

# Shell model calculations for neutrinoless double beta decay

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**Abstract.** The study of the neutrinoless double beta ( $0\nu\beta\beta$ ) decay is of great interest because of its potential to provide us with information about the lepton number conservation and neutrino properties as the neutrinos character (are they Dirac or Majorana particles?) and their absolute mass. Since the  $0\nu\beta\beta$  decay has not yet been discovered experimentally, one can only extract limits of the absolute neutrino mass. For that, one needs accurate calculations of both nuclear matrix elements (NMEs) and phase space factors (PSFs) which appear in the theoretical lifetime expressions, corroborated with experimental lifetime limits. In this paper I first present recent shell model (ShM) calculations of the NMEs and PSFs for  $0\nu\beta\beta$  decay performed by our group, in the hypothesis that the mechanism of its occurrence is the exchange of light Majorana neutrinos between two nucleons inside the nucleus. Also, the consensus on the use of different nuclear structure ingredients/parameters in the computation of the NMEs is discussed. Then, I present new limits of the Majorana neutrino mass parameter derived from the analysis of  $0\nu\beta\beta$  decay of nine isotopes.

## 1. Introduction

Double beta decay (DBD) is a nuclear natural decay by which an even-even nucleus transforms into another even-even nucleus with the same mass  $A$  but with its nuclear charge  $Z$  changed by two units. It occurs whatever single  $\beta$  decay can not occur due to energy reasons or if it is highly forbidden by angular momentum selection rules. There are several theoretical possible DBD modes, that can be classified according to the number and type of the leptons released in the decay. We have the so-called two neutrino double beta decays ( $2\nu\beta\beta$ ), where in the final states one finds two electrons/positrons and two anti-neutrinos/neutrinos, and the  $0\nu\beta\beta$  decay modes, where besides electrons/positrons one finds no neutrinos in the final states. The positron DBD modes can be accompanied by electron capture  $EC/\beta$  and/or  $ECEC$  decay modes. The  $2\nu\beta\beta$  decay modes conserve the lepton number ( $\Delta L = 0$ ) and are allowed in the initial formulation of the Standard Model (SM). They are already measured for eleven isotopes with values of the half-lives ranging between  $10^{18} - 10^{24}$  yr. By contrary, no  $0\nu\beta\beta$  decay is confirmed until present by independent measurements. The  $0\nu\beta\beta$  decay modes are particularly of great interest because they can provide us with information about the lepton number conservation and neutrino properties as, for example, the neutrinos character (are they Dirac or Majorana particles?) and their absolute mass [1]-[2]. Its discovery would imply that the lepton number may not conserve ( $\Delta L = 2$ ) and neutrino is a Majorana particle. They can theoretically occur through several mechanisms, the most investigated being the exchange of light left-handed neutrinos between two nucleons in the nucleus. In this paper we refer to the  $0\nu\beta\beta$  decay



which occur through this mechanism and where two electrons are released. The lifetimes for this decay mode can be expressed as a product of three factors: a phase space (PSFs) factor, depending on the  $Q_{\beta\beta}$  value of the decay and on the nuclear charge  $Z$ , a factor representing the NMEs and a Majorana neutrino mass parameter which is related to the elements of the neutrino mixing matrix and to absolute neutrino mass eigenvalues [1]-[2]. To derive accurate neutrino mass parameters and/or predict  $0\nu\beta\beta$  decay lifetimes, one needs precise calculations of both NMEs and PSFs. The computation of the NMEs is a challenge in the theoretical study of the DBD since long time. There are several nuclear structure methods for these calculations, the most employed being proton-neutron Quasi Random Phase Approximation (pnQRPA) [3]-[7], Interacting Shell Model (ISM) [8]-[11], Interacting Boson Approximation (IBA) [12]-[14], Projected Hartree Fock Bogoliubov (PHFB) [15] and Energy Density Functional (EDM) method [16]. There are still large differences in the literature between the NME values computed with different methods and by different groups, and these have been largely discussed in the literature (see for example [2], [17]). On the other hand PSFs have been calculated since long time using different approximations [18]-[23], and they were considered until recently to be calculated with enough precision. However, they were recently recalculated within an improved approach by using exact electron Dirac wave functions (w.f.), taking into account the finite nuclear size and electron screening effects [24], and differences were found especially for heavier nuclei. We also recalculated by developing new routines for computing the relativistic (Dirac) electron w.f. with inclusion of the nuclear finite size and screening effects and, in addition, with the use of a Coulomb potential derived from a realistic proton density distribution in the daughter nucleus [25]-[26], and found similar differences as compared with older calculations. So, it worth to take into account these new calculations for extracting the neutrino mass parameter.

In this paper I present recent ShM calculations of NMEs and PSFs performed by our group. The NMEs for three isotopes, i.e.  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$  and  $^{82}\text{Se}$ , have been computed with a Shell Model (ShM) code developed recently by our group (distinct from the Madrid-Strasbourg code) [27]-[28]. The values of the PSFs computed with the code described in Refs. [25]-[26] but with an improved numerical precision. Then, I present new up-date limits of the Majorana neutrino mass parameter derived from the analysis of  $0\nu\beta\beta$  decay of nine isotopes.

## 2. Formalism

The lifetimes for  $0\nu\beta\beta$  decay can be expressed as [1]:

$$\left(T_{1/2}^{0\nu}\right)^{-1} = G^{0\nu}(Q_{\beta\beta}, Z) g_A^4 |M^{0\nu}|^2 (\langle m_\nu \rangle / m_e)^2, \quad (1)$$

where  $G^{0\nu}$ , expressed in units of  $[yr]^{-1}$ , are the PSFs for this decay mode,  $Q_{\beta\beta}$  is the energy decay,  $Z$  is the nuclear charge,  $m_e$  is the electron mass and  $\langle m_\nu \rangle$  is the Majorana light neutrino mass parameter, which can be expressed as a linear combination of the light neutrino masses and the elements from the first row of the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) neutrino matrix [29].  $M^{0\nu}$  are the NMEs which depend on the nuclear structure of the parent and daughter nuclei:

$$M_\alpha^{0\nu} = \sum_{m,n} \left\langle 0_f^+ \parallel \tau_{-m} \tau_{-n} O_{mn}^\alpha \parallel 0_i^+ \right\rangle, \quad (2)$$

where  $\alpha = GT, F, T$  are the contributions associated with the Gamow-Teller ( $GT$ ), Fermi ( $F$ ) and Tensor ( $T$ ) two-body transition operators  $O_{mn}^\alpha$ , and the summation is performed over all the nucleon states. The computation of the reduced matrix elements of the operators  $O^\alpha$  can be decomposed into products of reduced matrix elements within the spin and relative coordinate spaces. Their explicit expressions are [1], [11]:

$$O_{12}^{GT} = \sigma_1 \cdot \sigma_2 H(r), \quad O_{12}^F = H(r), \quad O_{12}^T = \sqrt{2/3} [\sigma_1 \times \sigma_2]^2 \cdot (r/R) H(r) C^{(2)}(\hat{r}) \quad (3)$$

The most difficult is the computation of the radial part of these two-body transition operators, which contains the neutrino potentials, defined by integrals of momentum carried by the virtual neutrino exchanged between the two nucleons [6]:

$$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty j_i(qr) \frac{h_\alpha(q)}{\omega} \frac{1}{\omega + \langle E \rangle} q^2 dq \equiv \int_0^\infty j_i(qr) V_\alpha(q) q^2 dq, \quad (4)$$

where  $R = r_0 A^{1/3}$  fm ( $r_0 = 1.2$  fm),  $\omega = \sqrt{q^2 + m_\nu^2}$  is the neutrino energy and  $j_i(qr)$  is the spherical Bessel function ( $i = 0, 0$  and  $2$  for GT, F, and T, respectively). Usually, in calculations one uses the closure approximation which consists of the replacement of the excitation energies of the states in the intermediate odd-odd nucleus contributing to the decay, by an average expression  $\langle E \rangle$ . The detailed expressions of  $h_\alpha$  ( $\alpha = GT, F, T$ ) can be found, for example, in refs. [1], [6], [11]. They contain nuclear ingredients such as the finite nucleon size (FNS) and short range correlations effects, as well as inclusion of higher order terms in the nuclear currents (HOC), which are important for an accurate computation of the NMEs. FNS effect is taken into account through nucleon form factors,  $G_V$  and  $G_A$ , which depend on the momentum:

$$G_A(q^2) = g_A \left( \Lambda_A^2 / (\Lambda_A^2 + q^2) \right)^2, \quad G_V(q^2) = g_V \left( \Lambda_V^2 / (\Lambda_V^2 + q^2) \right)^2 \quad (5)$$

In calculations either the quenched ( $g_A = 1$ ) or unquenched ( $g_A = 1.25 - 1.273$ ), values of the axial-vector constant have been used, while the values of the cut-off parameters are  $\Lambda_V = 850$  MeV and  $\Lambda_A = 1086$  MeV [1]. The SRC effects are included by correcting the single particle w. f.:  $\psi_{nl}(r) \rightarrow [1 + f(r)] \psi_{nl}(r)$ . The correlation function  $f(r)$  can be parametrized in different ways: the Jastrow prescription with the i) Miller-Spencer (MS) parametrization [30] and the CCM parametrizations, derived with realistic ii) CD-Bonn and iii) AV18 NN potentials [6]:

$$f(r) = -c \cdot e^{-ar^2} (1 - br^2) \quad (6)$$

The three parametrizations mentioned above are associated with different sets of the a, b, c parameters. We mention that another method, unitary operator method (UCOM) is also successfully used to include SRC effects [4]. The inclusion of HOC brings additional terms in the  $h_{GT}$  component and leads to the appearance of the  $h_T$  component in the expressions of the neutrino potentials, as it is described in detail in refs. [1], [31]. Besides the nuclear effects mentioned above, a number of parameters as  $g_A$ ,  $r_0$ ,  $(\Lambda_A, \Lambda_B)$  and  $\langle E \rangle$  are involved as well, in the NME calculations. The use of different values for these parameters may result in important differences in the calculated NME values. For example, the use of a quenched or an unquenched value for  $g_A$  is still an unsolved matter.

### 3. Numerical results and discussions

The largest uncertainties in the derivation of the Majorana neutrino mass parameter and/or prediction of  $0\nu\beta\beta$  decay half-lives come from the NME computation. Their values depend on the method of calculation and on the different nuclear ingredients/parameters mentioned in the previous section. Fortunately, at present there is a consensus on the use of the nuclear ingredients/parameters [32] which helps to restrict the range of the different NME values from literature. For example, one recommends the inclusion in calculation of the HOC, FNS and the use of softer parametrizations like CCM [6] and UCOM [4] for SRCs. Concerning the input nuclear parameters, one recommends the use of an unquenched value for the axial vector constant ( $g_A = 1.25-1.273$ , [41]) and a value of  $r_0 = 1.2$  fm for the nuclear radius constant. Also, one can mention that the results are less sensitive to the changes (within a few MeV)

**Table 1.** The NMEs obtained with different methods and with softer SRC parametrizations, specified in the second column. In calculations an unquenched value of  $g_A$  was used.

Method	SRC	$^{48}\text{Ca}$	$^{76}\text{Ge}$	$^{82}\text{Se}$	$^{96}\text{Zr}$	$^{100}\text{Mo}$	$^{116}\text{Cd}$	$^{130}\text{Te}$	$^{136}\text{Xe}$	$^{150}\text{Nd}$
[28]ShM	CD-Bonn	0.81	3.13	2.88					2.21[34]	
[10]ShM	UCOM	0.85	2.81-3.52	2.64				2.65	2.19	
[14]IBM-2	CD-Bonn	2.38	6.16	4.99	3.00	4.50	3.29	4.61	3.79	2.88
[17]QRPA	CD-Bonn		5.93(3.27)	5.30(4.54)	2.19	4.67	3.72	4.80	3.00	3.16[35]
[40]QRPA	UCOM		5.36(4.11)	3.72	3.12	3.93	4.79	4.22	2.80	
[16]GCM	CD-Bonn	2.37	4.60	4.22	5.65	5.08	4.72	5.13	4.20	1.71
[15]PHFB	CD-Bonn				2.98	6.07		3.98		2.68

**Table 2.** Majorana neutrino mass parameters together with the other components of the  $0\nu\beta\beta$  decay halftimes: the  $Q_{\beta\beta}$  values, the experimental half-life limits, the PSFs and the nuclear matrix elements.

Nucleus	$Q_{\beta\beta}[\text{MeV}]$	$T_{exp}^{0\nu\beta\beta}[\text{yr}]$	$G^{0\nu\beta\beta}[\text{yr}^{-1}]$	$M^{0\nu\beta\beta}$	$\langle m_\nu \rangle [\text{eV}]$
$^{48}\text{Ca}$	4.272	$> 5.8 \cdot 10^{22}$ [42]	2.46E-14	0.81-0.90	$< [9.6 - 10.69]$
$^{76}\text{Ge}$	2.039	$> 2.1 \cdot 10^{25}$ [46]	2.37E-15	2.81-6.16	$< [0.24 - 0.52]$
$^{82}\text{Se}$	2.995	$> 3.6 \cdot 10^{23}$ [43]	1.01E-14	2.64-4.99	$< [1.09 - 2.05]$
$^{96}\text{Zr}$	3.350	$> 9.2 \cdot 10^{21}$ [44]	2.05E-14	2.19-5.65	$< [4.21 - 10.9]$
$^{100}\text{Mo}$	3.034	$> 1.1 \cdot 10^{24}$ [43]	1.57E-14	3.93-6.07	$< [0.41 - 0.63]$
$^{116}\text{Cd}$	2.814	$> 1.7 \cdot 10^{23}$ [51]	1.66E-14	3.29-4.79	$< [1.28 - 1.87]$
$^{130}\text{Te}$	2.527	$> 2.8 \cdot 10^{24}$ [50]	1.41E-14	2.65-5.13	$< [0.32 - 0.62]$
$^{136}\text{Xe}$	2.458	$> 1.6 \cdot 10^{25}$ [47]	1.45E-14	2.19-4.20	$< [0.16 - 0.31]$
$^{150}\text{Nd}$	3.371	$> 1.8 \cdot 10^{22}$ [45]	6.19E-14	1.71-3.16	$< [3.10 - 5.73]$

of the  $\langle E \rangle$  value, used in the closure approximation, and to (small) variations of the values of the cut-off parameters  $\Lambda_{V,A}$ . According to this consensus, we display in Table 1 the NME values obtained with different nuclear methods. The first row contains our results for  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$  and  $^{82}\text{Se}$ , performed with our code, described in more detail in [27], [28], while the NME values for the other nuclei are taken from different references, as it is specified. The NME values written in parenthesis for  $^{76}\text{Ge}$  and  $^{82}\text{Se}$  isotopes represent the values obtained with QRPA methods, when the s.p. energies were adjusted to the occupancy numbers reported in ref. [33]. One remarks, the NME values obtained with QRPA and ShM methods get closer when s.p. occupancies are adjusted to experiment. This is an important step in putting in agreement the ShM and QRPA results and should be verified for other isotopes as well. In Table 2 we present the results for the Majorana neutrino mass parameters ( $\langle m_\nu \rangle$ ), derived from Eq. (3) together with the values of  $Q_{\beta\beta}$ , the PSFs ( $G^{0\nu}$ ), NMEs ( $M^{0\nu}$ ) and experimental half-lives ( $T_{1/2}^{0\nu}$ ) for nine isotopes for which data exist. The ranges of the NME values were taken from Table 1. According to the consensus concerning the use of different nuclear ingredients/parameters, we reduced the interval of their spread to about a factor of 2 (with one exception). This results in reducing the uncertainty in deriving the constraints on the Majorana light neutrino mass parameters, while taking into account NME values obtained with all the main nuclear methods on the market. The PSF values were obtained by recalculating them with our code, described in [25], but with an improved numerical precision obtained by enhancing the number of the interpolation points on a case-to-case basis, until the results become stationary. We remark that the obtained values are close to both those reported previously in refs. [24] and [25], for

the most of isotopes, but differ from other previous calculations [20], [21], [23] by up to 28%. We mention that our method used for PSF calculations follows the method described in ref. [24], but we built up our own numerical routines. Our results confirm the PSF results reported in [24] obtained with a more rigorous method than those used previously [18]-[23]. Hence, we claim, it is justified the PSF re-calculation with improved methods and the use of the new values in the derivation the neutrino mass parameters. For the experimental half-lives we took the most recent results reported in literature. One observes that the stringent constraints are obtained from the  $^{136}\text{Xe}$  isotope, followed by the  $^{76}\text{Ge}$  one. This is due to both the experimental sensitivity of the experiments measuring these isotopes and to the reliability of the theoretical calculations of the corresponding PSF and NME quantities. One observes that the experiments measuring these isotopes are already exploring the quasi-degenerate scenarios for the neutrino mass hierarchy (which is around 0.5 eV).

#### 4. Conclusions

We reported new calculations of the NMEs and PSFs for the  $0\nu\beta\beta$  decay mode in the hypothesis that it occurs through the exchange of Majorana light neutrinos between the two nucleons inside the nucleus. The NMEs for three isotopes,  $^{48}\text{Ca}$ ,  $^{76}\text{Ge}$  and  $^{82}\text{Se}$ , were calculated with the code described in detail in [27], [28]. The PSFs were recalculated with an approach described in ref. [25] but with an improved numerical accuracy, with an increased number the number of interpolation points for each isotope. We used exact electron w.f. obtained by solving a Dirac equation when finite nuclear size and screening effects are included and, in addition, a Coulomb potential derived from a realistic proton distribution in the daughter nucleus has been employed. Then, we derived new upper limits of the Majorana light neutrino parameters from the analysis of  $0\nu\beta\beta$  decay of nine isotopes. For that, other NME values were selected from literature taking advance of the existed consensus in the community on the use of the nuclear ingredients involved in calculations, such as HOC, FHS and SRCs effects, and on the values of several nuclear input parameters. This allows us to restrict the range of spread of the NME values, reported in the literature, and to reduce the uncertainty in deriving constraints on the Majorana neutrino mass parameters. Our results may be useful to have an up-to-date image on the current neutrino mass sensitivities associated with  $0\nu\beta\beta$  decay measurements for different isotopes, and to better estimate the range of the neutrino masses that can be explored in the future DBD experiments.

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