

Shell model description of 0 and 1 $\hbar\omega$ states in neutron-rich N = 18 and 19 nuclei with Z = 14 to 19

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Abstract. The low excitation energy spectra of N = 18 and 19 sd-shell isotones are investigated. A detailed comparison experiment versus theory using the PSDPF interaction of the energy spectra for ³²Si and ³³Si, ³³P and ³⁴P, ³⁴S and ³⁵S, ³⁵Cl and ³⁶Cl, ³⁶Ar and ³⁷Ar, ³⁷K and ³⁸K will be presented. Based on this comparison, important predictions will be proposed and the general evolution of the main structure properties in these isotones will be discussed.

1. Introduction

The neutron-rich sd-shell nuclei around N = 20 are the subject of active experimental and theoretical studies and exhibit a rich variety of phenomena and physics such as the *island of inversion region*. Currently, much more data essentially obtained from Radioactive Ion Beam (RIB) facilities are available for nuclei near the N = 20 gap. The properties of these nuclei pose a challenge to conventional models of theoretical nuclear structure and a deeper understanding of the underlying physics. Study of such nuclei provides information about the evolution of their structure. For this purpose, we are interested to study this area of nuclei in the framework of the shell model. We remind that the spectra of sd-shell nuclei are characterized by the presence, at low excitation energies, of both normal positive and intruder negative parity states. Intruder 1 $\hbar\omega$ negative parity states are known throughout the sd-shell. These intruders result essentially from a one-nucleon jump from p to sd or from sd to pf shells. Our PSDPF interaction [1], mainly developed to describe such states, reproduces remarkably well the spectroscopic properties of 0 and 1 $\hbar\omega$ states in nuclei throughout the sd-shell and in particular in the upper shell closure region.

We performed shell model calculations using the PSDPF interaction in order to describe the spectroscopic properties of some neutron-rich nuclei such as the N = 20 isotones with Z = 14–16 [2], ³⁶S [3] and ^{34,35}P [4]. We also studied the Phosphorus and Sulfur isotopic chains; the results will be published later. In general, the calculations agree quite well with experiment and some predictions could also be proposed.

Recently, many experiments were implemented to investigate the properties of neutron-rich nuclei through fusion and multi-nucleon transfer reactions. Most of the obtained results could be interpreted in the framework of the shell model using the PSDPF interaction, particularly in the case of ³³Si [5],



^{36}Cl [6], ^{37}Cl [7], ^{35}S [8] and $^{34,35,36}\text{P}$ [9] for which all the low-lying spectroscopic properties are well described by PSDPF.

In this paper, we will present a comparison of our results using PSDPF to available experimental data of the spectroscopic properties for $N = 18$ and 19 isotones with $Z = 14$ to 19 .

Based on this comparison, predictions for unknown spin-parity of several states will be proposed and the general evolution of the main structure properties in these isotones will be discussed.

2. Results and discussion

The performed shell model calculations using the shell-code Nathan [10-12] of the energy spectra with the PSDPF interaction for $N = 18$ and 19 isotones with $Z = 14$ to 19 will be presented. We will discuss the results for each nucleus.

The ^{32}Si spectrum is known up to 7100 keV [13]. It contains nine well-identified positive parity states and seven negative parity states as shown on Table 1. The low levels are well described using PSDPF. This is reflected by the small energy ΔE differences. Above ~ 6000 keV, the calculated excitation energies become higher than the experimental ones. This is due to the fact that their configurations contain more than $0p-0h$ and $1p-1h$ components, which is expected in this high-energy range. The $J^\pi = 0^-_1, 3^+_1, 2^+_4$ and 1^-_2 assignments are proposed for the uncertain states at $5791, 5893, 6170$ and 6195 keV, respectively. We also propose $J^\pi = 1^+_1, 2^-_1$ and 0^+_3 values for the undetermined states at $5220, 5773$ and 5786 keV, respectively. PSDPF predicts a negative parity for the $J = 2$ state at 5956 keV.

Table 1. Comparison between experimental (Eexp) [13] and calculated (Eth) excitation energies (in keV) for ^{32}Si , with $\Delta E = \text{Eth} - \text{Eexp}$.

J^π	Eexp	J^π_i	Eth	ΔE	J^π	Eexp	J^π_i	Eth	ΔE
0^+	0	0^+_1	0	0	(0^-)	5791	0^-_1	5416	-375
2^+	1941	2^+_1	2043	102	(3^+)	5893	3^+_1	5602	-291
2^+	4231	2^+_2	4306	75	2	5956	2^-_2	6227	271
0^+	4984	0^+_2	5185	201	3^-	5967	3^-_2	6291	324
$(1-4)$	5220	1^+_1	5611	391	(2^+)	6170	2^+_4	6985	815
3^-	5289	3^-_1	5495	206	$1^{(-)}$	6195	1^-_2	6717	522
1^-	5413	1^-_1	5241	-172	0^+	6240	0^+_4	7943	1703
2^+	5427	2^+_3	5620	193	2^+	6388	2^+_5	7452	1064
4^+	5502	4^+_1	5589	87	3^-	6477	3^-_3	6395	-82
5^-	5581	5^-_1	5725	144	1^-	6705	1^-_3	6950	245
$(1,2,3)$	5773	2^-_1	5445	-328	3^-	6860	3^-_4	7435	575
$(0,1,2)^+$	5786	0^+_3	5531	-255	2^+	7083	2^+_6	7887	804

Table 2. Comparison between experimental [13] and calculated spectra (in keV) for ^{33}Si .

J^π	Eexp	J^π_i	Eth	ΔE
$3/2^+$	0	$3/2^+_1$	0	0
$1/2^+$	1010	$1/2^+_1$	798	-212
$7/2^-$	1435	$7/2^-$	1381	-54
$(3/2^-)$	1981	$3/2^-$	1703	-278
$(9/2^-)$	3159	$9/2^-$	3157	-2
$(11/2^-)$	4090	$11/2^-$	4129	39
$3/2^+, 5/2^+$	4341	$3/2^+_2$	4192	-149
$3/2^+, 5/2^+$	4341	$5/2^+_1$	4222	-119
$(11/2^-)$	4931	$11/2^-$	495	44

The ^{33}Si spectrum contains nine levels up to ~ 5000 keV [13]. Among them, only the three first states are well identified and quite nicely reproduced by PSDPF. Concerning the other states (see Table 2), all the proposed J^π agree with the PSDPF predictions. According to this comparison, we suggest that there is a doublet of states at 4341 keV with $J^\pi = 3/2^+$ and $5/2^+$.

The ^{33}P spectrum contains 25 levels up to ~ 5750 keV. Four states at 3990, 5207, 5221 and 5235 keV, were recently observed in the fusion-evaporation $^{18}\text{O}(^{18}\text{O}, 2\text{np})$ reaction [13]. Note that in this type of reactions, states with high spins are selectively populated. These states are not predicted by the PSDPF interaction (not in the valence space). Thus we do not find theoretical counterparts for these levels. However, all other observed states are quite well reproduced by PSDPF. The only exception is for the $5/2_4^+$ and $5/2_5^+$ at 4194 and 5049 keV, which are predicted to be higher than experiment by more than 800 and 1400 keV, respectively, and thus contain more than a $0h\omega$ configuration. The energy differences of the remaining levels vary from 9 keV for the $3/2_1^+$ to 244 keV in the cases of $7/2_1^-$ and $9/2_1^-$. For this $N = 18$ isotone, we propose $J^\pi = 5/2_1^-, 5/2_2^-, 9/2_1^+$ and $7/2_2^-$ for the unidentified states at 5191, 5411, 5498 and 5549 keV, respectively. Recently the experimental ^{34}P spectrum was extended and a detailed comparison with our PSDPF results has been discussed in Ref. [4].

We then examine the ^{34}S spectrum which is well known up to ~ 6500 keV and comprises 26 levels. The PSDPF results are in quite good agreement with experiment and all the reported states have their theoretical counterparts. Starting from ~ 6000 keV, the contribution of more complex than $0p-0h$ and $1p-1h$ configurations is dominant for the six states with $J^\pi = 0_4^+, 2_{6,7}^+, 4_2^+$ and $1_{2,3}^-$. These states are predicted to be higher than their experimental counterparts. Recent results, in particular for the high spin levels, are reported for ^{35}S . A comparison with shell model calculations using the PSDPF interaction and other Hamiltonians is discussed in Ref. [8].

Starting from ^{35}Cl , we approach the neutron and proton ~ 20 gaps with occupation of the $d_{3/2}$ sub-shell. The ^{35}Cl spectrum is well known up to ~ 6100 keV and contains a large number of states (~ 40). More than eleven states have negative parity. PSDPF describes reasonably well the observed excitation energies and important predictions for the uncertain states are proposed. We predict $J^\pi = 5/2_3^+$ and $3/2_4^+$ for the doublet of states at 4624 keV as well as $J^\pi = 1/2_3^+, 1/2_1^-, 1/2_2^-, 5/2_4^+, 3/2_3^-, 3/2_5^+, 7/2_4^-, 3/2_4^-, 3/2_5^-, 5/2_3^-$ and $5/2_4^-$ for states observed at 4839, 4854, 5010, 5216, 5404, 5600, 5645, 5683, 5758, 5806, 5823 keV, respectively. The parities of the $J = 7/2$ levels at 4769 and 4881 keV are proposed to be negative and positive, respectively. States at 3979, 5157, 5520 and 5633 keV do not have their theoretical counterparts. We think that these levels have more than 0 and 1 $h\omega$ configurations and thus are predicted by our model at higher excitation energies. This phenomenon is expected, especially, in nuclei near the gaps. New excited states, mainly those of high spins, have been populated in ^{36}Cl through fusion-evaporation reactions. The complete observed spectrum of ^{36}Cl has been compared to the PSDPF prediction and the results are discussed in Ref. [6].

We now discuss the main results of the $Z, N = 18$ and 19 nuclei, i.e., nuclei close to the $Z, N = 20$ gaps. The self-conjugate nucleus ^{36}Ar , is well studied, its spectrum comprises, up to ~ 8200 keV, 40 levels, 17 of them have negative parity. Good agreement is found between PSDPF results and experiment for both positive and negative parity states, except for the 4_2^- at 6835 keV, which is predicted at 6430 keV. As it was mentioned previously, when approaching the proton and neutron gaps, excitations of few particles across the gaps appear rapidly in the spectrum. Levels observed at 6137, 6611, 6837, 7139, 7432, 7672 and 7710 keV have more complex than $0p-0h$ and $1p-1h$ components. However, states at 4951, 6356, 7750, 7767 and 8132 keV are known to be purely collective. It was difficult to find candidates for states at 5194, 6646, 6867 and 7179 keV due to the same reasoning as in ^{35}Cl .

Regarding ^{37}Ar and ^{37}K mirror nuclei, the results are compared to the PSDPF predictions up to ~ 4000 keV on Table 3. In this excitation energy range, there is a one to one correspondence between mirror levels as well as with the PSDPF ones, except for the $11/2^-$ state not observed in ^{37}K . PSDPF

reproduces quite well the observed excitation energies. According to this comparison, the energy of the ^{37}K $7/2^+$ state is proposed to be 2285 keV. The third $3/2^+$, observed at 3937 keV in ^{37}Ar and known to be at 3839 keV in ^{37}K , is predicted at 4986 keV and is thus probably a more complex than $0p-0h$ state. We note that the ^{37}Ar nucleus is known up to 5440 keV.

Table 3. Comparison between experimental [13] and calculated excitation energies (in keV) of the ^{37}Ar and ^{37}K mirrors

^{37}Ar		PSDPF			^{37}K		
J^π	Eexp	J_i^π	Eth	ΔE	J^π	Eexp	ΔE
$3/2^+$	0	$3/2^+_1$	0	0	$3/2^+$	0	0
$1/2^+$	1410	$1/2^+_1$	1489	79	$1/2^+$	1371	118
$7/2^-$	1611	$7/2^-_1$	1433	-178	$7/2^-$	1380	53
$7/2^+$	2217	$7/2^+_1$	2416	199	$(5/2^+, 7/2^+)$	2285	131
$3/2^-$	2490	$3/2^-_1$	2788	298	$3/2^-$	2170	618
$5/2^+$	2796	$5/2^+_1$	2940	144	$5/2^+$	2750	190
$5/2^+$	3170	$5/2^+_2$	3179	9	$5/2^+$	3240	-61
$9/2^-$	3185	$9/2^-_1$	2982	-203		2967	15
$5/2^-$	3274	$5/2^-_1$	3109	-165	$5/2^-$	3082	27
$3/2^-$	3518	$3/2^-_2$	3383	-135	$3/2^-$	3315	68
$7/2^-$	3527	$7/2^-_2$	3638	111		3272	366
$3/2^+$	3602	$3/2^+_2$	3874	272	$3/2^+$	3622	252
$11/2^-$	3706	$11/2^-_1$	3813	107			
$3/2^+$	3937	$3/2^+_3$	4986	1049	$1/2^+, 3/2^+, 5/2^+$	3839	1147

The last isotone studied here is ^{38}K with $N=19$. PSDPF describes its energy spectrum between 0 and ~ 5300 keV. The np-nh excitations appear rapidly at low energy, this is the case of states at 3341, 3430, 3856 and 3931 keV. In this situation, it is almost impossible to find theoretical equivalents for states observed at 3458, 3668, 3704, 3739, 3977, 4175, 4317, 4394 and 5254 keV.

3. Evolution of the 3^- and $7/2^-$ states

In this section the evolution of the excitation energies of specific states in the $N=18$ and 19 isotones are presented. Let us start with the first excited positive parity state in these isotones. Figure 1 shows the evolution of the experimental versus calculated first positive parity states for even and odd A nuclei.

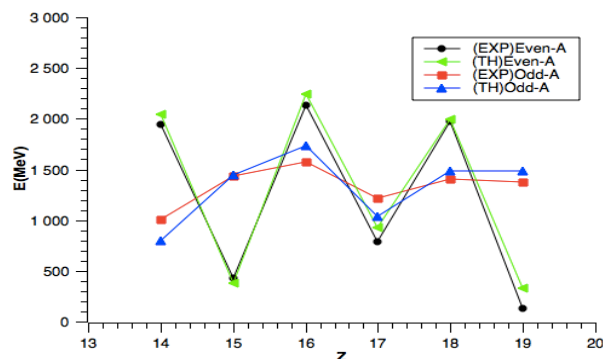


Figure 1. Excitation energy evolution of the first positive parity states for even and odd A in $N=18$ and 19 isotones (EXP versus TH).

As it is seen on this figure, the PSDPF interaction reproduces very well all the observed states and their energy variation throughout the sd-shell. In the case of the odd-A nuclei, the excitation energies are practically constant and vary between 1000 and 1500 keV for all the levels. As far as the even-A nuclei are concerned, the excitation energies, as expected for even-even nuclei with $N=18$, are higher than those for the odd-odd ones with $N=19$.

The evolution of the first 3^- and $7/2^-$ states in $N=18$ and 19 isotones is shown on Fig. 2. Note that the $7/2^-$ is the first excited negative parity state in odd-A nuclei. The 3^- is the first excited negative parity state in the even-A isotones except for the odd-odd ^{34}P and ^{36}Cl . Here also PSDPF describes very well the observed excitation energies of these states as well as their variation throughout the sd-shell. As can be seen in Fig. 2, the $7/2^-$ levels appear at low excitation energies close to the first excited positive parity states. This is not the case for the $N=18$ isotones ^{33}P and ^{35}Cl , where these states have high excitation energies. This fact is not observed in ^{37}K with $Z=19$.

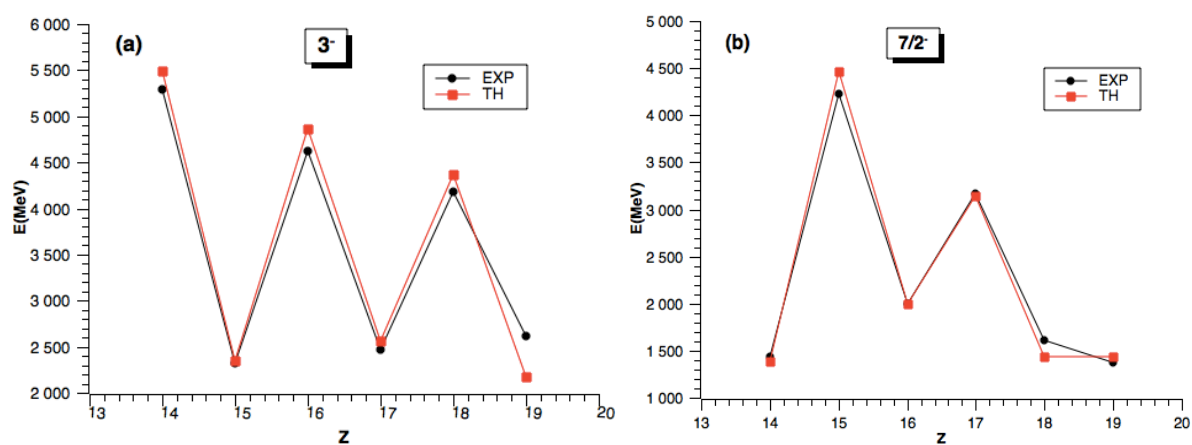


Figure 2. Experimental and calculated excitation energies of (a) 3^- states in $N=18$ and 19 isotones and (b) $7/2^-$ states in $N=18$ and 19 isotones.

4. Conclusion

The PSDPF interaction describing the 0 and $1\hbar\omega$ states throughout the sd-shell has been used. The results discussed here are for $N=18$ and 19 isotones, for which PSDPF reproduces quite well both positive and negative parity states. In the middle of the sd-shell, the $1\hbar\omega$ states have very fragmented (*competition of p-sd and sd-pf jumps*) wave functions, which complicate their description. We would like to retune our interaction in the near future using a new fitting code, developed by E. Caurier, where we can use much more data from nuclei in the middle of the sd-shell. It is also important that future studies cover cases approaching the N and $Z \sim 20$ gaps, where the np-nh states become rapidly important even at low energies. To describe them, a new interaction called SDPF-U-Mix, allowing such higher order jumps has to be developed.

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