

Shell-model study of spin modes in nuclei and nuclear forces

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Abstract. Spin-dependent modes in nuclei are studied by shell-model method with the use of new shell-model Hamiltonians which properly take into account important roles of tensor interactions. New Hamiltonians can describe spin degrees of freedom in nuclei remarkably well. Nuclear weak processes at stellar environments are investigated based on these successes. New neutrino-nucleus reaction cross sections on ^{12}C are applied to light-element synthesis in supernova explosions. The production rate for $^{11}\text{B}/^7\text{Li}$ is pointed out to be useful to determine ν -oscillation parameters, in particular, ν -mass hierarchy. New e-capture rates in Ni isotopes are obtained and implications for element synthesis are discussed. The monopole-based universal interaction is applied to study structure of p - sd shell nuclei and ^{40}Ar as well as ν -induced reactions on ^{40}Ar . Repulsive corrections in the isospin $T=1$ monopoles are shown to be important for proper shell evolutions in neutron-rich carbon isotopes. The repulsive correction is pointed out to be due to three-body forces, in particular, the Fujita-Miyazawa force. Roles of the three-body forces on the shell evolution of neutron-rich calcium isotopes, the closed-shell nature of ^{48}Ca and M1 transition in ^{48}Ca are studied on top of the two-body G-matrix obtained by including core-polarization effects in larger spaces ($\leq 24\hbar\omega$). Effects of the inclusion of $g_{9/2}$ -shell are also discussed.

1. Introduction

Spin modes in nuclei are studied by shell-model calculations, and important roles of tensor interaction and three-nucleon forces are discussed. Now, several new shell-model Hamiltonians with proper tensor components are available. They are found to be successful in describing proper shell evolutions and spin responses in nuclei. Gamow-Teller (GT) and magnetic dipole (M1) transitions as well as M1 moments in nuclei are well reproduced by the new Hamiltonians.

New shell-model Hamiltonians are applied to study nuclear weak processes in stars. In Sect. 2, neutrino-nucleus reaction cross sections and e-capture reaction rates at stellar environments are evaluated with the new Hamiltonians, and nucleosynthesis in supernova explosions are discussed.

In Sect. 3, the need for repulsive corrections in isospin $T=1$ monopoles is pointed out, and this repulsion is shown to be attributed to three-nucleon forces. Roles of three-nucleon forces in



neutron-rich isotopes are discussed.

2. Spin responses in nuclei and tensor forces

2.1. New shell-model Hamiltonians and roles of tensor interaction

The Hamiltonian for p -shell, SFO[1], can reproduce GT strengths in ^{12}C and ^{14}C , and M1 moments of p -shell nuclei very well. The GXPF1J[2] for fp -shell can reproduce GT strength in Ni isotopes and M1 strength in ^{48}Ca , ^{50}Ti , ^{52}Cr and ^{54}Fe . Recently, monopole-based universal interaction, VMU[3], with tensor force of $\pi+\rho$ -meson exchanges is proposed, and it has been found to be successful in description of structure of p - sd shell[4] and sd - fp shell nuclei[5].

The new interactions have a common feature that they have proper tensor components, that is, the monopole terms of the two-body matrix elements have a characteristic orbit dependence. The monopoles for $j_>-j_<$ ($j_> = \ell+1/2$, $j_< = \ell-1/2$) orbits are more attractive than those for $j_>-j_>$ or $j_<-j_<$ orbits. This feature comes from the general sign rule for the tensor interaction[6], and is also seen in microscopic G-matrices, whose tensor components are constructed from $\pi+\rho$ -meson exchanges. Proper shell evolution and change of magic numbers toward drip-lines are realized by this nature of the tensor interaction[3, 6].

2.2. Nuclear weak processes and nucleosynthesis

Spin dependent transition strengths evaluated with new shell-model Hamiltonians are applied to nuclear weak processes such as ν -nucleus and e-capture reactions, and nucleosynthesis in supernova explosions.

The SFO Hamiltonian, which can reproduce GT strength in ^{12}C , is applied to evaluate new ν - ^{12}C reaction cross sections[7]. New cross sections for ^{12}C and also for ^4He are used to study light-element synthesis in core-collapse supernova explosions. Production yields of ^{11}B and ^7Li are found to be enhanced compared with previous estimations[7, 8]. Effects of MSW matter oscillations in He/C layers on nucleosynthesis are also studied. The abundance ratio of $^7\text{Li}/^{11}\text{B}$ is shown to be sensitive to the ν -oscillation parameters, the mixing angle θ_{13} and ν -mass hierarchy[8, 9]. The ratio is enhanced for normal mass hierarchy at $\sin^2 2\theta_{13} > 0.02$. The mass hierarchy, therefore, can be determined from the ratio as $\sin^2 2\theta_{13} \sim 0.1$ [10]. Inverted mass hierarchy is pointed out to be statistically more favored[11] based on recent measurement on supernova SiC X-grains of the Murchison meteorite[12].

Liquid argon is a powerful target for ν -detection. GT strength in ^{40}Ar and cross section for $^{40}\text{Ar}(\nu, e^-)^{40}\text{K}$ are evaluated by shell-model calculations with the use of VMU[3]. GXPF1J and SDPF-M[13] interactions are used for fp - and sd -shells, respectively, and VMU is adopted for sd - fp cross-shells together with the two-body spin-orbit interaction of M3Y[14]. This interaction is referred as SDPF-VMU.

Calculated GT strength in ^{40}Ar is found to be consistent with the experimental data obtained by (p, n) reactions[15] as shown in figure 1(a). The present GT strength and calculated charge-exchange reaction cross section for ^8B solar ν [5] are found to be enhanced compared with the previous calculation[16]. Contributions from multipoles other than GT (1^+) and isobaric-analog (0^+) transitions evaluated by RPA become important at ν energies, $E_\nu \geq 50$ MeV[5] (see figure 1(b)).

Electron capture rates of fp -shell nuclei at stellar environments are evaluated with GXPF1J[17]. GT strengths in Ni isotopes obtained by shell-model calculations with GXPF1J are generally more fragmented than those by KB3G[18]. In particular, the GT strength in ^{56}Ni obtained with GXPF1J has two peaks while those for conventional Hamiltonians such as KB3G and KBF have only one peak as shown in figure 2(a). Recent (p, n) reaction experiment confirmed the two-peak structure of the GT strength[19]. E-capture rates at high densities and high temperatures are obtained by using the GT strengths of GXPF1J. The capture rates

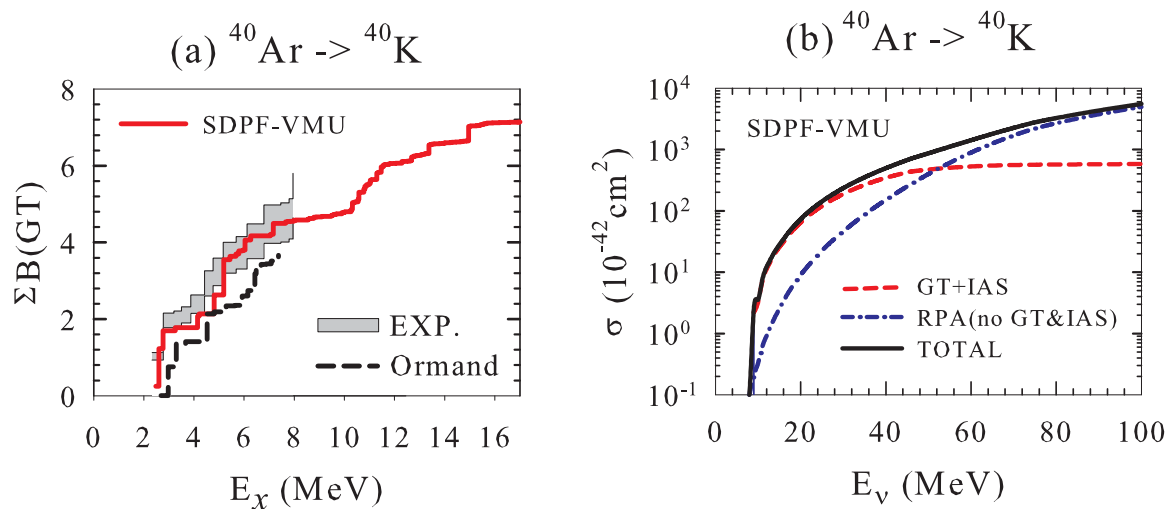


Figure 1. (a) Cumulative sum of $B(GT)$ in ^{40}Ar obtained by the present interaction (SDPF-VMU) and in Ref. [16]. Experimental data are taken from Ref. [15]. (b) Calculated cross sections for $^{40}\text{Ar}(\nu, e^-)^{40}\text{K}$. GT contributions are obtained by shell-model calculations with SDPF-VMU while RPA is used for multipoles other than 0^+ and 1^+ . Figures taken from Ref. [5].

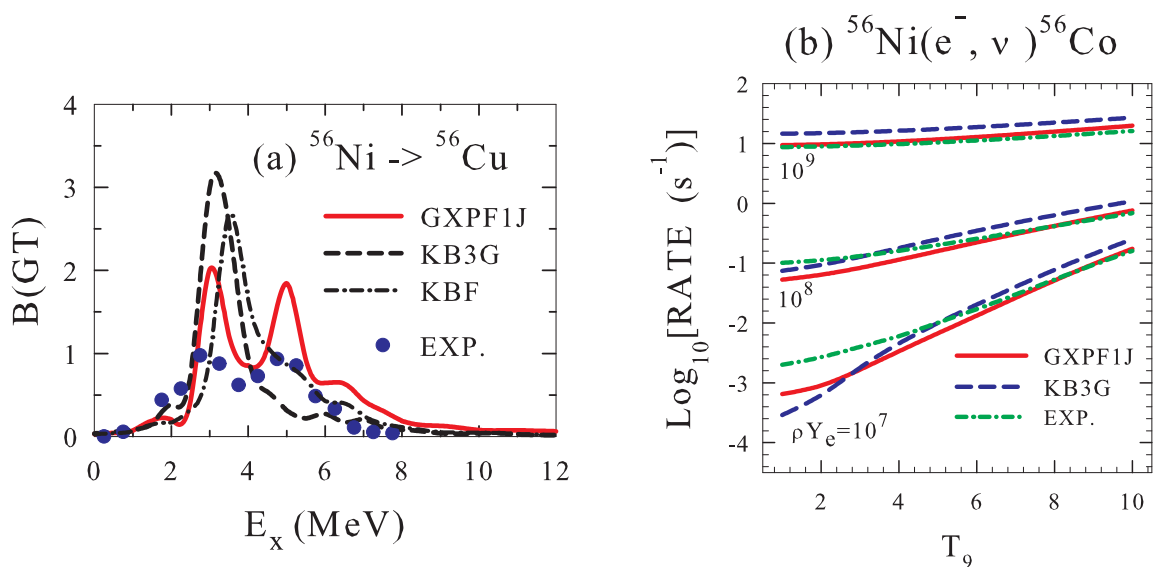


Figure 2. (a) GT strength in ^{56}Ni obtained with GXPF1J[2], KB3G and KBF[18]. Experimental data are taken from Ref. [19]. (b) E-capture rates for ^{56}Ni at densities $\rho Y_e = 10^7, 10^8$ and 10^9 g/cm^3 and at temperatures $T_9 = 1 \sim 10$ ($T = T_9 \times 10^9 \text{ K}$) obtained with GXPF1J and KB3G as well as with experimental data of Ref. [19]. Figure taken from Ref. [17].

become smaller than those with KB3G especially at higher densities as the strengths by GXPF1J are spread over higher excitation energy region (see figure 2(b)).

Accretion of matter to white-dwarfs from their binary stars ignites type-Ia supernova explosions when the white-dwarf mass exceeds the Chandrasekhar limit. An amount of ^{56}Ni is produced in type-Ia supernova explosions. As e-capture process on ^{56}Ni proceeds, neutron-rich nuclei are produced and the lepton-to-baryon ratio (or proton fraction) Y_e gets smaller. If e-capture rates on ^{56}Ni are smaller, production yields of neutron-rich isotopes such as ^{58}Ni decrease and Y_e remains to be a higher value in statistical equilibrium calculations. The problem of over-production of ^{58}Ni , ^{54}Cr and ^{54}Fe compared to the solar abundance[20] could be solved by using smaller e-capture rates of GXPF1J. This problem is now under investigation.

3. Three-nucleon forces and structure of neutron-rich isotopes

3.1. Carbon isotopes

Comparing microscopic G-matrices with good phenomenological shell-model interactions, one notices a general feature: monopole terms of phenomenological interactions are more repulsive (attractive) than those of microscopic ones in isospin $T=1$ ($T=0$) channels. The repulsive corrections to the monopoles in $T=1$ channels are important to reproduce proper shell evolutions in neutron-rich isotopes.

The SFO, whose sd -shell part is G-matrix elements, is modified to take into account this repulsive correction in $T=1$ monopoles in the sd -shell matrix elements. Tensor components of p - sd cross-shell part are also replaced by those of π - ρ -meson exchanges. The modified version of the interaction, referred as SFO-tls[21], is applied to study structure of neutron-rich carbon isotopes. Due to the repulsive correction in the $0d_{5/2}$ - $1s_{1/2}$ ($T=1$) monopole, effective single-particle energy (ESPE) of $1s_{1/2}$ orbit increases for $A=14\sim 20$ and crosses with the $0d_{5/2}$ orbit at $A=16$, resulting in lower ESPE for $0d_{5/2}$ than $1s_{1/2}$ at $A > 16$. The ground state energies for SFO-tls increase after $A=22$, and ^{22}C turns out to be the drip-line nucleus of carbon isotopes, consistent with the observation (see figure 3(a)).

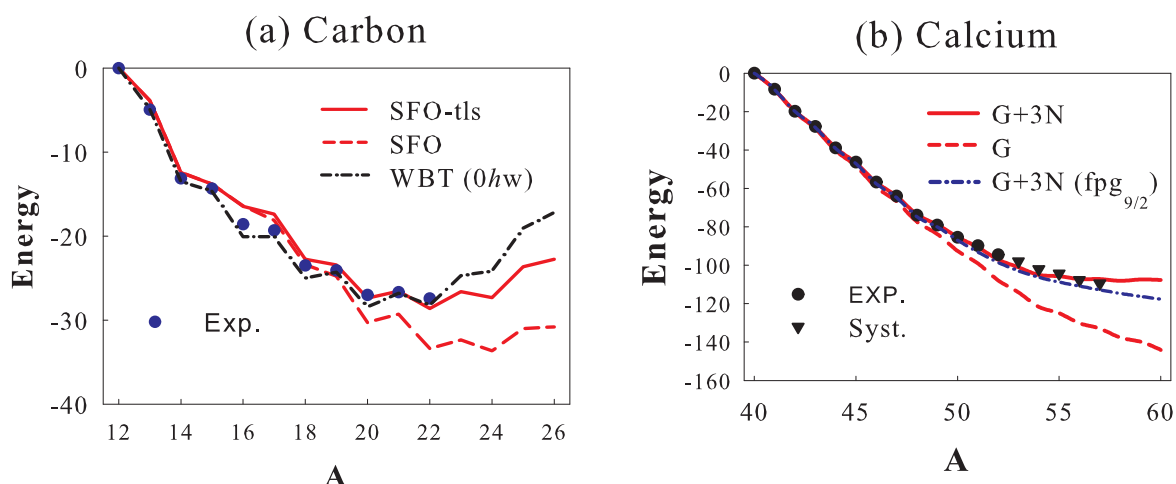


Figure 3. (a) Ground state energies of carbon isotopes obtained by shell-model calculations with SFO-tls, SFO and WBT[24]. (b) Ground state energies of calcium isotopes obtained by shell-model calculations with 2N (G-matrix) and 2N+3N (FM) interactions.

The SFO-tls can explain an anomalous M1 strength in ^{17}C ; the $B(M1)$ for $1/2^+ \rightarrow 3/2^+_{g.s.}$ is considerably suppressed[22]. Restricting to $d_{5/2}1s_{1/2}$ -space, $B(M1; 1/2^+ \rightarrow 3/2^+)$ is shown to vanish exactly[21]. One should note that ESPE of $0d_{5/2}$ and $1s_{1/2}$ orbits are nearly degenerate at $A=17$. The anomalous suppression of the $B(M1)$ strength in ^{17}C is well explained by SFO-tls[21]. Recently, similar suppression of $B(M1)$ is found to occur in ^{19}C [23].

3.2. Calcium isotopes

The origin of the repulsive correction in $T=1$ monopoles can be attributed to three-nucleon forces. In particular, the three-nucleon force induced by excitations of Δ_{33} -isobars through two-pion exchanges (Fujita-Miyazawa force[25]) is pointed out to give rise to repulsive corrections to the nucleon-nucleon interaction between valence neutrons[26]. This mechanism is found to be successful to explain the drip-line of oxygen isotopes at $A=24$ [26].

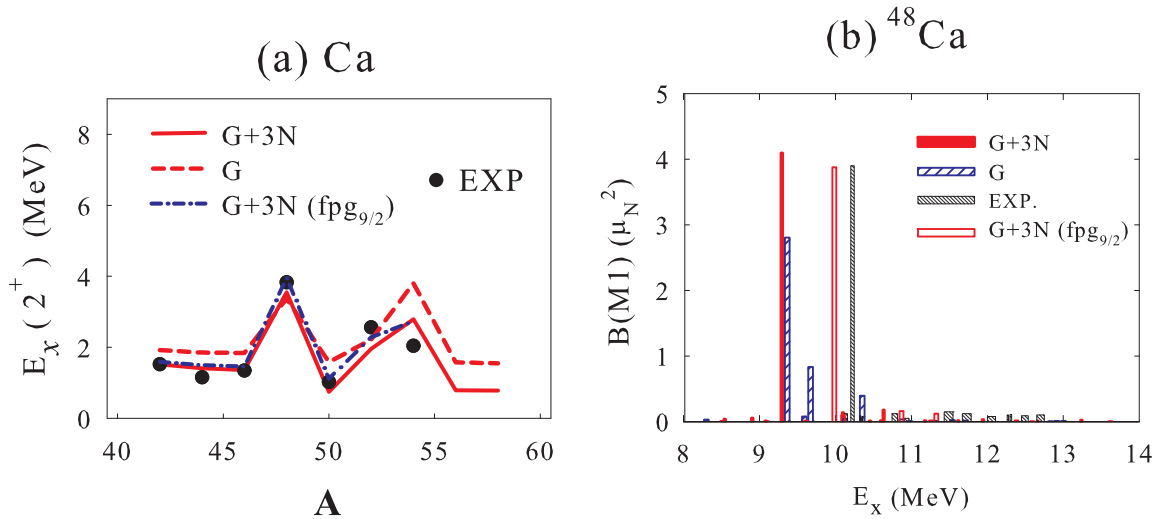


Figure 4. (a) Excitation energies of 2^+_1 states of calcium isotopes obtained with 2N and 2N+3N interactions. (b) $B(M1)$ strength in ^{48}Ca obtained with 2N and 2N+3N interactions. Experimental data are taken from Ref. [30].

Here, we study the effects of the three-nucleon forces in calcium isotopes[28]. G-matrix elements are used for the two-nucleon (2N) interaction while the Fujita-Miyazawa (FM) force is used for the three-nucleon (3N) interaction. Here, core polarization contributions are calculated by including up to third-order Q-box[27], and intermediate states are taken up to $24\hbar\omega$ excitations.

The ground state energies and excitation energies of the 2^+_1 states, $E_x(2^+)$, in calcium isotopes are shown in figure 3(b) and figure 4(a), respectively. Results for fp -shell and also for $fp g_{9/2}$ -shell are shown. In case of $fp g_{9/2}$ -shell, $g_{9/2}$ orbit is assumed to be degenerate with fp -shell orbits in the G-matrix calculation.

The repulsive contributions from the 3N forces push up the ground state energies, and experimental values are well reproduced with the 3N forces. The $E_x(2^+)$ are also well described with the 3N forces for $A=40\sim 50$, and a large value of $E_x(2^+)$ in ^{48}Ca close to the experimental value shows the closed-shell nature of ^{48}Ca . Calculated large values of $E_x(2^+)$ for ^{52}Ca and

^{54}Ca show new magic numbers at $N=32$ and $N=34$ [29]. The deviation between calculated and experimental values of $E_x(2^+)$ in ^{54}Ca obtained with the 2N forces is found to be reduced by inclusion of the 3N forces.

The experimental $B(M1)$ in ^{48}Ca is exhausted by a single state at $E_x = 10.23$ MeV[30]. Calculated $B(M1)$ strength obtained with/without the 3N forces are shown in figure 4(b). The concentration of the $B(M1)$ strength to almost a single peak is reproduced with the 3N forces. Thus, 3N interaction is important to realize the magicity at $N=28$. In case of $fp g_{9/2}$ -shell, the position of the peak comes closer to the experimental value.

Now, non-degenerate treatment of fp and $g_{9/2}$ shells by extended Kuo-Krenciglowa (EKK) method become possible[31]. Treatment of calcium isotopes in $fp g_{9/2}$ -shell by the non-degenerate method is now under investigation. Monopole terms obtained with non-degenerate and degenerate methods are found to be significantly different[32]. Nevertheless, main features found here, that is, the enhancement of $E_x(2^+)$ in ^{48}Ca , ^{52}Ca and ^{54}Ca and the concentration of $B(M1)$ in ^{48}Ca are common to both the methods.

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