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The role of symmetry in nuclear physics

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Abstract. The role of discrete symmetries in nuclear physics is briefly reviewed within the context of the algebraic cluster model (ACM). The symmetries D_3 (triangle) for 3α and T_d (tetrahedron) for 4α are discussed and evidence shown for their occurrence in ^{12}C (D_3) and ^{16}O (T_d).

1. Continuous symmetries

Continuous symmetries were introduced in nuclear physics as early as the 1930's. In 1932 Heisenberg introduced isospin, $SU_T(2)$, in 1937 Wigner combined isospin with spin to $SU(4) \supset SU_T(2) \otimes SU_S(2)$, in the 1940's Racah developed the group theory of the shell model in ℓs and jj coupling, $U((2\ell+1)(2s+1))$ and $U(2j+1)$, and in 1958 Elliott introduced the symmetry of mixed configurations, in particular the symmetry of the sd and pf shells. The use of continuous symmetries culminated in 1974 with the introduction of the interacting boson model, Arima-Iachello $U(6)$. The role of these symmetries in nuclear physics is well documented and will not be discussed here.

2. Discrete symmetries

Discrete symmetries were also introduced in the early days of nuclear physics, mostly within the framework of the α-particle model, by Wheeler in 1937 [1] and Dennison in 1954 [2] and later exploited by Brink [3], Robson [4] and others. We have recently readdressed this problem within an algebraic description of clustering, the Algebraic Cluster Model (ACM), and in this contribution some recent results will be reported.

The algebraic cluster model is a description of cluster states as representations of $U(3k-2)$, where k is the number of constituents. The elements of the algebra are the bilinear products of boson creation and annihilation operators which are a bosonic quantization of the Jacobi variables and their associated momenta plus an additional s boson. For $k=2$, there is only one Jacobi vector, $\boldsymbol{\rho} = \mathbf{r}_1 - \mathbf{r}_2$, for k=3 there are two vectors, $\boldsymbol{\rho} = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$, $\boldsymbol{\lambda} = (\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)/\sqrt{6}$, and for k=4 there are three vectors, $\boldsymbol{\rho} = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$, $\boldsymbol{\lambda} = (\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3)/\sqrt{6}$, $\boldsymbol{\eta} = (\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 - 3\mathbf{r}_4)/\sqrt{12}$, where $\mathbf{r}_i$ are the coordinates of the constituent particles. For two-body clusters, ACM was introduced years ago for applications to molecules [5] and nuclear molecules [6]. For three-body clusters ACM was introduced in [7], and discussed in [8], and for four-body clusters was introduced in [9], [10] and discussed, very recently, in [11], [12]. The discrete symmetries of the α-particle model discussed in those articles and here are shown in Table 1.

k	Nucleus	$U(3k-2)$	Discrete symmetry	Jacobi variables
2	^{8}Be	$U(4)$	Z_2	$\boldsymbol{\rho}$
3	^{12}C	$U(7)$	D_3	$\boldsymbol{\rho}, \boldsymbol{\lambda}$
4	^{16}O	$U(10)$	T_d	$\boldsymbol{\rho}, \boldsymbol{\lambda}, \boldsymbol{\eta}$

Table 1. Discrete symmetries of the α-particle model considered in this article.

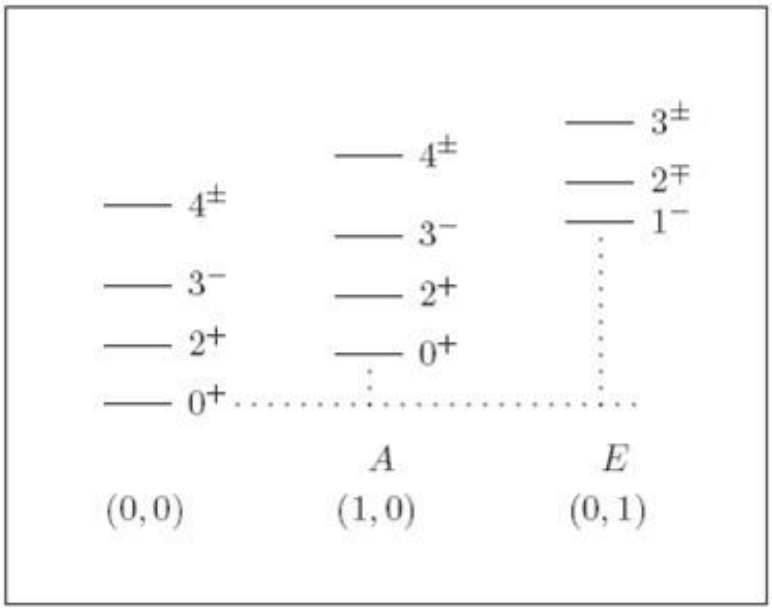

Figure 1. States of 3α with D_3 symmetry.

Within the algebraic cluster model (ACM) it is possible to derive many analytical results. In particular, for a rigid roto-vibrator one has explicit expressions for the energy levels in terms of the angular momentum L and the vibrational quantum numbers $v_i (i = 1, 2...)$. These are:

$$\begin{array}{cc} 2\alpha(Z_2) & E(v,L) = E_0 + \omega\left(v + \frac{1}{2}\right) + \kappa L(L+1) \\ 3\alpha(D_3) & E(v_1,v_2,L) = E_0 + \omega_1\left(v_1 + \frac{1}{2}\right) + \omega_2\left(v_2 + 1\right) \\ & \quad +\kappa L(L+1) \\ 4\alpha(T_d) & E(v_1,v_2,v_3,L) = E_0 + \omega_1\left(v_1 + \frac{1}{2}\right) + \omega_2\left(v_2 + 1\right) \\ & \quad +\omega_3\left(v_3 + \frac{3}{2}\right) + \kappa L(L+1) \end{array} . \quad (1)$$

Spectra are characterized by the representations of the discrete group G and consist in a set of rotation-vibration bands, with specific values of the angular momentum and parity. Representations can be labeled either by G or by the isomorphic group S_n (the permutation group). The conversion from G to S_n is: $G = Z_2 \sim S_2 \sim P$, $A \equiv [2]$; $G = D_3 \sim S_3$, $A \equiv [3]$, $E \equiv [21]$; $G = T_d \sim S_4$, $A \equiv [4]$, $F \equiv [31]$, $E \equiv [22]$. Fig.1 shows the expected rotation-vibration spectra of states for 3α with D_3 symmetry. For A representations, the rotational band has $L^P = 0^+, 2^+, 3^-, 4^{\pm}, ...$, while for E representations the rotational band has $L^P = 1^-, 2^{\mp}, 3^{\pm}, ...$ Note the unusual angular momentum and parity content of the rotational bands. This content is not the same as that of the rotational bands of a rotating ellipsoidal shape. [See, for example, the $SU(3)$ limit of the IBM].

In Fig.2, the expected spectrum of a 4α configuration with T_d symmetry is shown. For

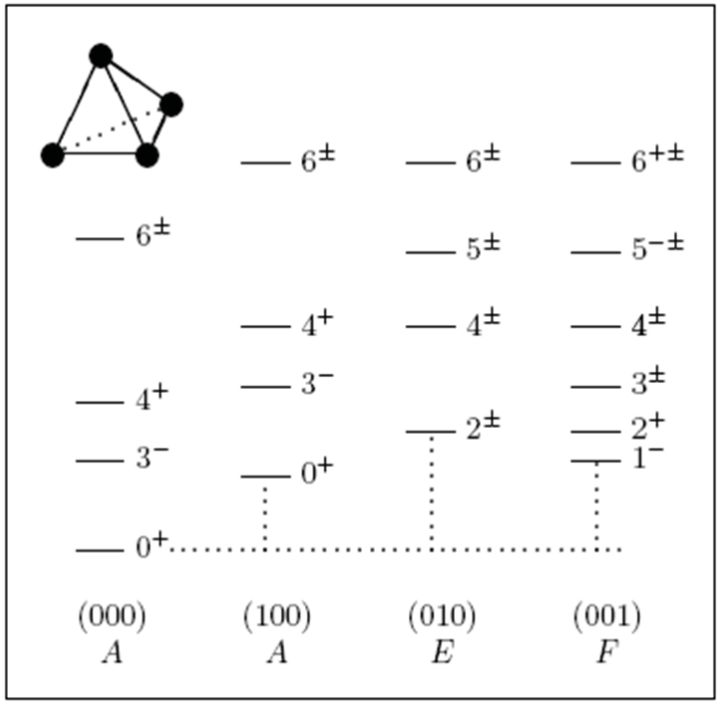

Figure 2. States of a 4α configuration with T_d symmetry.

A representations, the spectrum consists of a rotational band $L^P = 0^+, 3^-, 4^+, 6^{\pm}, ...$, for E representations $L^P = 2^{\pm}, 4^{\pm}, 5^{\pm}, ...$, and for F representations $L^P = 1^-, 2^+, 3^{\pm}, ...$

The breaking of representations of $U(3k-2)$ into those of S_n and thus the determination of the angular momentum content of each band is one of the novel results of ACM.

The occurrence of D_3 symmetry in ^{12}C has been confirmed by a very recent experiment [13], [14] and its evidence is shown in Figs. 3 and 4.

The occurrence of T_d symmetry in ^{16}O was discussed long ago by Robson [4], and it has been emphasized recently in [11].The evidence is shown in Figs. 5 and 6.

In addition to energy spectra one can also derive within the ACM, analytic expressions for other observables, most notably electromagnetic transition rates. For example, for 4α configurations with T_d symmetry, we have

$$B(EL; 0^+ \rightarrow L^P) = \left(\frac{Ze\beta^L}{4}\right)^2 \frac{(2L+1)}{4\pi} \left[4 + 12P_L\left(-\frac{1}{3}\right)\right] \tag{2}$$

and the form factors in electron scattering are

$$F_L\left(0^+ \rightarrow L^P\right) = c_L j_L\left(q\beta\right) \tag{3}$$

with $c_0^2 = 1; c_3^2 = \frac{35}{9}; c_4^2 = \frac{7}{3}; c_6^2 = \frac{32}{81}$.

The occurrence of T_d symetry in ^{16}O is confirmed by the $B(EL)$ values given in Table 2.

3. Conclusion

In conclusion, the role of discrete symmetry in cluster physics is that of providing benchmarks for energy levels and other obeservables that can be used to analyze data. In particular, by

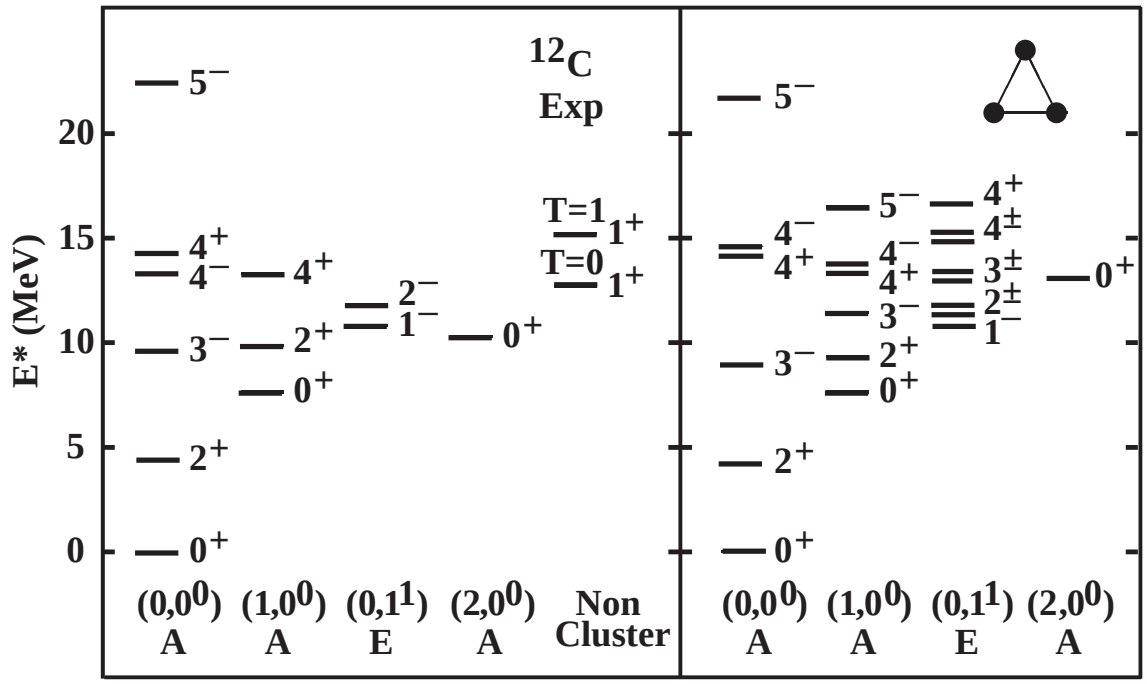

Figure 3. The experimental spectrum of ^{12}C(left) and its comparison with the theoretical spectrum with D_3 symmetry (right). The lowest non-cluster states are also shown. Reproduced with permission from [13].

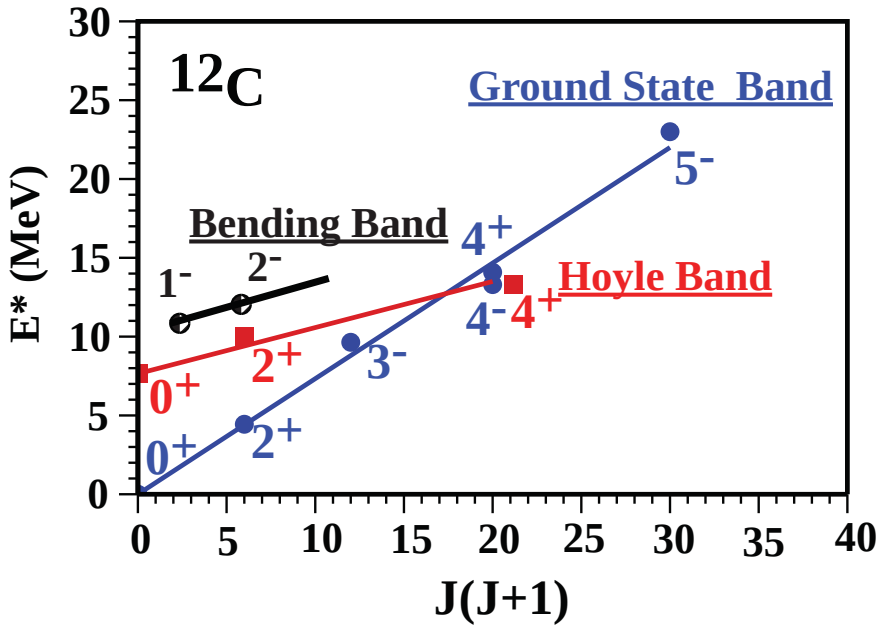

Figure 4. Observed cluster rotational bands in ^{12}C. Reproduced with permission from [13].

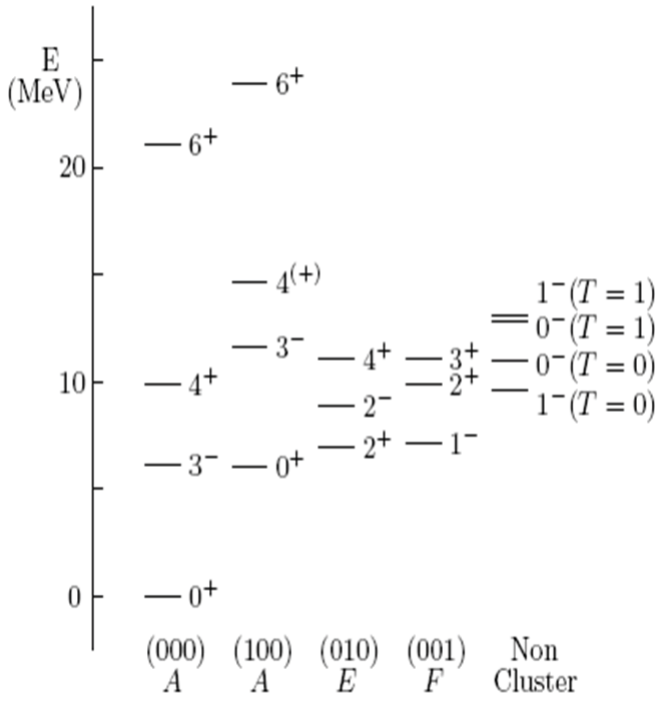

Figure 5. The experimental spectrum of ^{16}O showing evidence for T_d symmetry. The lowest non-cluster states are also shown.

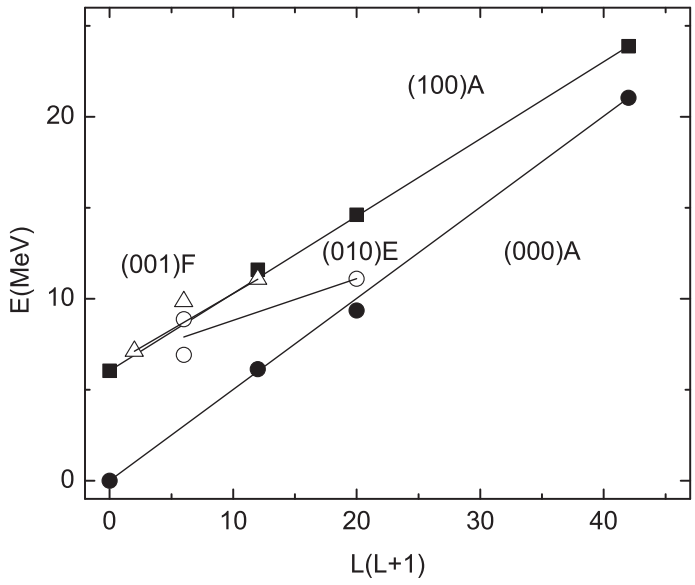

Figure 6. Observed cluster rotational bands in ^{16}O.

$B(EL; L^P \to 0^+)$	Th	Exp	$E(L^P)$	Th	Exp
$B(E3; 3^- \to 0^+)$	181	205±10	$E(3^-)$	6132	6130
$B(E4; 4^+ \to 0^+)$	338	378±133	$E(4^+)$	10220	10356
$B(E6; 6^+ \to 0^+)$	8245		$E(6^+)$	21462	21052

Table 2. $B(EL)$ values and energies in ^{16}O compared with those expected from a T_d symmetry. $B(EL)$ values in e^2fm^{2L} and E in keV. The theoretical energies in column 5 are calculated from $E(keV) = 511\ L(L+1)$. The value of β in Eq.(2) is extracted from the elastic form factor measured in electron scattering, $\beta = 2.0$fm.

providing analytic expression for rotation-vibration spectra as well as electromagnetic transition rates and form factors. Strong evidence for the occurrence of D_3 symmetry in ^{12}C and of T_d symmetry in ^{16}O has been presented. [There is evidence for Z_2 symmetry in ^{8}Be but this has not been presented here].

The occurrence of D_3 and T_d symmetry is also confirmed by microscopic calculations in the (i) molecular orbital method [15] and in (ii) lattice calculations [16].

Cluster states are instead very difficult to describe within the framework of the one-center shell model, where one needs multiparticle-multihole configurations and a model space with several $\hbar\omega$ quanta of excitation.

Acknowledgements
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