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Effects of Large-Scale Simple Velocity Shear on a Fluctuating Interplanetary Magnetic Field

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Abstract. The effects of large-scale simple velocity shear on a magnetic field in a low-β MHD fluid are examined in the limit that the magnetic field does not significantly affect the fluid flow. The large-scale fluid shear stretches the magnetic field and can cause significant field amplification. Application to the solar wind suggests that the latitudinal shear observed may contribute significantly to the observed periods of underwound or radial magnetic field.

1. Introduction and Background

Shear is commonly present in astrophysical fluids and affects the ambient magnetic field, which often can be regarded as being frozen in to the ambient thermal fluid. A particularly simple example of shear occurs when a quasi-steady flow in one direction (say, x or r) varies in a transverse direction (say, z or θ). Such a flow is observed in the solar wind near sunspot minimum, where the mainly radial wind flow changes from $\approx 300 - 400$ km/sec at low heliographic latitudes to ≈ 700 km/sec at high latitudes.

Clearly, in such a shear flow, one expects that any component of the magnetic field in the transverse direction, normal to the plane of the velocity shear, will be significantly affected, whereas the initial magnetic-field component parallel to the flow direction will initially, be unaffected. In this paper, we examine the effects of such such shear, neglecting the magnetic forces. We find that in this case, any initial transverse magnetic field results in amplification of the field parallel to the flow. Eventually, this may result in a situation where the effects of the magnetic field may not be neglected. We do not consider this limit in this paper.

2. Basic Considerations

We consider the effects of a prescribed simple shear on a magnetic field by considering a system where the magnetic field magnitude is small enough that it has negligible effect on the flow.

Consider a simple MHD flow and magnetic field system which illustrates the effects of shear. Let there be a steady fluid flow in the x direction which varies in the z direction, $\mathbf{U}(z) = U_x(z)\hat{\mathbf{e}}_x$. Then the magnetic-field evolution equation becomes:

$$\begin{aligned} \frac{\partial B_x}{\partial t} + U_x \frac{\partial B_x}{\partial x} &= B_z \frac{\partial U_x}{\partial z} \\ \frac{\partial By}{\partial t} + U_x \frac{\partial B_y}{\partial x} &= 0 \\ \frac{\partial Bz}{\partial t} + U_x \frac{\partial B_z}{\partial x} &= 0 \end{aligned} \quad (1)$$

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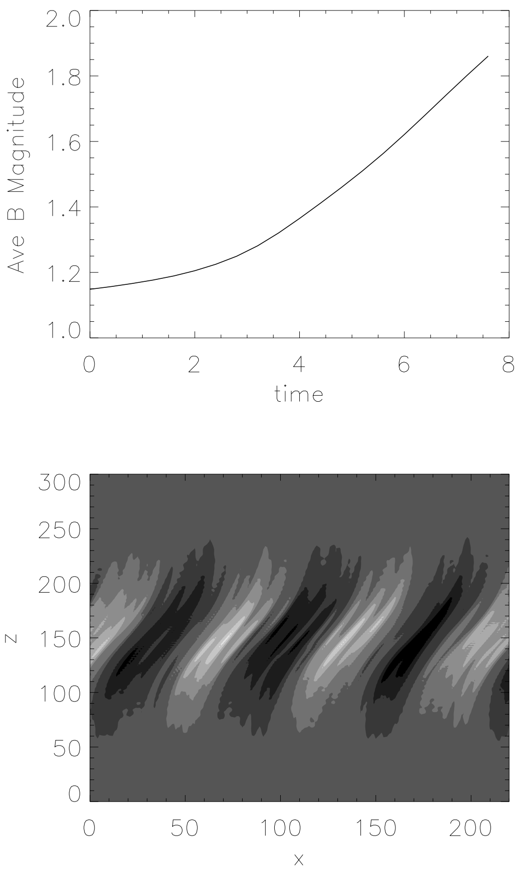

Figure 1. The solution in equation (3), where b_z is initially a simple cosine, showing the increase in the magnitude of the magnetic field with time (top) and contours of magnetic-field magnitude in the x,z plane.

These equations are straightforward to solve. The y and z components of the magnetic field are simply advected with the flow, whereas the x component in the direction of the flow vector changes because of the shear term $\partial U_x/\partial z$. Let there be an initial magnetic field, at t = 0,

$$\mathbf{B}(x, z, t_0) = B_0\hat{\mathbf{e}}_\mathbf{x} + b_z(x, t_0)\hat{\mathbf{e}}_\mathbf{z}. \tag{2}$$

Then, at later times the solution is

$$\mathbf{B}(x, z, t) = \left[B_0 + (t - t_0)\frac{\partial U_x}{\partial z} b_z(x - U_x(z)t, t_0)\right] \hat{\mathbf{e}}_\mathbf{x} + b_z(x - U_x t, t_0)\hat{\mathbf{e}}_\mathbf{z} \tag{3}$$

On the other hand, if the fluid at some plane $x = x_0$ is moved transversely in the z-direction at a varying velocity $U_z(t)$, the solution for $x > x_0$ is:

$$\mathbf{B}(x, z, t) = \left[B_0 + \frac{(x - x_0)}{U_x(z)}\frac{\partial U_x}{\partial z} b_z(z, x, t - (x - x_0)/U_x(z))\right] \hat{\mathbf{e}}_\mathbf{x} + b_z(x, (t - t0)/U_x(z))\hat{\mathbf{e}}_\mathbf{z}. \tag{4}$$

This case corresponds to a time-varying transverse fluid flow at the boundary $x = x_0$ which produces a time-varying transverse magnetic field which is then advected to $x > x_0$.

In both cases we see that the y component of the magnetic remains zero and the z component is advected in the x direction. The x component varies with time or position. In the first example

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of a specified initial magnetic field at zero time, the magnitude of the component of the magnetic field in the direction of the flow velocity increases without limit as the time increases to large values. Of course, this cannot happen because of the increasingly important dynamical effect of the magnetic field.

The behavior of the magnetic field for case 1 is illustrated in figure 1 for an initial magnetic field which is uniform and in the x direction and for a flow $U_x(z) = cos(z)$. The magnetic field grows so that at some point, the neglect of the magnetic-field stresses becomes inappropriate, and a more-complete analysis would be necessary.

The conclusions derived above in cartesian geometry also hold for a radial flow from the surface of a condensed object such as the Sun, where the radial flow speed varies with direction. If r is the radial direction and θ is the polar angle, the relevant magnetic-field evolution equations for latitudinal shear become:

$$\left[\frac{\partial}{\partial t} + U_r\frac{\partial}{\partial r}\right] rB_\theta \quad = \quad 0 \tag{5}$$

$$\left[\frac{\partial}{\partial t} + U_r\frac{\partial}{\partial r}\right] rB_\phi \quad = \quad 0 \tag{6}$$

$$\left[\frac{\partial}{\partial t} + U_r\frac{\partial}{\partial r}\right] r^2B_r \quad = \quad rB_\theta\frac{\partial U_r}{\partial \theta} + \frac{rB_\phi}{sin\theta}\frac{\partial U_r}{\partial \phi} \tag{7}$$

Defining the quantities $S_r = r^2B_r$ and $S_\theta = rB_\theta$, we obtain equations for S_r and S_θ which are very similar to the equations obtained above for the cartesian case. For the axisymmetric case, where θ is the polar angle, we find

$$\frac{\partial S_\theta}{\partial t} + U_r(\theta)\frac{\partial S_\theta}{\partial r} \quad = \quad 0 \tag{8}$$

$$\frac{\partial S_r}{\partial t} + U_r(\theta)\frac{\partial S_r}{\partial r} \quad = \quad \frac{\partial U_r}{\partial \theta}S_\theta \tag{9}$$

If we have a radial field B_0 and a transverse velocity $U_t(t)$ at $r = r_0$, the solution for $r > r_0$ can be written

$$S_\theta \quad = \quad B_0r_0\frac{U_t(t - \frac{(r-r_0)}{U_r(\theta)})}{U_r(\theta)} \tag{10}$$

$$S_r \quad = \quad B_0r_0^2 + \frac{(r-r_0)}{U_r(\theta)}\frac{\partial U_r}{\partial \theta}S_\theta \tag{11}$$

3. Velocity Shear in the Solar Wind and Underwound Magnetic Field

Observations of the solar wind over the past four decades (eg, Coles and Maagoe, 1972) have established that the solar wind at high heliographic latitude, and during the several years around sunspot minimum, has speed much larger than that in at low latitudes. The Ulysses observations during the first orbit of Sun showed (McComas, etal, 2000) that above a latitude of about 33^o the solar wind speed was greater than 750 km/sec, whereas, at lower latitudes below about 17^o, the speed varied more but was typically approximately 400 km sec. This implies a significant latitudinal gradient of approximately 1000 km/sec/radian.

Using these values in the above equations results in a significant enhancement of the radial component of the interplanetary magnetic field above that obtained for the Parker spiral corresponding to the solar-wind velocity at a given point.

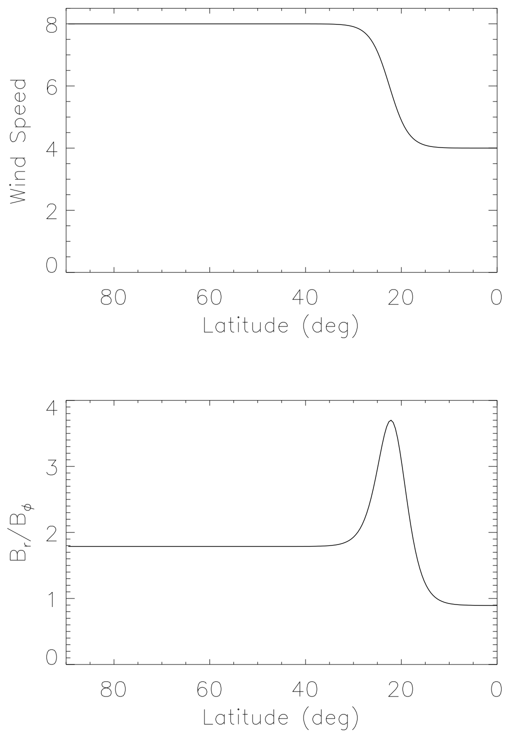

Figure 2. Illusration of a solution to equation (7) for plausible solar-wind parameters, showing a model latitudinal variation of the radial solar-wind velocity and the resulting ratio of the radial and azimuthal components of the magnetic field. In this case the shear was centered at approximately 20^o latitude and the transverse velocity was 0.5 km/sec.

Schwadron(2002), Murphy etal (2002), Gosling and Skoug (2002), Schwadron and McComas (2005) and Lario and roelof (2010) have reported analyses of observations which show periods of time occur in which, for several hours to a few days, what is termed an "underwound" interplanetary magnetic field occur. That is, in contrast to the expected ratio of azimuthal to radial field components of order unity at on AU, the ratio was significantly larger. This would occur if the radial magnetic field were enhanced. Corresponding "overwound" periods are not reported. Murphy et al(2002) and Gosling and Skoug (2002) and Schwadron and McComas (2005) proposed models which essentially called upon temporal changes in the solar-wind velocity or rarefaction regions to change the field angle. These may indeed be operative.

We propose, here, that these underwound magnetic field periods may also be in part the result of the latitudinal shear discussed above. Schwadron (2002) and Schwadron and McComas (2003) discussed models which also involved latitudinal velocity shear, although their approach was different from that suggested here.

Because of the random walk or meandering of the interplanetary magnetic field any latitudinal component of the magnetic field will be acted upon by the latitudinal shear in the wind to increase the radial component of the magnetic field, as discussed above.

$$\frac{\partial U_r}{\partial \theta} \approx 1000 \frac{km}{sec\ radian}. \tag{12}$$

Consider a latitudinal interplanetary magnetic field produced at the Sun by a supergranulation velocity of approximately .5 km/sec. This produces a latitudinal meandering magnetic field in the solar wind. Jokipii, et al, 1995, demonstrated that this field could account for the magnetic field observed by Ulysses. This transversed field will be acted upon by the latitudinal shear in the solar wind as discussed above, and result in an amplified radial magnetic field. Figure (2) illustrates the ratio of radial to azimuthal field at 1 AU for reasonable parameters. Note in particular the region of underwound magnetic field.

4. Discussion and Conclusions

We have discussed the effects of a prescribed simple velocity shear on a magnetic field which is weak enough not to affect the flow. The shear is such that in cartesian coordinates, $\mathbf{U} = U_x(z)\hat{\mathbf{e}}_\mathbf{x}$ or in spherical coordinates $\mathbf{U} = U_r(\theta)\hat{\mathbf{e}}_\mathbf{r}$. In each case, the effect of the shear is to increase the magnitude of the magnetic field in the direction of the flow velocity if there is an initial magnetic field component in the direction of the velocity change.

Application of this to the observed solar-wind, where the flow is observed to be nearly radial but with a variation in latitude results in an enhanced radial magnetic field. In the range of latitudes where there is latitudinal shear, the radial component of the magnetic field is significantly enhanced over the case with no shear. It is suggested that this effect may contribute to the observed periods of "underwound" interplanetary magnetic field. These changes to the magnetic field will also have consequences for energetic-particle transport, as discussed previously by Intriligator, etal (2001).

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References

Coles, W. A. and S. Maagoe, *J. Geophys. Res.*, **77**, 5622, 1972.
Gosling, J. T. and R. M. Skoug, *J. Geophys. Res.*, **107**, No A10, 1327, 2002.
Intriligator, D. S., et al, J. Geophys. Res., **106**, 10,625, 2001.
Jokipii, J. R., et al, Geophys. Res. Lett, **22**, 3385, 1995.
Lario, D. and E. C. Roelof, AIP Conference Proceedings 1216, page 639, 2010.
McComas, D. J., et al, J. Geophys. Res, bf 105, 10,419, 2000.
Murphy, N., Smith, E. J., and Schwadron, N. A., Geophys. Res. Lett., **29**, No 22, 2066, 2002.
Schwadron, N. A., Geophys. Res. Lett., **29**, No 14, 1863, 2002.
Schwadron, N. A. and D. J. McComas, Geophys. Res. Letters, 30, 1587, 2003.
Schwadron, N. A. and D. J. McComas, Geophys. Res. Letters, 32, 3112, 2005.