

# Triangular $\mathcal{D}_{3h}$ Symmetry in the Rotation-Vibration Spectrum of $^{12}\text{C}$

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**Abstract.** Our recent measurements of new states in  $^{12}\text{C}$  including the second  $2^+$  at 10 MeV and the high spin  $5^-$  state at 22.4 MeV allow us to study the Rotation-Vibration spectrum of  $^{12}\text{C}$  from which evidence for a new ( $\mathcal{D}_{3h}$ ) geometrical symmetry emerges. The data fit very well to the predicted (ground state) rotational band of an oblate equilateral triangular spinning top with a  $\mathcal{D}_{3h}$  symmetry characterized by the sequence of states:  $0^+, 2^+, 3^-, 4^\pm, 5^-$  with almost degenerate  $4^+$  and  $4^-$  (parity doublet) states. Such a  $\mathcal{D}_{3h}$  symmetry was observed in triatomic molecules, and it is observed in  $^{12}\text{C}$  for the first time in nuclear physics. The triatomic like structure in nuclei is reminiscent of the discovery of diatomic  $\alpha + ^{14}\text{C}$  structure in  $^{18}\text{O}$ . We discuss a classification of other rotation-vibration bands in  $^{12}\text{C}$  such as the ( $0^+$ ) Hoyle band and the ( $1^-$ ) bending mode band and suggest measurements in search of the predicted (“missing”) states that may shed new light on clustering in  $^{12}\text{C}$  and light nuclei. In particular, the observation (or non observation) of the predicted (“missing”) states in the Hoyle band will allow us to conclude the geometrical arrangement of the three alpha particles composing the Hoyle state at 7.654 MeV in  $^{12}\text{C}$ .

## 1. Introduction

Molecular states are characterized by the separation vector connecting the constituent objects. For diatomic like object one single (separation) vector is required leading to the predicted U(4) symmetry [1] that was observed in  $^{18}\text{O}$  via the characteristic enhanced E1 decays [2]. Triatomic symmetric spinning tops are characterized by the two Jacobbi vectors shown in figure 1 leading to the predicted U(7) with the geometrical  $\mathcal{D}_{3h}$  symmetry [3, 4].

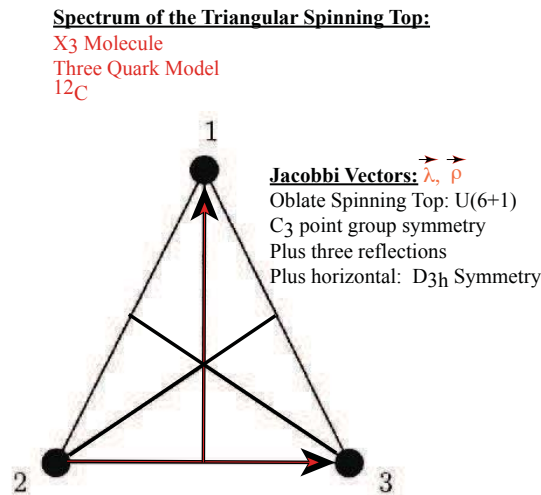
The recent observation of the  $2^+$  Hoyle rotational excitation in  $^{12}\text{C}$  [5] allows for the first time the study of the Rotation-Vibration Structure of  $^{12}\text{C}$ . These new data and the recently discovered  $4^-$  [6, 7] and  $4^+$  [8] states in  $^{12}\text{C}$  are in agreement with the predicted spectrum of an oblate spinning top with a  $\mathcal{D}_{3h}$  symmetry [3, 4]. It was predicted [3, 4] that the three alpha-particle system of  $^{12}\text{C}$  leads to the ground state rotational band including the most unusual sequence of states:  $0^+, 2^+, 3^-, 4^\pm, 5^-$ . The new high spin  $5^-$  state [9] as well the previously published  $4^-$  state [6, 7] lead to a  $J(J+1)$  trajectory as predicted by this U(7) model [9, 10] including the nearly degenerated  $4^-$  and  $4^+$  states as shown in figure 2.

## 2. The Algebraic Cluster Model

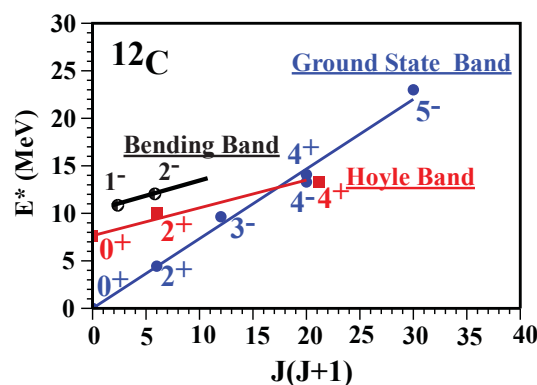
The spectrum of  $^{12}\text{C}$  predicted by the Algebraic Cluster Model (ACM) [3, 4] is shown in figure 3 where it is also compared to the measured spectrum of  $^{12}\text{C}$  [9]. In addition to the ground state rotational band this U(7) model [3, 4] predicts the Hoyle state at 7.65 MeV in  $^{12}\text{C}$  to be the first vibrational breathing mode of the three alpha-particle equilateral configuration leading to



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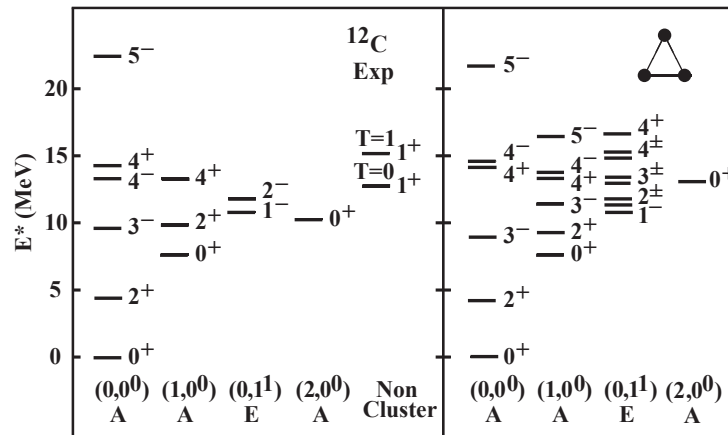
**Figure 1.** The Jacobbi vectors characterizing the triatomic symmetric spinning tops



**Figure 2.** Rotational band structure of the ground-state band, the Hoyle band and the bending vibration band in  $^{12}\text{C}$ . The spin-parity of the ground-state band are indicated (in blue), the Hoyle band (in red) and the bending mode band (in black). For each band the straight line fit to a  $J(J+1)$  curve is shown. Note the predicted "missing"  $3^-$  and  $4^-$  states in the Hoyle band shown in figure 3.

the same rotational structure albeit with a larger moment of inertia (by a factor of 2). Recent measurements revealed the  $2^+$  [5] and  $4^+$  [8] members of the predicted Hoyle rotational band and we are currently searching [11] for the  $4^-$  predicted by the ACM to be nearly degenerated with the  $4^+$  state and the  $3^-$  (broad) state that was suggested to lie between 11 and 14 MeV [6]. The observation (or lack there) of these "missing" states will allow us to determine whether the Hoyle state is composed of three alpha-particles in an equilateral triangle arrangement [9, 12, 13] or an obtuse triangle [14] or whether it is better described as vibrational excitation of a "diffuse gas" of three alpha-particles [15].

The  $U(7)$  model also predicts the  $1^-$  state at 10.84 MeV to be the vibrational bending mode with a rotational band including the  $1^-$  and a degenerate  $2^\pm$  states. We are searching [11] for the third  $2^+$  of  $^{12}\text{C}$  that is predicted by the  $U(7)$  model to lie near the observed  $2^-$  state at 11.8 MeV.



**Figure 3.** Comparison between the low-lying experimental spectrum of  $^{12}\text{C}$  and the energies of the oblate symmetric top. The levels are organized in columns corresponding to the ground state band and the vibrational bands with  $A$  and  $E$  symmetry [3, 4] of an oblate top with triangular symmetry. The last column on the left-hand side, shows the lowest observed non-cluster ( $1^+$ ) levels. The levels are characterized by angular momentum and parity  $L^P$ , and the vibrational labels  $(v_1, v_2)$  as discussed in [3].

In conclusion the ACM appears to open a new chapter in cluster physics of light nuclei and it presents an opportunity for further experimental investigation of light nuclei.

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### References

- [1] Iachello F 1981 *Phys. Rev. C* **23**, 2778
- [2] Gai M, Ruscev M, Hayes A C, Ennis J F, Keddy R, Schloemer E C, Sterbenz S M and Bromley D A, 1983 *Phys. Rev. Lett.* **50** 239
- [3] Bijker B and Iachello F 2000 *Phys. Rev. C* **61** 067305
- [4] Bijker R and Iachello F 2002 *Ann. Phys.* **298** 334
- [5] Itoh M *et al.* 2004 *Nucl. Phys.* **A738** 268; Itoh M *et al.* 2011 *Phys. Rev. C* **84** 054308; Freer M *et al.* 2009 *Phys. Rev. C* **80** 041303(R); Zimmerman W R *et al.* 2011 *Phys. Rev. C* **84** 027304(BR); Zimmerman W R *et al.* 2013 *Phys. Rev. Lett.* **110** 152502
- [6] Freer M *et al.* 2007 *Phys. Rev. C* **76** 034320
- [7] Kirsebom O S *et al.* 2010 *Phys. Rev. C* **81** 064313
- [8] Freer M *et al.* 2011 *Phys. Rev. C* **83** 034314
- [9] Marin-Lambarri D J, Bijker R, Freer M, Gai M, Kokalova Tz, C. Wheldon, 2014 *Phys. Rev. Lett.* **113** 012502.
- [10] Popular press of Ref. [9]: Kevin Dusling, Physics, Synopsis: <http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.113.012502>, Hamish Johnston, Physicsworld.com: <http://physicsworld.com/cws/article/news/2014/jul/08/carbon-nu>.
- [11] A  $^{12}\text{C}(e,e')$  experiment in progress at Darmstadt, P. von Neumann-Cosel, private communication, 2013.
- [12] Chernykh M, Feldmeier H, Neff T, von Neumann-Cosel P, and Richter A 2007 *Phys. Rev. Lett.* **98** 032501
- [13] Yoshiko Kanada-Enyo 2007 *Prog. of Theo. Phys.* **117** 655
- [14] Epelbaum E, Krebs H, Lee D, Meissner Ulf-G 2012 *Phys. Rev. Lett.* **106** 192501; Epelbaum E, Krebs H, Lahde T, Lee D, and Meissner Ulf-G 2012 *Phys. Rev. Lett.* **109** 252501
- [15] Funaki Y, Horiuchi H, von Oertzen W, Ropke G, Schuck P, Tohsaki A, and Yamada T 2009 *Phys. Rev. C* **80** 64326 and references therein