi particle:

$$\frac{E_{cr}}{Q_{cr}} \cong \frac{3.53\pi}{16 \ln 2} \Delta_0 = \Delta_0. \quad (5)$$

This result is remarkable: it indicates that, if we consider the excitation energy as the independent variable, the energy cost per quasi particle is constant as expected for 1<sup>st</sup> order transition. This is in contrast with what one observes when the temperature is used as the independent variable, when a distinct 2<sup>nd</sup> order phase transition is visible. In the same spirit, we note that the heat capacity increases nearly exponentially with temperature up to  $T_{cr}$ . This means that most of the energy is absorbed near  $T_{cr}$ . From the constant energy cost  $\Delta$  per quasiparticle, it follows that the entropy per quasiparticle is

$$\frac{\delta S}{\delta Q} = \frac{\Delta_0}{T_{cr}} = \frac{3.53}{2} = 1.77 \quad (6)$$

to be compared with the empirical, vertical shift as discussed above (see table 1). To summarize, if we use the energy rather than the temperature as the independent variable, we observe the progressive creation of quasiparticles, in number proportional to the energy, like the amount of ice melted is proportional to the absorbed heat, independent of the amount of previously melted ice. This independence, together with the constant entropy per quasiparticle, gives clear evidence of a first-order phase transition.

We can also calculate the entropy at the critical point:  $S_{cr} = 2 \frac{\pi^2}{3}g T_{cr}$  and the overall entropy per quasi particle:

$$\frac{S_{cr}}{Q_{cr}} = \frac{\pi^2}{6 \ln 2} = 2.374. \quad (7)$$

Correcting for the BCS discontinuity in the specific heat of a factor 2.43

$$\frac{S'_{cr}}{Q_{cr}} = 2.374 - \frac{1}{2} \ln 2.43 = 1.92 \quad (8)$$

Alternatively

$$\frac{S'_{cr}}{Q_{cr}} = \frac{E/Q}{T_{cr}} = \frac{3.54\Delta}{2\Delta} = 1.77 \quad (9)$$

in excellent agreement.

The number of the states associated with each quasi particle is then approximately constant and given

by  $N = e^{\frac{S'}{Q}} = 6.8$ .

Given that this transition is “nearly” 1<sup>st</sup> order, we infer that the level density should be “nearly” exponential:  $\rho(E) \approx \exp \frac{E}{T}$  where  $T$  should be “about”  $T_{cr}$ .

#### 4. Even-Odd effects in level densities

In the pairing picture, an odd nucleus possesses one quasi particle in its ground state, which should control the level density at low energy. Otherwise, the odd- $A$  nucleus should look like an even-even one except for an energy shift which should correspond to the even-odd mass difference  $\Delta$ . A simple check for this is to verify that the level densities of two adjacent nuclei overlap if a horizontal shift  $\Delta$  is applied to the odd nucleus. According to the considerations made above, this shift  $\Delta$  can be related to the level density slope by the expression  $T_{exp} \cong T_{cr} = \frac{2\Delta}{3.53}$ .

The next check can be made by overlapping the two level densities by means of a vertical shift. This vertical shift  $\Delta S$  should be compared with the entropy per quasi particle  $\frac{S'}{Q} \cong 1.77$ .

These checks are done in table 1 where a substantial consistency is observed. However, somewhat unexpectedly, the linearity of  $\ln Q$  with  $E$  is also observed at low energy near the magic regions, so for these regions the cause of the linear behaviour must be looked for elsewhere.

#### 5. Spectra with any gap

As discussed above, the origin of the linear dependence is due to the constant energy cost for the production of a quasi particle and a constant entropy per quasi particle. A similar situation occurs for a magic system with a gap in the single particle spectrum. Here, the cost to promote a nucleon and thus create a quasi particle (particle, hole excitation) is constant, at least for a while. This can be illustrated with a simple model.

Let excitations (quasi particles) be created into a state of degeneracy  $N$  at the cost  $\delta$  per excitation. The excitation energy is:  $E = n\delta$  and the associated number of states  $\Omega$  is  $\Omega \cong N^n$  for  $n \ll N$ . It follows that  $S = n \ln N = \frac{E}{\delta} \ln N = \frac{E}{T}$  where  $T = \frac{\delta}{\ln N}$  and  $\rho(E) = \exp \left( \frac{E}{T} \right)$ .

Thus an exponential spectrum is expected if a gap is present irrespective of its origin.

#### 6. Consistency between “Pairing gap” and “any gap”

The entropy per quasi particle in the pairing model is:  $\frac{S_{cr}}{Q_{cr}} = \frac{\Delta}{T_{cr}} = 1.77$

For the “any gap” model we have:  $\frac{\delta S}{\delta n} = \ln N$

Let us put pairing  $\Delta$  into  $\delta$  and equate  $T$  with  $T_{cr}$ :  $T = \frac{\Delta}{\ln N} = \frac{2\Delta}{3.53}$

From this we obtain:  $\ln N = 1.77$  in exact agreement with equation (6), and in good agreement with the average, experimental values from table 1 giving  $\Delta S_{ave} = 2.0(1)$  Further, the number of available states per quasiparticle is  $\exp(\Delta S) = \exp(1.77) \approx 6$ , again agreeing well with the experimental  $\Delta S$  values in table 1,  $\exp(\Delta S_{ave}) = 8(1)$ .

#### 7. Conclusion

In conclusion, we have shown that the low-energy level densities (below the neutron separation energy) portray a strong signature of a first-order phase transition, completely consistent with the BCS framework. The coexistence of a superfluid with a vapor of quasiparticles is easily characterized

thermodynamically, especially through the comparison of even-even and odd-A nuclei. In particular, it is shown that the even-odd mass difference is sufficient to determine the level density in absolute

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