A simple model of hysteresis behavior using spreadsheet analysis

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Abstract. Hysteresis loops occur in many scientific and technical problems, especially as field dependent magnetization of ferromagnetic materials, but also as stress-strain-curves of materials measured by tensile tests including thermal effects, liquid-solid phase transitions, in cell biology or economics. While several mathematical models exist which aim to calculate hysteresis energies and other parameters, here we offer a simple model for a general hysteretic system, showing different hysteresis loops depending on the defined parameters. The calculation which is based on basic spreadsheet analysis plus an easy macro code can be used by students to understand how these systems work and how the parameters influence the reactions of the system on an external field. Importantly, in the step-by-step mode, each change of the system state, compared to the last step, becomes visible. The simple program can be developed further by several changes and additions, enabling the building of a tool which is capable of answering real physical questions in the broad field of magnetism as well as in other scientific areas, in which similar hysteresis loops occur.

1. Introduction

Each student of physics knows the typical ferromagnetic hysteresis loop in different shapes, depending on material, temperature, sample shape and other parameters. However, general models of hystereses often include sophisticated mathematical descriptions, such as the Bouc-Wen model [1,2], or are hard to bring into agreement with the physical situation, such as the Preisach hysteresis model [3].

What is offered here, instead, is a simple program, based on a spreadsheet analysis plus an easy macro code, to enable testing the influence of system parameters on the resulting hysteresis loops. The program is optimized for Microsoft Excel®, included in Microsoft Office 2007 or higher.

In this way, students can get a feel for the importance of the different parameters, for their effects on coercivities (loop width) and loop shapes. Our article is targeted on developing an intuitive approach of these influences, which is why – additional to automatically creating a complete hysteresis loop – a step-by-step mode is included, allowing for examination of each single step after changing the external field. While magnetism is surely one of the most important research areas for such a model to be applied [4], we have intentionally avoided all special mathematical magnetism-related descriptions in the model, to allow for using it for a broader spectrum of topics without unnecessary restrictions. The model also allows for examination of other systems showing hysteresis loops, such as of ferroelectric

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spins [5], hysteretic phase transformations in Lennard-Jones fluids [6], adsorption / desorption from a fluid reservoir in a porous material [7], and further effects in physics, chemistry or material sciences.
The article is structured as follows: In section 2, the simple mathematical base of the program is described. Section 3 shows a small selection of possible results, calculated with our program, showing the influence of the different parameters. Section 4, finally, offers some additional ideas, allowing for the students to develop the program further, before the article is concluded in Section 5.

2. Mathematical Model

The following mathematical rules are the base for the program described here:
The system under investigation is subdivided into uniformly distributed cells. Each cell (depicting e.g. one magnetic moment or one ferroelectric spin) can have two discrete states, -1 or +1, as usual in Ising models [8]. Each cell is influenced by its four nearest neighbors, further interactions between diagonally neighboring cell or others are excluded in the basic version. Here, Ising regions of 16 x 16 cells have been chosen (Fig. 1), ten of which are used to calculate an average value of the respective system parameter (e.g. the overall magnetization) and an averaged hysteresis loop.
The first input parameter is the weight of the switching energy. Its value describes the relative importance of the switching energy, i.e. for the maximum value of 1, switching is independent of the neighbors, while for smaller values, the influence of the magnetization of the neighbors becomes larger. The rarely met value of 0 should not be used since it results in too many +1 values.
The second input parameter, the probability of the "wrong" magnetization, describes the influence of the temperature in a real system. For this probability = 0, a completely deterministic, stationary system is depicted. A probability of 0.5 describes – taking into account a magnetic system – a paramagnet, values between 0 and 0.5 result in ferromagnetic particles (softer ferromagnets for higher values). Values between 0.5 and 1 describe antiferromagnetic particles, which show a chessboard-like structure in the step-by-step processing for small external magnetic fields.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	-1	-1	1	1	-1	1	1	1	1	-1	-1	1	1	1	1	0
0	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	-1	1	-1	0
0	1	1	1	1	1	1	1	1	1	-1	-1	1	1	1	1	1	0
0	-1	-1	1	1	1	1	1	1	1	-1	-1	1	-1	1	1	1	0
0	1	1	1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	1	0
0	-1	1	1	-1	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	0
0	-1	1	-1	-1	1	1	1	-1	-1	-1	-1	1	-1	-1	-1	-1	0
0	-1	1	-1	-1	1	1	1	-1	-1	-1	-1	1	-1	-1	1	1	0
0	-1	1	-1	-1	-1	1	1	-1	-1	-1	1	-1	-1	-1	1	1	0
0	1	1	-1	-1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	0
0	1	1	1	1	1	-1	-1	-1	1	-1	1	1	1	1	1	1	0
0	1	1	1	1	1	-1	-1	1	1	-1	1	1	1	1	1	-1	0
0	1	-1	-1	-1	-1	-1	1	1	1	-1	-1	-1	1	1	1	-1	0
0	1	1	1	1	-1	1	1	1	-1	1	-1	-1	-1	-1	-1	-1	0
0	-1	1	1	1	-1	1	1	1	1	1	1	1	1	-1	-1	-1	0
0	1	-1	1	1	-1	1	-1	1	1	-1	-1	1	-1	-1	-1	-1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 1. Example of the cells moments at zero external field in one 16 x 16 "particle".

The third input parameter, the value of the external switching field (i.e. the magnetic field in case of magnetic particles), can be chosen between ± 1. At these borders, the external switching field is stronger than all possible interactions within the particle. The hysteresis loops, however, only sweep up to switching fields of ± 0.1 (± 0.2), since in most cases, saturation is reached at these values. It must be mentioned that the field sweeping speed – i.e. the number of calculation steps per field unit controlled by the user – strongly influences the resulting hysteresis loops, thus the consequences of changes in different parameters should be examined with identical field sweeping speeds, i.e. with identical field step sizes. In the extreme case of a quasi-static loop (i.e. for very low sweeping speeds), this model is well-known to result in vanishing hysteresis loops [9,10]; a behavior which can be examined by the students themselves by means of more advanced spreadsheet models which are available on request from the authors.

In Fig. 2 the idea of simulation algorithm is described. In general, interaction with the four closest neighbors determines the resulting +1/-1 state, governed via the above mentioned parameters w_{cr}, P_{wm}, and H_{ext}, i.e. the weight of switching energy, the probability of the "wrong" state, and the external switching field, respectively, as well as a random number *R* between 0 and 1.

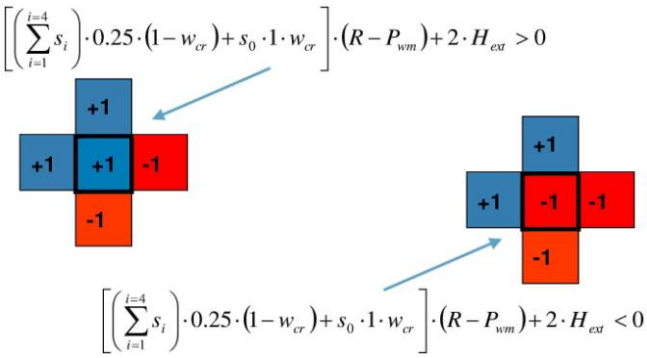

Figure 2. The two main conditions of switching for a cell s_0, surrounded by the four neighbors s_i.

The general condition of switching of a single cell s_0 is based on the following expression

$$\left[\left(\sum_{i=1}^{i=4} s_i\right)\cdot 0.25\cdot(1-w_{cr})+s_0\cdot 1\cdot w_{cr}\right]\cdot(R-P_{wm})+2\cdot H_{ext} \tag{1}$$

The easiest possibility to work with the program is to choose manual processing. After choosing the desired input values, a calculation step is manually started (by pressing F9 in Microsoft Excel®). Continuously starting new calculation steps – by keeping the respective key depressed – will result in a stable state of the simulated particles. Due to the included probability of the "wrong" state, it can be easily seen that each of the ten particles shown in the program looks different, even if they are all set to saturation (i.e. high external switching field to achieve completely blue or red particles) before. The overall system parameter under examination (e.g. magnetization) of the ten particles is steadily shown in the orange field "Magnetization".
For the hysteresis loops, a macro is needed. For this, the developer mode has to be switched on, by clicking (in Microsoft Excel®) on the Microsoft Office Button → Excel Options → Popular → Show Developer Tab in the Ribbon. Now the hysteresis loop can be carried out by pressing "Ctrl + m" for a "small" loop (external magnetic field in the range of ± 0.1) or "Ctrl + M" for a "large" loop (± 0.2). Depending on the computer speed, the recalculation will take a few seconds or a little longer time. Each hysteresis loop can be copied or exported as a graph, or the x-y-values can be copied into another document.

3. Results

In this section, a few results are shown to describe the influence of the different parameters.
Figure 3 depicts hysteresis loops calculated for a constant probability of the "wrong" state = 0.1 and a weight of the switching energy of 0.01, 0.1, and 0.5, respectively. While the coercive fields (the external fields for which the system parameter becomes 0, i.e. the intersection points with the x-axis) are broadest for a middle weight of the switching energy, larger weights lead to a change in the shape of the hysteresis loop.
In Fig. 4, the influence of the probability of a "wrong" state is shown for a constant weight of the switching energy. Apparently, the coercive fields are largest for small probabilities, while for larger probabilities, the system becomes softer, until a paramagnet-like state (completely reversible, without a visible hysteresis loop) is reached for a probability of 0.5.

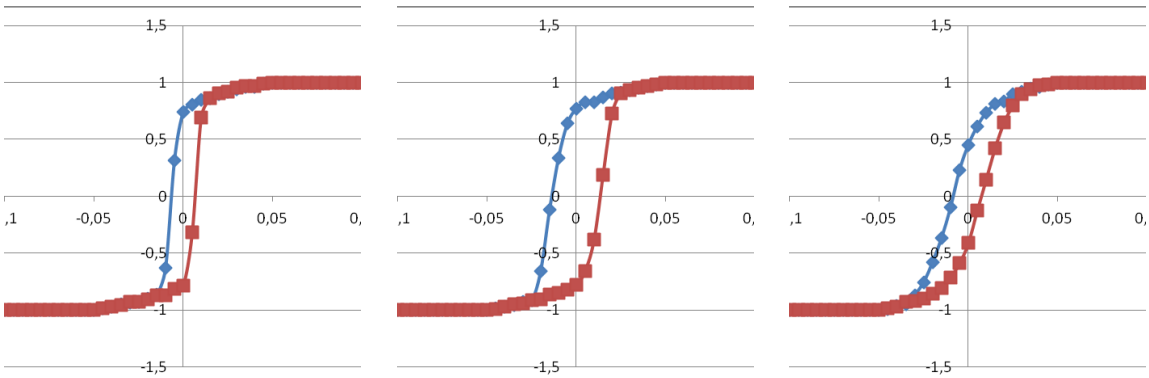

Figure 3. Hysteresis loops, calculated for a probability of "wrong" state $P_{wm} = 0.1$ and a weight of the switching energy $w_{cr} = 0.01$ (left panel), 0.1 (middle panel), and 0.5 (right panel).

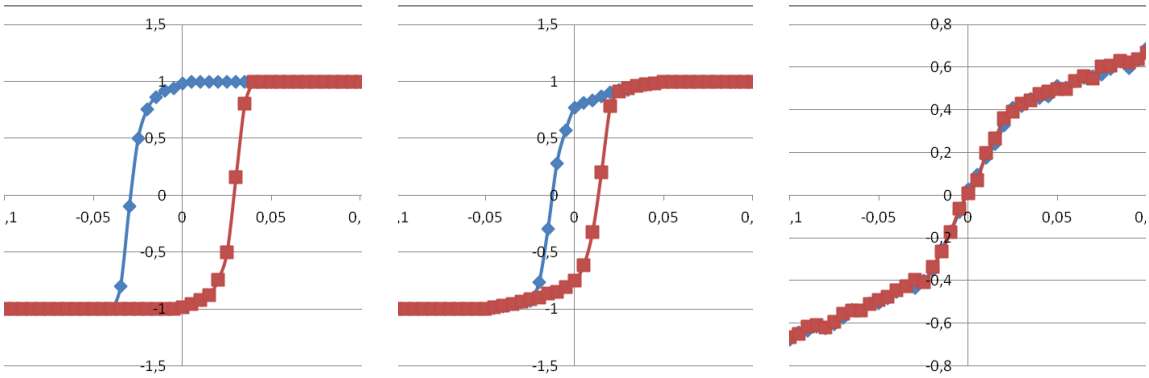

Figure 4. Hysteresis loops, calculated for a weight of the switching energy $w_{cr} = 0.1$ and a probability of "wrong" state P_{wm} of 0.01 (left panel), 0.1 (middle panel), and 0.5 (right panel, paramagnetic).

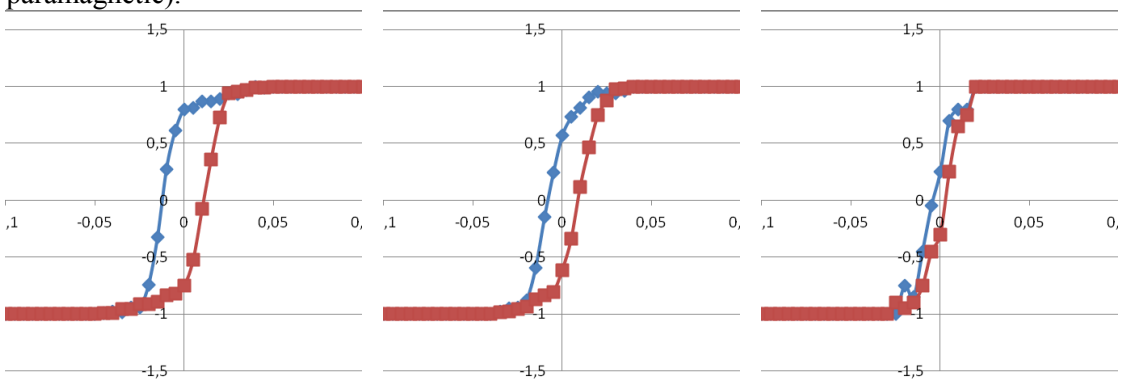

Figure 5. Hysteresis loops, calculated for a weight of the switching energy = 0.1 and a probability of "wrong" state of 0.1, for system dimensions of 4 x 16 cells (left panel), 2 x 16 cells (middle panel), and 1 x 4 cells (right panel).

Finally, Fig. 5 shows the influence of a more advanced change. Here, the system dimensions have been reduced, from 16 x 16 cells, as in the original program version, to significantly smaller dimensions. It should be mentioned here that, due to the circular reference which is necessary to introduce the switching energy, deleting columns or rows will always lead to errors in the neighboring cells of the particles. To overcome this problem, the desired number of cells should be copied from one of the unchanged particles into the changed one with the broken references. Smaller particles show, on the one hand, stronger deviations from "smooth" loops. Recalculation can always be used to

identify if any unexpected loop shapes are real or just arbitrary, due to the small number of cells involved in the simulation. On the other hand, the hysteresis loops show smaller coercive fields for smaller particles – which is equivalent to real magnetic systems, in which the so-called super-paramagnetic limit exists for very small particles, which lose their ferromagnetic behavior below a certain number of magnetic moments per particle.

4. Further Developments

Besides the simple basic examples shown above, several other parameters can be tested by the students, since the program is completely open. For getting clearer hysteresis loops with more points on the sides of the loops, the macros can easily be changed from the recent version with equidistant field values to varied field values by simply exchanging the respective values (twice per block). In this way, it is possible to test the influence of the field sweeping speed on the resulting hysteresis loop.

Besides changing the dimensions of square or rectangular particles, it is also possible to test different forms, such as circles, donuts, triangles etc. or quite long, thin particles, e.g. to simulate hysteresis in yarn tension measurements.

While the recent program version offers the possibilities to simulate magnetic particles such as ferromagnets, paramagnets and antiferromagnets, a ferrimagnet could be created by, e.g., exchanging +1 by +2 and leaving the -1 unchanged. In the same way, non-magnetic hysteresis loops with different behavior can be defined.

A more sophisticated further development introduces additional layers, influencing their direct neighbors in out-of-plane direction and summing the magnetization over all layers. Another further development is the introduction of two additional possible states, by defining vectors (+1,+1), (+1,-1) etc., as often seen in magnetic systems with fourfold anisotropies or as to simulate multi-axial stress-strain measurements. Additionally, it is possible to allow for the magnetic moments to be oriented in each in-plane direction at random angles, as in the Stoner-Wohlfarth model [11].

As can be seen, the program offers a broad variety of simple and also more sophisticated possibilities to change parameters or calculation methods due to the students' own ideas.

5. Conclusion

An Ising particle with variable parameters has been built using Microsoft Excel® and a simple macro. While the step-by-step mode supports understanding of the parameters' influence and getting a feel for the changes in each step, the macro mode results in complete hysteresis loops.

As we have shown, the program can be developed further by easier or more sophisticated changes and additions, enabling the building of a tool which is capable of answering real physical questions in the broad field of magnetism as well as in other scientific areas, in which similar hysteresis loops occur.

6. References

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