

# Inter-band B(E2) transitions strengths in $^{160-170}\text{Dy}$ nuclei

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**Abstract.** The rare earth region of the nuclear landscape is characterized by a large collectivity observed. The microscopic studies are difficult to perform in the region due to the enormous size of the valence spaces. The use of symmetries based models avoids that problem, because the symmetry allows to choose the most relevant degrees of freedom for the system under consideration. We present theoretical results for electromagnetic properties in  $^{160-168}\text{Dy}$  isotopes employing the pseudo-SU(3) model. In particular, we study the B(E2) inter-band transition strengths between the ground state,  $\gamma$  and  $\beta$ -bands. The model successfully describes in a systematic way rotational features in these nuclei and allows to extrapolate toward the midshell nucleus  $^{170}\text{Dy}$ .

## 1. Introduction

The rare-earth region of the nuclear landscape provides an excellent opportunity to study unique aspects of nuclear structure around midshell region as new modes of excitation and collectivity, the role of single-particle levels [1] and even possible changes in the shell structure. Nevertheless, despite the recent progress in experimental techniques, the rare-earth neutron-rich nuclei remains a region of the nuclear chart which has been seldom studied. For example, in heavy-ion induced multinucleon transfer reaction, the combination of stable projectiles and stable targets has unfavorable kinematic matching conditions leading to heavy neutron-rich nuclei, resulting in small cross sections that limits its use for populating neutron-rich nuclei [2].

Many low-lying collective nuclear properties are expected to appear in this region, because there nuclei have large numbers of valence protons and neutrons. The neutron-rich nuclei Er, Yb and Hf exhibit a minimum in the  $2^+$  energies and a maximum of deformation at  $N=104$  [3]. Assuming standard spherical shell gaps,  $^{170}\text{Dy}$  has the larger number of valence particles of any nucleus lighter than the doubly magic lead, and hence it is expected to be the most collective nucleus in the region [4, 5, 6, 7]. Experimental works in the lighter dysprosium isotopes have shown that there is an enhancement of the collectivity in  $^{164}\text{Dy}$ , a relative flatness of the moment of inertia in  $^{166}\text{Dy}$ , and  $^{168}\text{Dy}$  shows again an apparent enhancement of deformation [8]. Nevertheless, to date the energies of the levels in  $^{170}\text{Dy}$  have not been measured, and hence its degree of collectivity is still unknown. In addition, the known saturation of B(E2) in the region [5, 6, 7] and the decrease in moment of inertia between  $N=98-100$  in the gadolinium isotopes [9]



open the following question of the collectivity in the region: is  $^{170}\text{Dy}$  indeed the most collective of the region or is a little of the collectivity lost when we reach  $N=104$ ?

There are very different channels for producing the dysprosium neutron-rich nuclei. Between these, C. Wu and coworkers have used quasielastic two-neutron transfer reaction to study the neutron-rich  $^{166}\text{Dy}$  [2], two-proton pick-up reaction has allowed to populate  $^{168}\text{Dy}$  [8, 10], and projectile fragmentation of  $^{208}\text{Pb}$  was used [11, 12] to synthesize the midshell  $^{170}\text{Dy}$ .

Regarding the use of theoretical schemes, the large valence spaces associated with heavy-nuclei have implied a slow progress, limiting the number of models capable of dealing with  $^{170}\text{Dy}$ . A shell model description of heavy nuclei requires assumptions that include a systematic and proper truncation of the Hilbert space. The symmetry based SU(3) shell model [13, 14] has been successfully applied in light nuclei, where a harmonic oscillator mean-field and a residual quadrupole-quadrupole interaction can be used to describe dominant features of the nuclear spectra. However, the strong spin-orbit interaction renders the SU(3) model useless in heavier nuclei, while at the same time pseudo-spin emerges as a good symmetry [15, 16]. This symmetry has its origin in the relativistic mean field for heavy-nuclei [17, 18]. The consistency of this symmetry has determined the success of the pseudo-SU(3) model [19].

Four decades have passed since the pseudo-SU(3) model was first introduced. During that time period, and especially over the last few years, the pseudo-SU(3) model has been used to describe a wide variety of very different nuclear phenomena. Initially it was considered the pseudo-spin as a dynamical symmetry [20]. With the development of a computer code it has been possible to calculate reduced matrix elements of physical operators (i.e. the breaking-symmetry Nilsson single-particle energies and pairing correlations) between different SU(3) irreps [21]. This enabling technology allows, among others, fully microscopic studies of energy levels belonging to normal parity bands, BE2 intra and interband transitions and the analysis of the interplay between orbital and spin components of the scissors mode [22, 23, 24, 25] in both, even-even and A-odd nuclei. The high density of  $0^+$  states in  $^{158}\text{Gd}$  has also been described with this model [26]. Despite this long history, however, the model has been rarely used to predict the phenomenology in nuclei far from the stability region. Nevertheless, in a recent work a systematic parametrization of the interaction in the frame of the pseudo-SU(3) model [27] for the  $^{160-168}\text{Dy}$  chain of dysprosium isotopes has allowed to calculate energies in the ground state,  $\gamma$  and  $\beta$ -bands. The calculations for these known nuclei were then extrapolated towards the neutron-rich nucleus  $^{170}\text{Dy}$ .

Our starting points are the results obtained in a previous study [27]. In the present work, we use the eigenfunctions and eigenvalues of the Hamiltonian (2) of Ref. [27] in order to calculate the electric quadrupole transition strengths between states belonging to different bands and with values larger than  $0.00001 e^2b^2$ . Our goal is to study the evolution of collectivity along the dysprosium isotopic chain ( $Z=66$ ) starting at  $N = 94$  and ending with the  $N = 104$  midshell  $^{170}\text{Dy}$  nucleus [28].

In contrast to the results reported [27] for the B(E2) strengths (between states belonging to the same band), in this contribution we are interested in the evaluation of cross transitions, that is between states belonging to different bands (inter-band). The calculation of these values is important in order to understand the nature of the excited  $0^+$  and  $2^+$  bands, which are usually referred as  $\beta$  and  $\gamma$  bands, respectively. As it has been pointed out by P. E. Garrett [29], the nomenclature used for the excited  $0^+$  bands should be done attending their properties, which involves the assessment of some B(E2) transitions between ground state,  $\gamma$  and  $\beta$  bands. In this way, the present calculations help to discriminate which of the  $0^+$  excitations in  $^{160-170}\text{Dy}$  correspond to  $\beta$  oscillations.

This contribution is organized as follows: The group theory that underlies the pseudo-SU(3) model is reviewed in the first part of the next section. Only a very limited number of results are given because a rather extensive discussion of the correspondence between the quantum

numbers, Casimir operators, and pseudo-SU(3) wave functions can be found in previously published articles [20, 30]. The pseudo-SU(3) Hamiltonian and a choice for the corresponding interaction strength parameters are also discussed, as they are of special importance for a theoretical description of the  $^{160-170}\text{Dy}$  isotopes. In the final part of Sect. 2, definitions of experimentally measured quantities are reviewed. The pseudo-SU(3) model results for inter-band B(E2) transition strengths are presented in Sect. 3. A summary as well as a brief conclusion are given in Sect. 4.

## 2. The model

The pseudo-SU(3) model is a many-particle shell-model based theory that takes full advantage of pseudospin symmetry, which in heavy nuclei is manifest in the near degeneracy of the orbital pairs  $[(l-1)_{j=l-1/2}, (l+1)_{j=l+1/2}]$ . It is also a theory that takes full account of the Pauli Exclusion principle. Like most other shell-model schemes, the proton and neutron configuration spaces of the pseudo-SU(3) model are usually restricted to a single oscillator shell. A recent extension of the pseudo-SU(3) model takes the  $\tilde{S} = 0$  and 1 (proton and neutron) spin degrees of freedom into account in a full proton-neutron formalism [25]. This extended theory was used to investigate the effect of different nuclear interactions on the low-energy properties of even-even and  $A$ -odd nuclei, where the influence of pairing and the remaining spin-orbit [22] correlations are known to play an important role.

The pseudo-SU(3) scheme is an algebraic shell-model theory that exploits powerful group theoretical methods in the construction of basis functions and for the calculation of required matrix elements. Specifically, basis states are labeled by eigenvalues of Casimir operators of the underlying symmetry groups and additional indices that are required to resolve multiplicities in the group reductions. Since a full discussion of these matters can be found elsewhere [7, 20], only the results will be included in this contribution. In terms of the space ( $U(N) \leftrightarrow [f]$ ), shape ( $SU(3) \leftrightarrow (\lambda, \mu)$ ), orbital ( $SO(3) \leftrightarrow L$ ), spin ( $S$ ) and total angular momentum ( $J$ ) as well as the various multiplicities ( $\alpha$  for the  $U(N) \supset SU(3)$  reduction,  $\rho$  for the product of two SU(3) irreps, and  $\kappa$  for the  $SU(3) \supset SO(3)$  reduction) the basis states have the form (subscript  $\pi$  for protons and  $\nu$  for neutrons):

$$|\{\alpha_\pi(\lambda_\pi, \mu_\pi)\kappa_\pi S_\pi, \alpha_\nu(\lambda_\nu, \mu_\nu)\kappa_\nu S_\nu\}\rho(\lambda, \mu)\kappa LS; JM\rangle. \quad (1)$$

The occupation numbers for protons are constant along the chain. In shell model applications, the dysprosium isotopes are considered to have 16 protons out of the  $Z = 50$  inert core, 10 of these in normal and 6 in abnormal  $h_{11/2}$  parity levels. For neutrons, the occupations numbers in normal parity levels change from 8 to 14.

The Hamiltonian contains spherical Nilsson single-particle terms for the protons and neutrons ( $H_{sp,\pi[\nu]}$ ), the quadrupole-quadrupole ( $\tilde{Q} \cdot \tilde{Q}$ ) and pairing ( $H_{pair,\pi[\nu]}$ ) interactions parametrized systematically, as well as three rotorlike terms ( $K^2$ ,  $J^2$  and  $C_3$ ) that are diagonal in the SU(3) basis:

$$H = \sum_{\alpha=\pi,\nu} \{H_{sp,\alpha} - G_\alpha H_{pair,\alpha}\} - \frac{1}{2} \chi \tilde{Q} \cdot \tilde{Q} + a K^2 + b J^2 + c \tilde{C}_3. \quad (2)$$

A detailed analysis of each term of this Hamiltonian and its parametrization can be found in Ref. [22]. The first row contains the basic components of any realistic Hamiltonian: the single-particle levels, pairing correlations and the quadrupole-quadrupole interaction, essential in the

description of deformed nuclei. They have been widely studied in nuclear physics literature, allowing to fix their respective strengths by systematics. The rotorlike terms in Hamiltonian (2) are used to fine tune the spectra. Their three parameters  $a$ ,  $b$ , and  $c$  were determined [27] by applying best fit to the experimental data for the five even-even isotopes  $^{160-168}\text{Dy}$ . The states from  $J^\pi = 0^+$  to  $6^+$  belonging to ground,  $\gamma$  and  $\beta$  bands were used in the best fit procedure. In the present contribution, we are using the values  $a = 20$ ,  $b = -3.2$  and  $c = 0.033$  (all in [keV]).

Before discussing comparisons of electromagnetic properties to experimental data, it is important to write down definitions for transition operators and other quantities.

The electric quadrupole transition operator, following the notation of Ref. [20], is defined as

$$T_{2\mu}(E2) \equiv b_0^2 \left( e_\pi \sum_{i_\pi} \sqrt{\frac{16\pi}{5}} r_\pi^2(i) Y_{2\mu}(\theta_i^\pi, \phi_i^\pi) \right) + b_0^2 \left( e_\nu \sum_{i_\nu} \sqrt{\frac{16\pi}{5}} r_\nu^2(i) Y_{2\mu}(\theta_i^\nu, \phi_i^\nu) \right).$$

The SO(3) reduced matrix elements of a tensor operator  $T_{JM}$  between states of initial (final) angular momentum and projection  $J_i$  and  $M_i$  ( $J_f$  and  $M_f$ ) is defined by

$$\langle \gamma_f J_f M_f | T_{JM} | \gamma_i J_i M_i \rangle \equiv \langle \gamma_f J_f M_f | T_{JM} | \gamma_i J_i M_i \rangle$$

where  $\gamma_i$  and  $\gamma_f$  represent additional quantum numbers that are required to uniquely define the initial and final states, and  $\langle \gamma_f J_f M_f | T_{JM} | \gamma_i J_i M_i \rangle$  stands for the reduced matrix element. It follows from this, that the reduced transition probability for electric quadrupole transition [30] is given by

$$B(E2; \gamma_i J_i \rightarrow \gamma_f J_f) \equiv \frac{2J_f + 1}{2J_f - 1} \langle \gamma_f J_f M_f | Q_2 | \gamma_i J_i M_i \rangle^2.$$

The definition of the electric quadrupole moment is given by the last expression when  $\gamma_i = \gamma_f$  and  $J_i = J_f$ . In that case, the electric quadrupole moment is given by

$$Q(\gamma J) \equiv \sqrt{\frac{16\pi}{5}} \sqrt{\frac{J(2J-1)}{(J+1)(2J+3)}} \langle \gamma J M | Q_2 | \gamma J M \rangle.$$

### 3. Inter-band B(E2) transition strengths

As indicated above, in this contribution the pseudo-SU(3) model is used to investigate electric quadrupole transition strengths in  $^{160-170}\text{Dy}$  and to compare with experimental data. The experimental energy spectra and their corresponding theoretical values have been published in [27].

Using the basis states, Hamiltonian and parametrization presented in Reference [27], we present the results for inter-band B(E2) strengths between states of ground state,  $\gamma$  and  $\beta$ -bands in the  $^{160-170}\text{Dy}$  isotopes in Table 1.

The effective charges used in the electric quadrupole transition operator  $Q_\mu$  are  $e_\pi = 2.3$  and  $e_\nu = 1.3$ . These values are the same used in the pseudo-SU(3) studies up to now allowing to describe both intra- and inter-band B(E2)s. They are larger than those used in standard calculations of quadrupole transitions [28] due to the absence of nucleons in intruder levels, and they were not varied to fit any particular value. These results may be the starting point to realize a systematic analysis of the  $0_2^+$  excitations and to determine whether they are or not  $\beta$

$J_{i,band}^{\pi} \rightarrow J_{f,band}^{\pi}$	B(E2) [ $e^2b^2 \times 10^{-2}$ ]					
	$^{160}\text{Dy}$	$^{162}\text{Dy}$	$^{164}\text{Dy}$	$^{166}\text{Dy}$	$^{168}\text{Dy}$	$^{170}\text{Dy}$
$0_{g.s.b.}^+ \rightarrow 2_{\gamma}^+$	14.401	14.603	16.501	2.306	1.703	18.401
$2_{g.s.b.}^+ \rightarrow 4_{\gamma}^+$	1.577	1.602	0.672	0.346	0.358	0.507
$4_{g.s.b.}^+ \rightarrow 5_{\gamma}^+$	4.034	4.070	3.369	0.041	0.037	4.734
$4_{g.s.b.}^+ \rightarrow 6_{\gamma}^+$	0.190	0.186	0.081	0.105	0.102	0.081
$6_{g.s.b.}^+ \rightarrow 7_{\gamma}^+$	2.133	2.166	1.081	*0.001	*0.001	2.596
$6_{g.s.b.}^+ \rightarrow 8_{\gamma}^+$	0.021	0.153	1.227	0.037	0.013	0.259
$8_{g.s.b.}^+ \rightarrow 9_{\gamma}^+$	0.643	0.659	0.121	0.048	0.050	1.333
$8_{g.s.b.}^+ \rightarrow 10_{\gamma}^+$	0.021	0.912	2.479	0.042	0.144	1.039
$0_{g.s.b.}^+ \rightarrow 2_{\beta}^+$	0.196	0.017	0.011	1.044	1.115	0.408
$2_{g.s.b.}^+ \rightarrow 4_{\beta}^+$	0.313	*0.001	0.024	0.386	0.347	0.078
$4_{g.s.b.}^+ \rightarrow 6_{\beta}^+$	0.068	0.003	0.063	0.024	0.027	0.035
$6_{g.s.b.}^+ \rightarrow 8_{\beta}^+$	0.005	0.007	0.091	0.198	0.206	*0.001
$0_{\beta}^+ \rightarrow 2_{\gamma}^+$	4.356	4.383	3.442	180.534	173.640	0.426
$2_{\beta}^+ \rightarrow 3_{\gamma}^+$	1.374	1.080	1.775	108.443	113.550	19.116
$2_{\beta}^+ \rightarrow 4_{\gamma}^+$	0.390	0.281	1.696	26.626	27.701	11.543
$4_{\beta}^+ \rightarrow 5_{\gamma}^+$	0.317	0.320	1.968	1.092	0.996	26.994
$4_{\beta}^+ \rightarrow 6_{\gamma}^+$	0.211	0.015	1.667	*0.001	0.014	33.639
$6_{\beta}^+ \rightarrow 7_{\gamma}^+$	0.005	0.726	1.644	1.411	1.106	35.167
$6_{\beta}^+ \rightarrow 8_{\gamma}^+$	0.043	0.002	1.640	1.139	0.374	57.099
$8_{\beta}^+ \rightarrow 9_{\gamma}^+$	0.107	1.212	0.803	1.860	3.104	43.599

**Table 1.** Theoretical B(E2;  $J_i^+ \rightarrow J_f^+$ ) [given in  $e^2b^2 \times 10^{-2}$ ] inter-band transitions in  $^{160-170}\text{Dy}$  nuclei. Effective charges are  $e_{\pi}=2.3$  and  $e_{\nu}=1.3$ . The values with \* are for  $B(E2; J_i^+ \rightarrow J_f^+) < 0.001 \times 10^{-2} e^2b^2$ .

oscillations [29]. As it has been pointed out in this Reference, the  $B(E2; 0_{gs}^+ \rightarrow 2_{0_2}^+)$  display a wide variation of values over well deformed nuclei, with variations of orders of magnitude over a single isotopic chain. That means that we can not continue our practice of labelling the  $0_2^+$  state by  $\beta$ , irrespective of its properties, because that practice has reduced the value of the term, and the original definitions of Bohr, Mottelson, Faessler and Greiner of the properties of a  $\beta$  vibration have been lost.

The calculation of the electric quadrupole moment and B(E2) transition strengths with realistic effective charges ( $e_{\pi} = 1.4$  and  $e_{\nu} = 0.6$ ) can be done if the intruder sector is explicitly considered. As discussed in Ref. [27], it can be obtained a simple evaluation of its effect by considering the quasi-SU(3) symmetry of the intruder sector, as proposed in Refs. [31, 32]. Having established for a given deformation the number of nucleons in the abnormal parity levels, their contribution to  $Q_0$  is easily obtained by filling the orbits in the right panel of Fig. 34 in Ref. [33]. By summing this contribution and that of the normal parity sector, the B(E2) transitions are calculated with realistic values for effective charges  $e_{\pi}$  and  $e_{\nu}$ . Nevertheless, that method does not allow to estimate the contribution of the intruder sector in each band; this means that the method only allows to calculate the constant contribution of the intruder sector for a given nucleus, and not allowing to differentiate its contribution in each band.

#### 4. Summary and conclusions

For the first time, employing a systematically parametrized Hamiltonian and the best fit of three parameters for a set of nuclei [27], the model was used to study systematically inter-band B(E2) transitions in  $160 < A < 168$  dysprosium isotopes and to predict the values between the ground,  $\gamma$  and  $\beta$ -bands in  $^{170}\text{Dy}$ . The results show very collective bands and some degree of mixing between ground state and  $\gamma$  and  $\beta$  bands. This shows that the pseudo-SU(3) scheme is a very creditable theory for achieving a microscopic description of heavy deformed rare-earth nuclei. These results may be the starting point to realize a systematic analysis of the  $0_2^+$  excitations and to determine when they are or not  $\beta$  oscillations [29].

The results presented confirm the adequacy of the model to predict properties of rare-earth exotic nuclei. Experimental information for the neutron-rich nuclei is highly desired to understand nuclear structure around the midshell region. The ability of the model to describe complementary properties of heavy deformed nuclei such as g-factors, quadrupole moments and magnetic dipole transitions will be explored in the future.

##### 4.1. Acknowledgments

This work was supported in part by CONACyT (México) under grant CB-2012-01-177519.

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