

Dominant Modes in Light Nuclei – *Ab Initio* View of Emergent Symmetries

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Abstract. An innovative symmetry-guided concept is discussed with a focus on emergent symmetry patterns in complex nuclei. In particular, the *ab initio* symmetry-adapted no-core shell model (SA-NCSM), which capitalizes on exact as well as partial symmetries that underpin the structure of nuclei, provides remarkable insight into how simple symmetry patterns emerge in the many-body nuclear dynamics from first principles. This *ab initio* view is complemented by a fully microscopic no-core symplectic shell-model framework (NCSpM), which, in turn, informs key features of the primary physics responsible for the emergent phenomena of large deformation and alpha-cluster substructures in studies of the challenging Hoyle state in Carbon-12 and enhanced collectivity in intermediate-mass nuclei. Furthermore, by recognizing that deformed configurations often dominate the low-energy regime, the SA-NCSM provides a strategy for determining the nature of bound states of nuclei in terms of a relatively small subspace of the symmetry-reorganized complete model space, which opens new domains of nuclei for *ab initio* investigations, namely, the intermediate-mass region, including isotopes of Ne, Mg, and Si.

1. Introduction

The *ab initio* symmetry-adapted no-core shell-model (SA-NCSM), with results that corroborate and are complementary to those enabled within the framework of the no-core shell model (NCSM) [1], and which can be used to facilitate *ab initio* applications to challenging lower *sd*-shell nuclei, reveal that bound states of light nuclei are dominated by high-deformation and low-spin configurations [2]. The applicable symmetries reveal the nature of collectivity in such nuclei and provide a description of bound states in terms of a relatively small fraction of the complete space when the latter is expressed in an $(LS)J$ coupling scheme with the spatial configurations further organized into irreducible representations (irreps) of SU(3). That SU(3) plays a key role tracks with the seminal work of Elliott [3, 4, 5], and is further reinforced by the fact that SU(3) also underpins the microscopic symplectic model [6, 7], which provides a theoretical framework for understanding deformation-dominated collective phenomena in nuclei [7].

While applications to *p*-shell and selected heavier nuclei [2, 9, 10, 11] illustrate the success of the *ab initio* approach, a very simple algebraic interaction, which reduces to the Elliott SU(3) model [3, 4, 5] in the single-shell limit, augmented by the SU(3) symmetry breaking spin-orbit interaction, reproduces characteristic features of the low-lying 0^+ states in ^{12}C as well as ground-



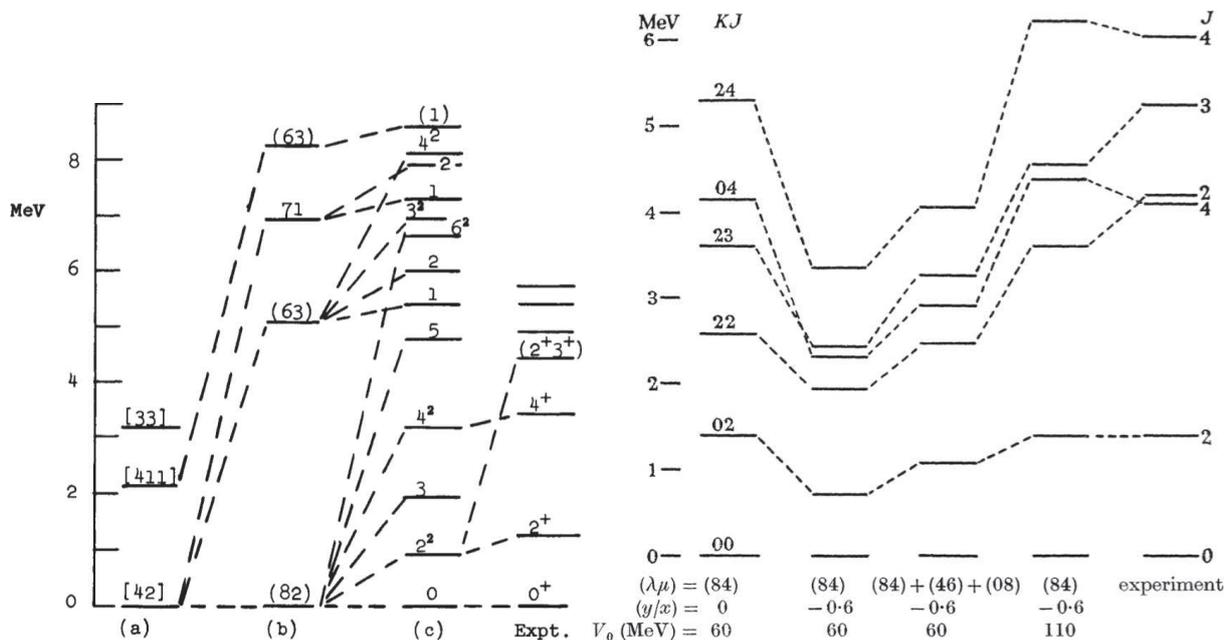


Figure 1. Elliott’s SU(3) model applied to sd -shell nuclei. Left panel: Spectrum of ^{22}Ne (or ^{22}Mg) (a) with a Majorana potential, (b) with the addition of the second-order SU(3) Casimir invariant, C_2^{su3} , and (c) with the Majorana potential plus an attractive $Q \cdot Q$ interaction [or (b) with the addition of L^2]. Figure taken from [8]. Right panel: Spectrum of ^{24}Mg with a Gaussian central force. Figure taken from [5]. The vertical axis in both figures represents energy in MeV. Note the importance of the most deformed SU(3) configuration (82) in ^{22}Ne and (84) in ^{24}Mg for reproducing the experimental low-lying states.

state rotational bands in p and sd -shell nuclei (from Be to Si) [12, 13, 14]. The study of ^{12}C includes the elusive first excited 0_2^+ state, known as the Hoyle state that was predicted based on observed abundances of heavy elements in the universe [15], and which has attracted much recent attention (e.g., see [16, 17]). An implication of the latter is that efforts to reproduce the structure of ^{12}C using a ‘bottom up’ *ab initio* effective interaction theory may benefit from ‘top down’ algebraic considerations that serve to expose emergent properties in terms of simple interaction forms that seem to dominate the structure of deformed nuclei, especially the 0^+ states of ^{12}C .

2. Shell models and collectivity-driven models

In the 1950s, two simple models of nuclear structure were advanced that are complementary in nature, namely, the independent-particle model of Mayer and Jensen [18, 19], and the collective model of Bohr and Mottelson [20]. The first of these, which is microscopic in nature, recognizes that nuclei can be described by particles independently moving in a mean field, with the harmonic oscillator (HO) potential being a very good first approximation to the average potential experienced by each nucleon in a nucleus. The second of these, which is collective in nature, recognizes that deformed shapes dominate the dynamics. For example, deformed configurations are found to be important even in a nucleus such as ^{16}O , which is commonly treated as spherical in its ground state, but 20% of the latter is governed by deformed shapes; in addition, the lowest-lying excited 0^+ states in ^{16}O and their rotational bands are dominated by large deformation [21]. Bohr & Mottelson offered a simple but important description of nuclei in terms of the deformation of the nuclear surface and associated vibrations and rotations.

The seminal work of Elliott [3] focused on the key role of SU(3), the exact symmetry of the three-dimensional spherical HO. Within a shell-model framework, Elliott's model utilizes an SU(3)-coupled basis that is related via a unitary transformation to the basis used in the conventional shell model. The new feature here is that SU(3) divides the space into basis states of definite $(\lambda\mu)$ quantum numbers of SU(3) linked to the intrinsic quadrupole deformation [22, 23, 24]. E.g., the simplest cases, (00), $(\lambda 0)$, and (0μ) , describe spherical, prolate, and oblate deformation, respectively. For SU(3)-symmetric interactions, the model can be solved analytically. But regardless whether a simple algebraic interaction is used, such as $H = H_{\text{HO}} - \frac{\chi}{2} Q \cdot Q$ within a shell (see, e.g., Fig. 1, left), or an SU(3)-symmetry breaking interaction (see, e.g., Fig. 1, right), the results have revealed a striking feature, namely, the dominance of a few most deformed configurations. This has been shown for *sd*-shell nuclei, such as ^{18}Ne , ^{20}Ne , ^{22}Ne , ^{22}Mg , ^{24}Mg , and ^{28}Si , that have been known to possess a clear collective rotational structure in their low-lying states [5, 8]. Furthermore, the pairing interaction, that can be microscopically incorporated into the Elliott model, also breaks the SU(3) symmetry. Nonetheless, it has been shown [25] that this results in mixing of only several different $(\lambda\mu)$ configurations and yields outcomes close to experiment and to the energies obtained using full *sd*-shell-model calculations. The SU(3)-symmetry dominance has been also observed in heavier nuclei, where pseudo-spin symmetry and its pseudo-SU(3) complement have been shown to play a similar role in accounting for deformation in the upper *pf* and lower *sdg* shells, and in particular, in strongly deformed nuclei of the rare-earth and actinide regions [26].

Another very significant advance is the microscopic symplectic model [6, 7], developed by Rowe and Rosensteel, which provides a theoretical framework for understanding deformation-dominated collective phenomena in atomic nuclei [7] that involves particle excitations across multiple shells. The significance of the symplectic $\text{Sp}(3, \mathbb{R})$ symmetry, the embedding symmetry of SU(3) [$\text{Sp}(3, \mathbb{R}) \supset \text{SU}(3)$], for a microscopic description of a quantum many-body system of interacting particles naturally emerges from the physical relevance of its 21 generators, which are directly related to the particle momentum ($p_{s\alpha}$) and coordinate ($r_{s\alpha}$) operators, with $\alpha = x, y,$ and z for the 3 spatial directions and s labeling an individual nucleon, and realize important observables. Namely, $\text{Sp}(3, \mathbb{R})$ -preserving operators include: the many-particle kinetic energy $\sum_{s,\alpha} p_{s\alpha}^2/2m$, the HO potential, $\sum_{s,\alpha} m\Omega^2 r_{s\alpha}^2/2$, the mass quadrupole moment $Q_{(2\mathcal{M})} = \sum_s q_{(2\mathcal{M})s} = \sum_s \sqrt{16\pi/5} r_s^2 Y_{(2\mathcal{M})}(\hat{\mathbf{r}}_s)$, and angular momentum L operators, together with multi-shell collective vibrations and vorticity degrees of freedom for a description from irrotational to rigid rotor flows. Indeed, the symplectic $\text{Sp}(3, \mathbb{R})$ symmetry underpins the symplectic shell model, which provides a microscopic formulation of the Bohr-Mottelson collective model and is a multiple oscillator shell generalization of the successful Elliott SU(3) model. The symplectic model with $\text{Sp}(3, \mathbb{R})$ -preserving interactions¹ has achieved a remarkable reproduction of rotational bands and transition rates without the need for introducing effective charges, while only a single $\text{Sp}(3, \mathbb{R})$ configuration is used [7, 28]. A shell-model study in a symplectic basis that allows for mixing of $\text{Sp}(3, \mathbb{R})$ configurations due to pairing and non-degenerate single-particle energies above a ^{16}O core [27] has found that using only seven $\text{Sp}(3, \mathbb{R})$ configurations is sufficient to achieve a remarkable reproduction of the ^{20}Ne energy spectrum (Fig. 2a) as well as of $E2$ transition rates without effective charges (Fig. 2b).

One of the most successful particle-driven models is the no-core shell model (NCSM), which, in principle, can straightforwardly accommodate any type of inter-nucleon interaction. Specifically, for a general problem, the NCSM adopts the intrinsic non-relativistic nuclear plus Coulomb interaction Hamiltonian defined as follows:

$$H = T_{\text{rel}} + V_{NN} + V_{NNN} + \dots + V_{\text{Coulomb}}, \quad (1)$$

¹ An important $\text{Sp}(3, \mathbb{R})$ -preserving interaction is $\frac{1}{2} Q \cdot Q = \frac{1}{2} \sum_s q_s \cdot (\sum_t q_t)$, as this realizes the physically relevant interaction of each particle with the total quadrupole moment of the nuclear system.

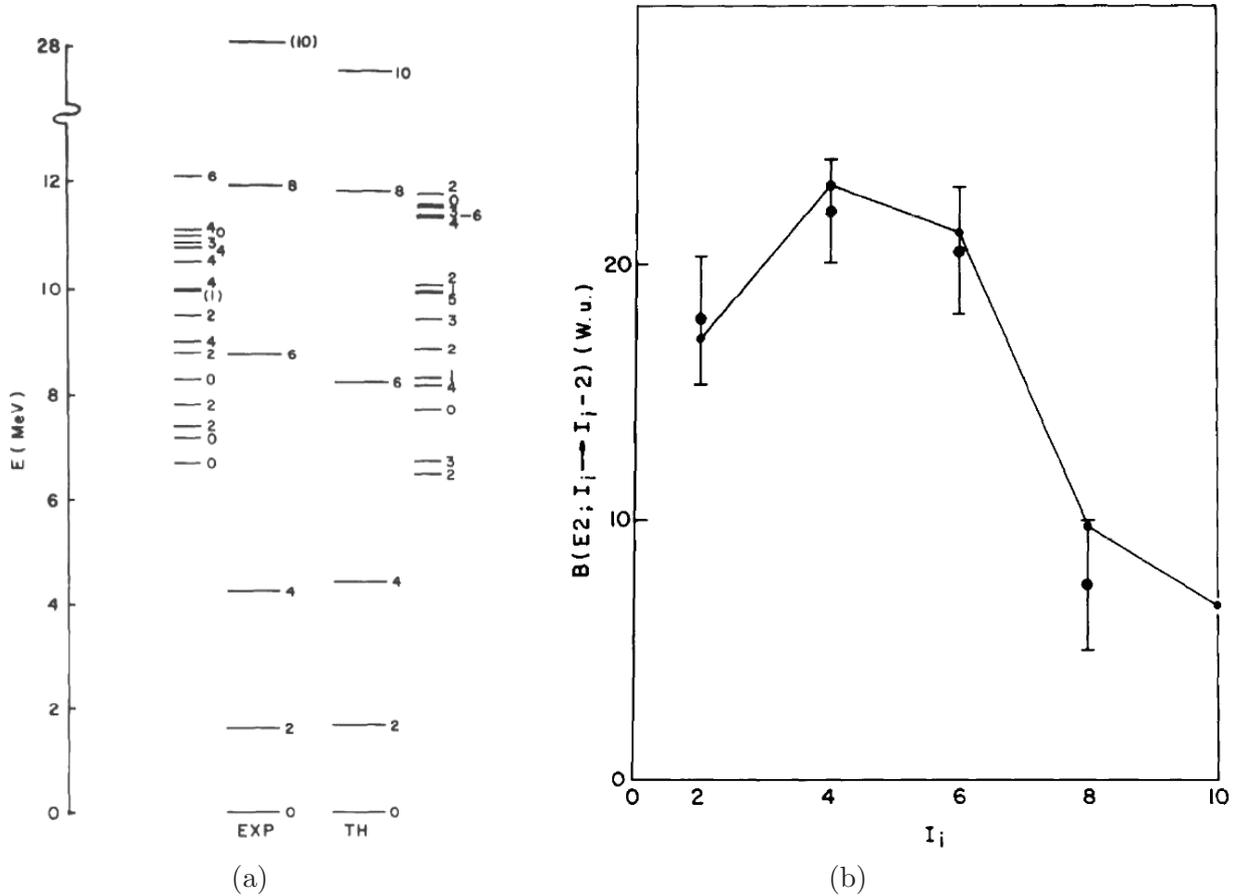


Figure 2. Microscopic symplectic model with a set of effective single-particle energies, a $Q \cdot Q$ -type interaction+pairing for ^{20}Ne . (a) Energy spectrum and (b) $B(E2 \downarrow)$ transition strengths for $J^\pi = 2^+, 4^+, 6^+$, and 8^+ (labeled as I_i in the figure), which are calculated *without effective charges* and are found to fall within the uncertainties of the corresponding experimental measurements. Figures taken from [27].

where the V_{NN} nucleon-nucleon and V_{NNN} 3-nucleon interactions are included along with the Coulomb interaction between the protons. The Hamiltonian may include additional terms such as multi-nucleon interactions among more than three nucleons simultaneously and higher-order electromagnetic interactions such as magnetic dipole-dipole terms. It adopts the HO single-particle basis characterized by the $\hbar\Omega$ oscillator strength and retains many-body basis states of a fixed parity, consistent with the Pauli principle, and limited by a many-body basis cutoff N_{\max} . The N_{\max} cutoff is defined as the maximum number of HO quanta allowed in a many-body basis state above the minimum for a given nucleus. It divides the space in “horizontal” HO shells and is dictated by particle-hole excitations (this is complementary to the microscopic symplectic model, which divides the space in vertical slices selected by collectivity-driven rules). It seeks to obtain the lowest few eigenvalues and eigenfunctions of the Hamiltonian (1). The NCSM has achieved remarkable descriptions of low-lying states from the lightest s -shell nuclei up through ^{12}C , ^{16}O , and ^{14}F , and is further augmented by several techniques, such as NCSM/RGM [29], Importance Truncation NCSM [30] and Monte Carlo NCSM [31]. This supports and complements results of other first-principle approaches, such as Green’s function Monte Carlo (GFMC) [11], Coupled-cluster (CC) method [32], In-medium SRG [33], and Lattice Effective Field Theory (EFT) [17].

The next-generation *ab initio* symmetry-adapted no-core shell model (SA-NCSM) [2] combines the first-principle concept of the NCSM with symmetry-guided considerations of the collectivity-driven models. For the first time, in the framework of the SA-NCSM, we show the emergence of orderly patterns from first principles [2]. These patterns are linked to the SU(3) and symplectic Sp(3, \mathbb{R}) symmetries. This novel feature, in turn, enables the SA-NCSM – by using symmetry-dictated subspaces – to reach new domains of nuclear structure currently inaccessible by *ab initio* calculations, such as isotopes of Ne, Mg, and Si. The model and its recent findings are described in the next section.

Furthermore, also detailed in the next section, a fully microscopic no-core symplectic shell model, NCSpM, is explored to study emergent simple patterns. It utilizes an Sp(3, \mathbb{R})-preserving $Q \cdot Q$ -type interaction plus a symmetry-breaking $l \cdot s$ interaction. And while the coupling constants of the two-body interaction we employ can be tied to realistic nucleon-nucleon interactions, we include many-nucleon interactions of a simple form ($e^{-\gamma Q \cdot Q}$) controlled by a parameter γ , the only adjustable parameter in the model. The study has revealed the importance of ultra-large shell-model spaces that are imperative to provide a successful description of the ^{12}C Hoyle state and low-lying states in nuclei from Be to Si [12, 13] (including energy spectra, $E2$ transition strengths, quadrupole moments, and matter rms radii without effective charges).

3. Emergent phenomena

3.1. *Ab initio* SA-NCSM – mechanism of simple pattern formation from first principles

The *ab initio* symmetry-adapted no-core shell model (SA-NCSM) [2] adopts the first-principle concept and utilizes a many-particle basis that is reduced with respect to the physically relevant SU(3) \supset SO(3) subgroup chain (for a review, see [21]). This allows the full model space to be down-selected to the physically relevant space. The significance of the SU(3) group for a microscopic description of the nuclear collective dynamics can be seen from the fact that it is the symmetry group of the Elliott model [3], and a subgroup of the Sp(3, \mathbb{R}) symplectic model [6].

The basis states of the SA-NCSM are based on HO single-particle states and for a given N_{max} , are constructed in the proton-neutron formalism using an efficient construction based on powerful group-theoretical methods. The SA-NCSM basis states are related to the NCSM basis states through a unitary transformation (hence, the SA-NCSM results obtained in a complete N_{max} space are equivalent to the N_{max} -NCSM results). They are labeled by the SU(3) \supset SO(3) subgroup chain quantum numbers $(\lambda \mu) \kappa L$, together with proton, neutron, and total intrinsic spins S_p , S_n , and S . The orbital angular momentum L is coupled with S to the total orbital momentum J and its projection M_J . Each basis state in this scheme is labeled schematically as $|\tilde{\gamma}(\lambda \mu) \kappa L; (S_p S_n) S; J M_J\rangle$. The label κ distinguishes multiple occurrences of the same L value in the parent irrep $(\lambda \mu)$, and $\tilde{\gamma}$ distinguishes among configurations carrying the same $(\lambda \mu)$ and $(S_p S_n) S$ labels.

The *ab initio* SA-NCSM results for p -shell nuclei reveal a dominance of configurations of large deformation in the $0\hbar\Omega$ subspace. For example, the *ab initio* $N_{\text{max}} = 8$ SA-NCSM results with the bare N^3LO realistic interaction [34] (similarly, for the bare JISP16 realistic interaction [35]) for the 0^+ ground state (*g.st.*) and its rotational band for ^8Be reveal the dominance of the $0\hbar\Omega$ component with the foremost contribution coming from the leading (40) $S = 0$ irrep (Fig. 3). Furthermore, we find that important SU(3) configurations are then organized into structures with Sp(3, \mathbb{R}) symplectic symmetry, that is, the (40) symplectic irrep gives rise to (60), (41), and (22) configurations in the $2\hbar\Omega$ subspace and so on, and those configurations indeed realize the major components of the wavefunction in this subspace. This further confirms the significance of the symplectic symmetry to nuclear dynamics. Similar results are observed for other p -shell nuclei, such as ^6Li , ^6He , and ^{12}C .

The outcome points to a remarkable feature common to the low-energy structure of nuclei that has heretofore gone unrecognized in other first-principle studies; namely, the emergence, without

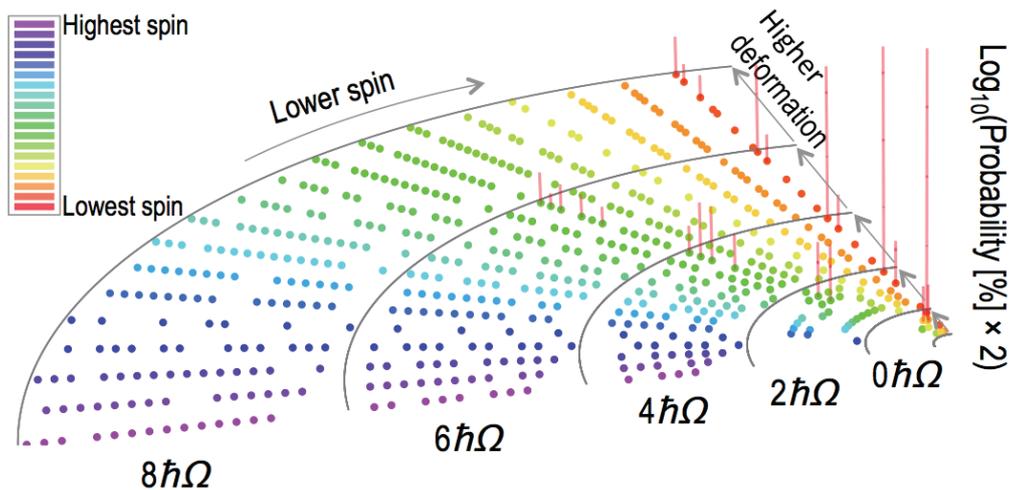


Figure 3. Probability distributions for proton, neutron, and total intrinsic spin components ($S_p S_n S$) across the Pauli-allowed deformation-related ($\lambda\mu$) values and for $N\hbar\Omega$ subspaces for the calculated 0^+ ground state of ${}^8\text{Be}$ obtained for $N_{\text{max}} = 8$ and $\hbar\Omega = 25$ MeV with the N^3LO bare interaction. The concentration of strengths to the far right demonstrates the dominance of collectivity in the calculated eigenstates.

a priori constraints, of simple orderly patterns that favor strongly deformed configurations and low spin values. This feature confirms the dominant role the $\text{SU}(3)$ and $\text{Sp}(3, \mathbb{R})$ symmetries play in nuclear dynamics and is central to expanding the reach of first-principle studies to heavier nuclei. *Ab initio* investigations of nuclei in the *sd*-shell region are now feasible and the first *ab initio* calculations for ${}^{24}\text{Si}$ and its mirror nucleus ${}^{24}\text{Ne}$ (Fig. 3.1) have been achieved in the framework of the SA-NCSM with chiral interactions in an $N_{\text{max}} = 6$ symmetry-winnowed space of 3.5×10^6 dimensionality (compare to the inaccessible space of 8×10^9 dimensionality required for the corresponding conventional NCSM calculations) [36].

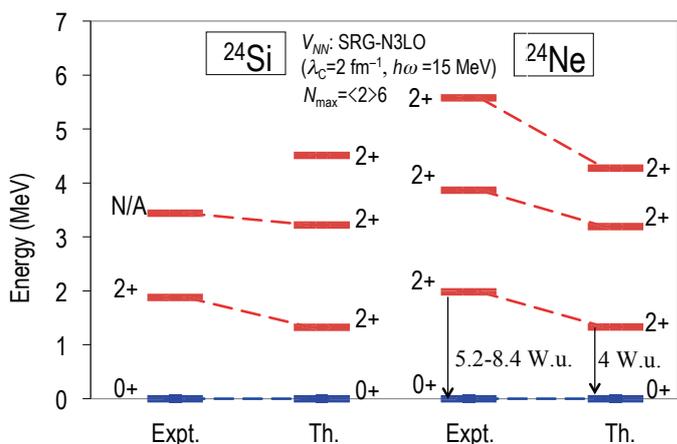


Figure 4. First *ab initio* SA-NCSM calculations for open-shell intermediate-mass nuclei, ${}^{24}\text{Si}$ and ${}^{24}\text{Ne}$, obtained for a selected $N_{\text{max}} = \langle 2 \rangle 6$ model space (that is, the complete space up through $2\hbar\Omega$ and selected large-deformation/low-spin configurations in $4\hbar\Omega$ and $6\hbar\Omega$ subspaces). Calculations are performed for $\hbar\Omega = 15$ MeV and using the SRG- N^3LO NN interaction for an SRG cutoff $\lambda_c = 2 \text{ fm}^{-1}$.

In short, the SA-NCSM advances an extensible microscopic framework for studying nuclear structure and reactions that capitalizes on advances being made in *ab initio* methods while exploiting symmetries – exact and partial, known to dominate the dynamics.

3.2. NCSpM model – physics responsible for simple pattern formation

The no-core symplectic shell model (NCSpM) is a fully microscopic no-core shell model that uses a symplectic $Sp(3, \mathbb{R})$ basis and $Sp(3, \mathbb{R})$ -preserving interactions. The NCSpM employed within a full model space up through a given N_{\max} coincides with the NCSM for the same N_{\max} cutoff. However, in the case of the NCSpM, the symplectic irreps divide the space into ‘vertical slices’ that are comprised of basis states of a definite deformation $(\lambda \mu)$. Hence, the model space can be reduced to only a few important configurations that are chosen among all possible $Sp(3, \mathbb{R})$ irreps within the N_{\max} model space. The NCSpM, while selecting the most relevant symplectic configurations, is employed to provide shell model calculations beyond current NCSM limits, namely, up through $N_{\max} = 20$ for ^{12}C , the model spaces we found sufficient for the convergence of results [12].

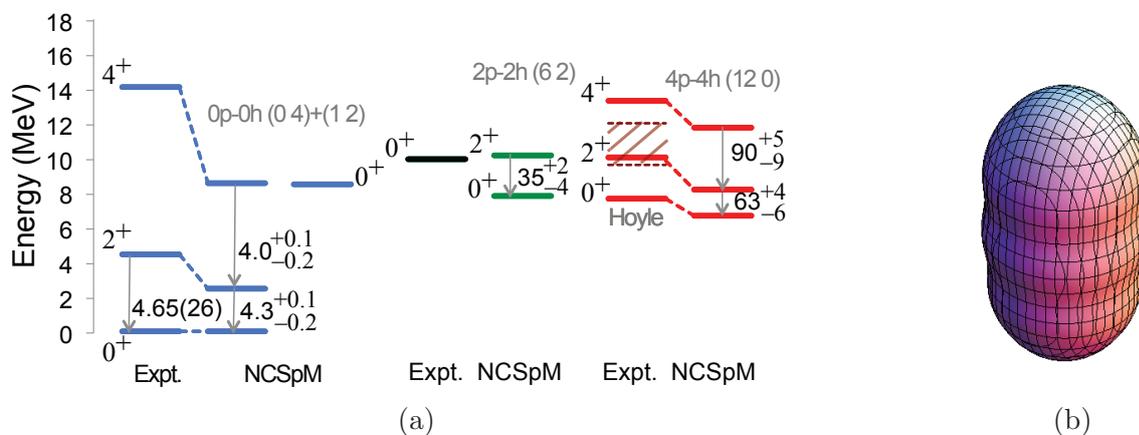


Figure 5. (a) Energy spectra calculated by the NCSpM with $\gamma = -1.71 \times 10^{-4}$ for ^{12}C in an $N_{\max} = 20$ model space and compared to experiment (“Expt.”). Figure taken from Ref. [12]. (b) Matter-density profile of the Hoyle state in ^{12}C (with respect to the intrinsic frame) showing the formation of three clusters within a no-core shell-model framework.

We employ a very simple Hamiltonian with an effective interaction derived from the long-range expansion of the two-body central nuclear force together with a spin-orbit term,

$$H_{\text{eff}} = H_0 + \frac{\chi}{2} \frac{(e^{-\gamma Q \cdot Q} - 1)}{\gamma} - \kappa \sum_{i=1}^A l_i \cdot s_i. \quad (2)$$

This includes the spherical HO potential, which together with the kinetic energy yields the HO Hamiltonian, $H_0 = \sum_{i=1}^A \left(\frac{\mathbf{p}_i^2}{2m} + \frac{m\Omega^2 \mathbf{r}_i^2}{2} \right)$, and the $Q \cdot Q$ quadrupole-quadrupole interaction not restricted to a single shell. For the latter term, the average contribution, $\langle Q \cdot Q \rangle_n$, of $Q \cdot Q$ within a subspace of n HO excitations is removed [37], that is, the trace of $Q \cdot Q$ divided by the space dimension for a fixed n . Hence, the large monopole contribution of the $Q \cdot Q$ interaction is removed, which, in turn, helps eliminate the considerable renormalization of the zero-point energy, while retaining the $Q \cdot Q$ -driven behavior of the wavefunctions. This Hamiltonian in its zeroth-order approximation (for parameter $\gamma \rightarrow 0$) and for a valence shell goes back to the established Elliott model. We take the coupling constant χ to be proportional to $\hbar\Omega$ and, to leading order, to decrease with the total number of HO excitations, as shown by Rowe [38] based on self-consistent arguments. The success of such an effective nuclear interaction is not unexpected, as the spherical HO potential and the $Q \cdot Q$ interaction directly follow from the second and third term, respectively, in the long-range expansion of

any two-body central force, e.g., like the Yukawa radial dependence, $V^{(2)} = \sum_{i<j} V(r_{ij}/a) = \sum_{i<j} (\xi_0 + \xi_2 r_{ij}^2/a^2 + \xi_4 r_{ij}^4/a^4 + \dots)$ [8], for a range parameter a .

As the interaction and the model space are carefully selected to reflect the most relevant physics, the outcome reveals a quite remarkable agreement with the experiment [12]. The low-lying energy spectrum and eigenstates for ^{12}C were calculated using the NCSpM with H of Eq. (2) for $\hbar\Omega = 18$ MeV given by the empirical estimate $\approx 41/A^{1/3} = 17.9$ MeV and for $\kappa \approx 20/A^{2/3} = 3.8$ MeV (e.g., see [20]). The results are shown for $N_{\max} = 20$, which we found sufficient to yield convergence. This N_{\max} model space is further reduced by selecting the most relevant symplectic irreps, namely, the spin-zero ($S = 0$) $0\hbar\Omega$ 0p-0h (04), $2\hbar\Omega$ 2p-2h (62), and $4\hbar\Omega$ 4p-4h (120) symplectic bandheads together with the $S = 1$ $0\hbar\Omega$ 0p-0h (12) and all multiples thereof up through $N_{\max} = 20$ of total dimensionality of 6.6×10^3 . In comparison to the experimental energy spectrum (Fig. 5a), the outcome reveals that the lowest 0^+ , 2^+ , and 4^+ states of the 0p-0h symplectic slices calculated for $\gamma = -1.71 \times 10^{-4}$ closely reproduce the *g.st.* rotational band, while the calculated lowest 0^+ states of the $4\hbar\Omega$ 4p-4h (120) and the $2\hbar\Omega$ 2p-2h (62) slices are found to lie close to the Hoyle state and the 10-MeV 0^+ resonance (third 0^+ state), respectively. The model successfully reproduces other observables for ^{12}C that are informative of the state structure, such as mass rms radii, electric quadrupole moments and $B(E2)$ transition strengths (Fig. 5a).

A preponderance of the (04) $S = 0$ configuration and also (12) $S = 1$ configuration is observed for the ground-state rotational band, thereby indicating an oblate shape. The Hoyle-state rotational band includes shapes of even larger deformations but prolate, with the largest contribution of (160) (Fig. 5b).

While the model includes an adjustable parameter, γ , this parameter only controls the presence of many-body interactions that cannot be informed by existing realistic NN and NNN interactions. The entire many-body apparatus is fully microscopic and no adjustments are possible. Hence, as γ varies, there is only a small window of possible γ values that, for large enough N_{\max} , closely reproduces the relative positions of the three lowest 0^+ states, as well as mass rms radii, electric quadrupole moments and $B(E2)$ transition strengths.

The outcome of the present analysis is not limited to ^{12}C . The model we find is also applicable to the low-lying states of other p -shell nuclei, such as ^8Be , as well as sd -shell nuclei without any adjustable parameters [13, 14]. In particular, using the same $\gamma = -1.71 \times 10^{-4}$ as determined for ^{12}C , we describe selected low-lying states in ^8Be in an $N_{\max} = 24$ model space with only 3 spin-zero $0\hbar\Omega$ (40), $2\hbar\Omega$ (60), and $4\hbar\Omega$ (80) symplectic irreps. Furthermore, we have successfully applied the NCSpM without any adjustable parameters to the ground-state rotational band of heavier nuclei, such as ^{20}O , $^{20,22,24}\text{Ne}$, $^{20,22}\text{Mg}$, and ^{24}Si [14]. This suggests that the fully microscopic NCSpM model has indeed captured an important part of the physics that governs the low-energy nuclear dynamics and informs key features of the interaction and nuclear structure primarily responsible for the formation of such simple patterns.

In short, the NCSpM findings reveal that, to explain emergent simple patterns of deformation and clustering, shell-model spaces well beyond the current limits (up through 22 major HO shells for the Hoyle state) are vital to accommodate particle excitations that appear critical to highly-deformed spatial structures and the convergence of associated observables. The outcome also points to the importance of simple many-body interactions and of the long-range part of the NN interaction, especially through its link to the HO potential and the interaction of individual particles with the total quadrupole moment of the nuclear system.

4. Conclusion

Symmetries in atomic nuclei, that have been long recognized, have been recently utilized and further understood in the framework of the *ab initio* symmetry-adapted no-core shell model SA-NCSM as well as of the microscopic no-core symplectic model NCSpM that combine the

shell-model and collectivity-driven concepts. The findings pointed to a remarkable new insight, namely, understanding the mechanism on how such simple structures emerge from a fundamental level – the *ab initio* view; but also, understanding the primary physics responsible for the emergent simplicity – the *ab exitu* view.

Furthermore, by utilizing symmetries that are found to underpin nuclear dynamics, the SA-NCSM, while building upon the *ab initio* foundation of the NCSM, has extended its reach to heavier nuclear systems, such as isotopes of Ne, Mg, and Si.

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