

# Spectroscopic properties of neutron rich nuclei beyond $^{132}\text{Sn}$ and seniority mixing

H. Naïdja<sup>†,‡,★</sup>, F. Nowacki<sup>†</sup> and K. Sieja<sup>†</sup>

<sup>†</sup> Université de Strasbourg, IPHC, 23 rue du Loess 67037 Strasbourg, France  
CNRS, UMR7178, 67037 Strasbourg, France.

<sup>‡</sup> GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany

<sup>★</sup> Université Constantine 1, (LPMS) 1 route Ain El Bey, 25000 Constantine, Algeria

E-mail: [houda.naidja@iphc.cnrs.fr](mailto:houda.naidja@iphc.cnrs.fr)

**Abstract.** Within the shell model, our calculations have been focused on the study of the spectroscopic properties of the even-even tin isotopes around  $^{132}\text{Sn}$ . In particular the calculations of the low-lying states  $0^+, 2^+, 4^+, 6^+$  energies, and different electromagnetic transitions  $B(E2, J + 2 \rightarrow J)$  have been carried out. Two important points are discussed: the effect of the core excitations on the structure and transition rates in this region, and the possible closure of the sub-shell at  $N=90$ .

## 1. Introduction

The heavy mass region of neutron-rich nuclei around  $^{132}\text{Sn}$ , represents an interesting area for experimental and theoretical nuclear structure research. Theoretically, lot of studies have been performed in this region, within the mean field using RPA or QRPA [1, 2, 3], or within the shell model [4, 5, 6], using realistic and empirical interactions. Sensitivity studies of the (*r* – *process*) show that the nuclear structure data in the mass  $A = 140$  region is most impactful on the calculated abundances.

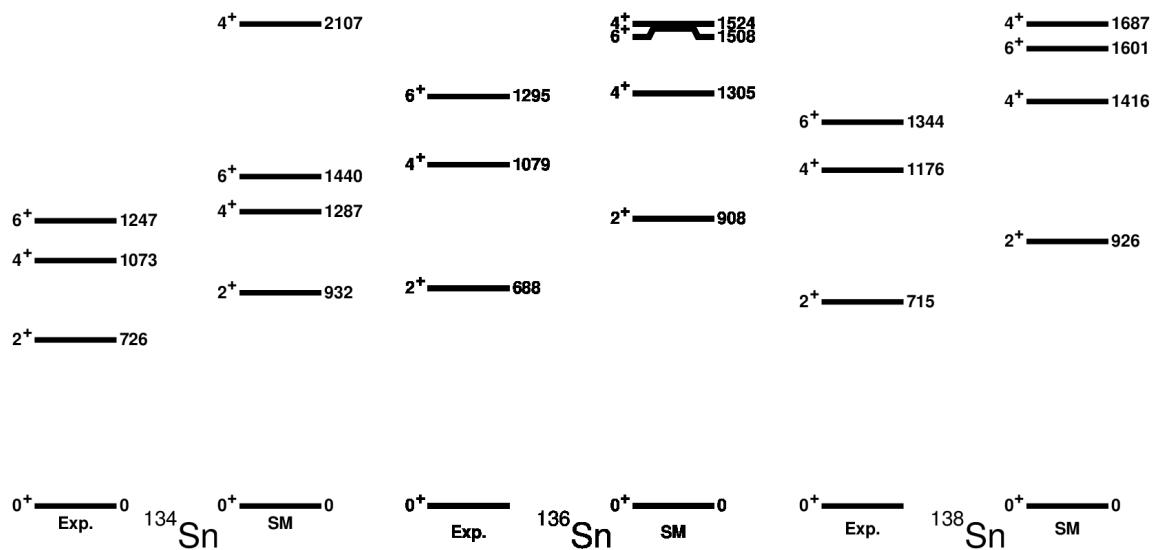
In this exotic region, new experimental data has been obtained recently in RIKEN [7], by the observation of three delayed gamma rays each from  $^{136}\text{Sn}$  and  $^{138}\text{Sn}$ , originating from  $6^+$  isomeric state, and are the first gamma ray transitions observed in these very neutron rich nuclei. These results provided us valuable information to study the nuclear structure of this mass region, and for the development of our empirical shell model interaction.

## 2. Shell Model tools

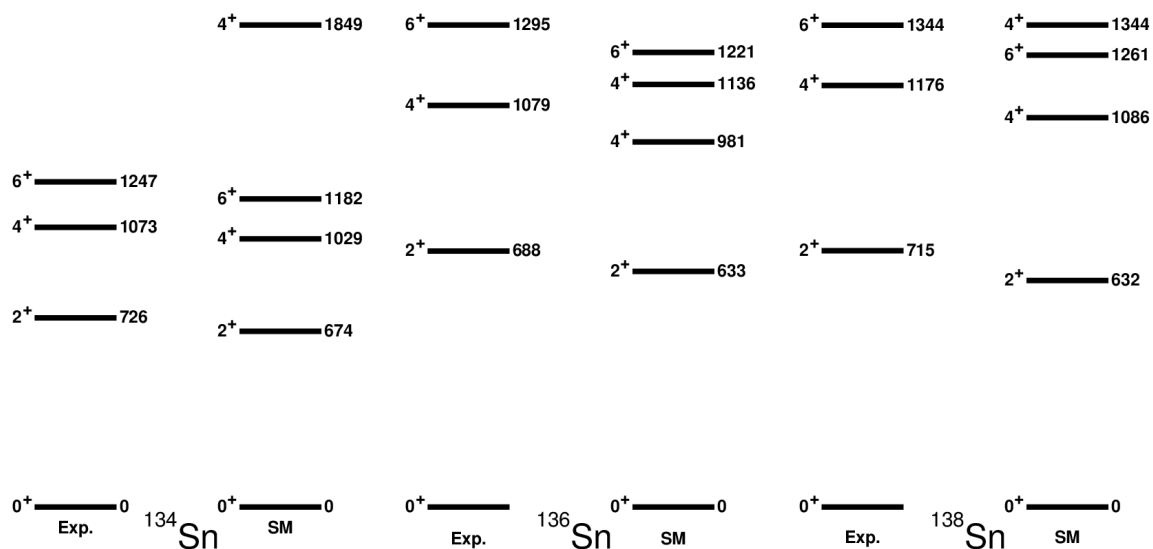
Our calculations have been performed using shell model codes ANTOINE and NATHAN [8, 9], including the  $0h_{11/2}, 1f_{7/2}, 0h_{9/2}, 1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 0i_{13/2}$  orbitals for neutrons and  $0g_{9/2}, 0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}$  orbitals for protons, which are taken above the closed core of  $^{110}\text{Zr}$ . The advantage to take such a model space, is the fact that we have the possibility to do calculations using  $^{110}\text{Zr}$  or  $^{132}\text{Sn}$  as cores, just by closing or opening  $0h_{11/2}$  orbit for neutrons and  $0g_{9/2}$  orbit for protons. Taking into account core excitations constitutes a challenge in the diagonalisation of Hamiltonian matrix due to its extremely large size.

Here we have employed an interaction, which is derived from CD-Bonn potential, renormalized following the so-called  $V_{low-k}$  approach with a cutoff momentum of  $\Lambda = 2.2\text{fm}^{-1}$ ,





**Figure 1.** Level schemes of the tin isotopes, calculated using NNS110 interaction.



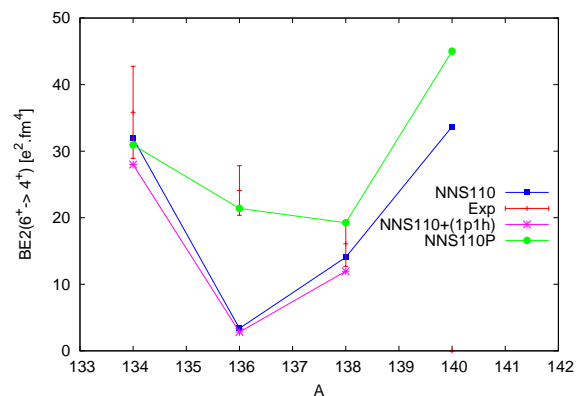
**Figure 2.** Level schemes of the tin isotopes, calculated using NNS110P interaction, obtained by reducing the pairing force in NNS110 interaction.

and adopted to the model space by many body perturbation theory techniques, including all the diagrams up to the second order [11].

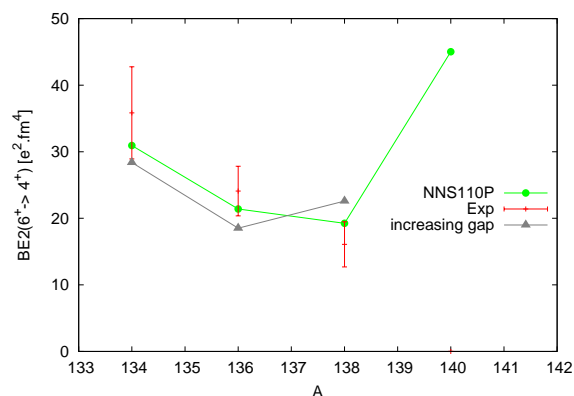
### 3. Applications and results

#### 3.1. Energy levels and transition rates in $^{134,136,138}\text{Sn}$

In the calculations with  $^{110}\text{Zr}$  core, we used as input single particle energies from GEMO model [15], which is the J. Duflo and A.P. Zuker systematic parametrisation of single particle and single hole energies over the nuclear chart. Further, monopole corrections have been applied to  $V_{low-k}$  realistic interaction, to reproduce the experimental structure of  $^{133}\text{Sn}$  and  $^{133}\text{Sb}$  nuclei [10]. As it is displayed in figure 1, this interaction noted by NNS110 does not reproduce correctly the excitation energies of the tin isotopes compared to experimental data [10, 7].



**Figure 3.** Variation of the electromagnetic transitions as function of the mass number, calculated using: NNS110, opening  $^{132}\text{Sn}$  core (1p1h excitations), and NNS110P interactions.



**Figure 4.** The same as figure 3, but increasing the  $f_{7/2} - p_{3/2}$  gap, compared to NNS110P results.

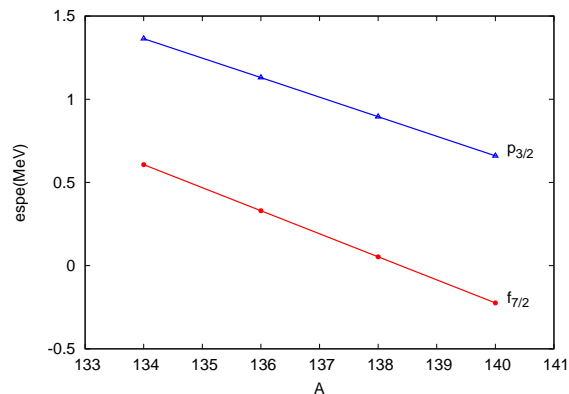
$6^+$  ————— 1661

$4^+$  ————— 1178

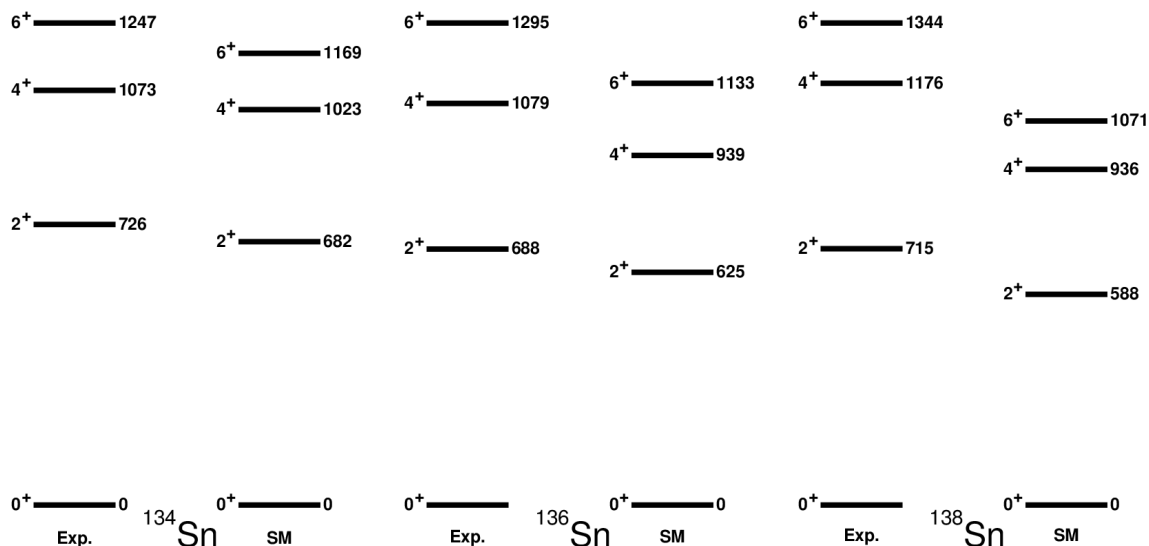
$2^+$  ————— 780

$0^+$  ————— 0  
 $^{140}\text{Sn}$

**Figure 5.** Energy levels of  $^{140}\text{Sn}$  calculated using NNS110P interaction.



**Figure 6.** Variation of the gap between  $f_{7/2}$  and  $p_{3/2}$  as function of the mass number of tin isotopes.



**Figure 7.** The effect of increasing the  $f_{7/2} - p_{3/2}$  gap, to the level schemes of the tin isotopes.

The variation of the electromagnetic transition rates from  $6^+$  to  $4^+$  states, as function of the mass number calculated within the shell model, and compared to experimental data [7, 10] is reported in figure 3. As one can notice, with effective charge  $e_n = 0.64e$ , our calculations predict well the observed transitions from  $^{134}\text{Sn}$  and  $^{138}\text{Sn}$ , but they underestimate it for the mid-shell nucleus  $^{136}\text{Sn}$ . We have performed additional calculations, including  $1p1h$  excitations from  $h_{11/2}$  neutrons and  $g_{9/2}$  protons to the above valence space, with effective charges  $e_n = 0.5e$  and  $e_p = 1.5e$  for neutrons and protons, respectively. In this case, the transition rates in  $^{134,138}\text{Sn}$  are good enough without enhanced effective charges, but still fail to reproduce the exact observed value in  $^{136}\text{Sn}$ . It appears that reducing the diagonal and off diagonal pairing strength of the  $f_{7/2}$  matrix elements by about 130 keV (denoted hereafter by NNS110P), gives us a good agreement with the measured value of  $^{136}\text{Sn}$  transition.

We try to explain this result in the seniority model, where in the two first cases (using NNS110 interaction and opening the core), the  $v = 2$  is the dominant component of the first excited states in  $^{136}\text{Sn}$  (82% in  $4_1^+$  state, and 95% in  $6^+$  state). It is well known that in the seniority selection

rule, transitions between  $6^+$  and  $4_1^+$  states with the same seniority is forbidden, which leads to a vanishing transition strength between them. Reducing the pairing force in the realistic interaction modifies the seniority structure and leads to a more complex two  $4^+$  states: Their wave functions contain  $4_1^+$  (53% of  $\nu = 2$  and 46% of  $\nu = 4$ ), and  $4_2^+$  (43% of  $\nu = 2$  and 57% of  $\nu = 4$ ), leading to a larger transition rate.

Now, our calculations using the NNS110P interaction are in fair agreement with the observed energy levels of the tin isotopes (figure 2). The  $(f_{7/2})^n$  ( $n$  is the valence neutron number varying from 2 to 6) is the principal component of the ground state and the first excited states for all the tin isotopes, where its percentage decreases from 90% to about 50%, going from  $^{134}\text{Sn}$  to  $^{138}\text{Sn}$ . The inversion between the two states  $6^+$  and  $4_2^+$  states in  $^{136}\text{Sn}$  constitutes another consequence of the seniority mixing discussed in previous paragraph. A very similar effect was observed in [14], where the structure and location of the  $6_1^+(\nu = 2)$  and  $6_2^+(\nu = 4)$  states in  $^{72,74}\text{Ni}$ ,  $^{94}\text{Ru}$  and  $^{96}\text{Pb}$  has important implications for the isomeric properties of the  $8^+(\nu = 2)$  state.

Opening  $^{132}\text{Sn}$  core and taking 1p1h excitations into account, has a minor effect ( $\simeq 50$  keV) on the excitation energies, compared to NNS110 interaction.

### 3.2. Sub-shell closure at $N=90$

The evolution of the shell structure is a hot topic in modern nuclear structure studies. The authors of [5] have proposed the closure of the sub-shell  $N=90$  in  $^{140}\text{Sn}$ , by the creation of an important gap between  $f_{7/2}$  and  $p_{3/2}$  orbitals. The trend is very similar to that observed in the  $^{22}\text{O}$  and  $^{48}\text{Ca}$ , where neutron shell gaps increase by adding the neutrons in the high  $j$  orbit ( $1d_{5/2}$  in  $^{22}\text{O}$  and  $1f_{7/2}$  in  $^{48}\text{Ca}$ ). It has been shown in Refs.[12, 13] that the 3-body forces play an important role in creation of these gaps.

Figure 5 displays our predictions of the level scheme of  $^{140}\text{Sn}$  calculated using NNS110P interaction. Excited states are characterized by excitations from  $f_{7/2}$  to  $p_{3/2}$  shell, which leads to an increased transition probability  $B(E2, 6^+ \rightarrow 4^+)$  in this nucleus (see figure 3).

As one can notice from figures 2 and 5, the spacing between  $0^+$  and  $2^+$  remains nearly constant ( $\simeq 700$  keV) for all the tin isotopes. There is just small increase of the  $2^+$  and  $4^+$  states in  $^{140}\text{Sn}$ , and it becomes more significant in  $6^+$  state, which has different configuration  $(f_{7/2})^{-2}(p_{3/2})^2$ . At the same time, the spacing between  $f_{7/2} - p_{3/2}$  shells remains nearly constant, when moving from  $^{134}\text{Sn}$  to  $^{140}\text{Sn}$  (see figure 6). These results constitute an indication of possible no sub-shell closure in  $^{140}\text{Sn}$ .

However, the results in [5] show that using an empirical interaction SMPN, there is a sudden increase of the  $2^+$  at  $N=90$ , and the gap  $f_{7/2} - p_{3/2}$  grows by about 2.2MeV, leading to double magicity in  $^{140}\text{Sn}$ .

We have imposed this spacing by changing the monopole part. As a consequence, the  $2^+$  energy increases in  $^{140}\text{Sn}$  to 1815 keV, but at the same time, we lose the spectroscopic properties (figure 7) and the transition rates (figure 4) of the remaining tin isotopes.

## 4. Conclusions

The discovery of new isomers in very exotic magic nuclei [7] is a key for the development of the theory far from stability, the nuclear shell model in particular. Here, we have investigated excitation energies and transition rates in  $^{134,136,138}\text{Sn}$  isotopes, and our results using NNS110P interaction seem to be in fair agreement with experiment.

Reducing the pairing force of the realistic interaction was necessary to reproduce the observed transition rates in  $^{136}\text{Sn}$ , which leads to a seniority mixing effect. This slight modification of the pairing force has an apparent effect on the excitation energies, and to the position of the second  $4^+$  state.

For the first time, we have also studied the effect of the opening of  $^{132}\text{Sn}$  core on the transition rates, which showed that it plays a minor role in the evolution of the excitation energies and transition rates, confirming the strong magicity of  $^{132}\text{Sn}$ .

Finally, we have studied how the modification of the  $f_{7/2} - p_{3/2}$  gap influences the results and we conclude that a large  $N = 90$  gap is incompatible with the observed excitation energies and  $B(E2)$  transitions in the tin isotopes.

### Acknowledgments

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