# Sharpening technique to decrease comb passband deviation

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The decimation filter is necessary to prevent aliasing during the process of decreasing sampling rate, which is called decimation. Due its simplicity, the most popular decimation filter is the comb filter. However, this filter exhibits a high passband droop in the passband magnitude characteristic, which must be avoided to prevent the distortion of the decimated signal. In this paper we present the sharpening technique to improve comb passband characteristic. In order to decrease complexity introduced by sharpening, we also consider two-stage structure where the sharpening is applied only at the second stage. Different examples are included to illustrate the benefits of the proposed method.

## 1. Introduction

The current trend in the wireless radio receivers is to move the analog-to-digital converters (A/D) as close as possible to the antenna to do most of the signal processing in the digital domain. This imposes demands on design of innovative low-power analog-to-digital converters (A/D), known as the oversampling ΣΔ-A/D converters. The oversampling ΣΔ-A/D converter samples the analog information with a frequency much larger than the Nyquist frequency, which is the minimum sampling frequency required for preserving the quality of the information. The rate of the oversampled signal must be decreased to the Nyquist frequency to be efficiently processed by the DSP ( Digital Signal processing). This process is performed in a digital format and is called down-sampling or decimation. Down-sampling may introduce the aliasing effects that deteriorate the signal and have to be eliminated by the decimation filter, as shown in Figure1, where $M$ is the decimation factor [1].

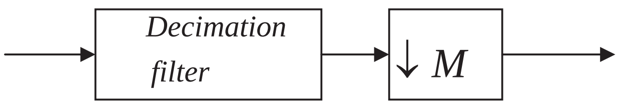


**Figure 1**. Decimation.

The decimation of a highly oversampled signal is usually accomplished using a cascade of two or more stages. Because the filter in the first stage must be very simple, it is usually used comb filter, which does not require the multipliers. The passband magnitude characteristic of the decimation filter must be flat to avoid the distortion of the decimated signal. However, the comb passband characteristic exhibits a high passband droop which must be compensated. The main objective of this work is to propose a novel procedure to obtain a flat passband comb characteristic considering a wide passband region, defined by the passband frequency,

$$\omega_p = \pi/(2M)\,. \tag{1}$$

The rest of the paper is organized in the following way. Next section briefly describes the magnitude characteristic of comb filters. Section 3 presents the compensation filter [2] and the sharpening technique. The proposed procedure is described in Section 4 and illustrated with examples. Two-stage structure is elaborated in Section 5 and compared with the comb, method [2] and the one-stage sharpened structure.

## 2. Comb filters

The system function of a comb filter can be expressed, either in the recursive form,

$$H(z) = \left[\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right]^K, \tag{2}$$

or in recursive form,

$$H(z) = \left[\frac{1}{M}\sum_{k=0}^{M-1} z^{-k}\right]^K, \tag{3}$$

where $M$ is a decimation factor and $K$ is order of the filter and presents the number of the cascaded comb filters.

The magnitude response of the filter is in the sin$x$/$x$ form :

$$\left|H(e^{j\omega})\right| = \left|\frac{\sin(\omega M/2)}{M\sin(\omega/2)}\right|^K. \tag{4}$$

Figure 2 shows the magnitude characteristics of the comb filter for $M$=18 and $K$=1, 3, and 5. The passband zooms, shown in Figure 2 demonstrate that the comb filter exhibits a high passband droop which is increases with the increasing of the parameter $K$.

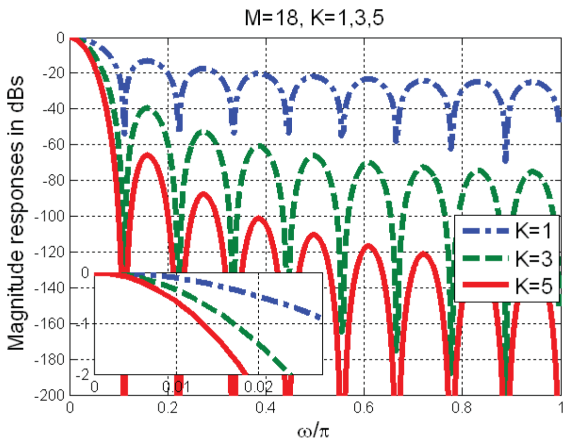


**Figure 2.** Magnitude responses of the comb filter for $M$=18 and the different values of $K$.

Different methods have been proposed to decrease the passband droop of the comb filters, using simple filters called compensators [2]-[8]. The method proposed here is the improvement of the method proposed in [2], which is briefly presented in next session, along with the brief review of the sharpening method.

## 3. Wideband comb compensator [2] and sharpening technique

### 3.1. Wideband compensator [2]

The compensation filter is a cascade of two compensators, both working at low rate:

$$G_1(z^M) = -2^{-(b+2)}\left[1-(2^{b+2}+2)z^{-M}+z^{-2M}\right], \tag{5}$$

where $b$ is the integer which depends on the comb parameter $K$, and

$$G_2(z^M) = [-2^{-4}[z^{-M}-(2^4+2)z^{-2M}+z^{-3M}]]^{K_1}, \tag{6}$$

where $K_1$ depends on the comb parameter $K$ in the following way:

$$K_1 = \begin{cases} K & for \quad 1<K\le 3 \\ K-1 & for \quad K>3 \end{cases}. \tag{7}$$

Both filters can be moved to a low rate, which is $M$ times less than the high input rate, thus becoming:

$$G_1(z) = -2^{-(b+2)}\left[1-(2^{b+2}+2)z^{-1}+z^{-2}\right]. \tag{8}$$

$$G_2(z) = [-2^{-4}[z^{-1}-(2^4+2)z^{-2}+z^{-3}]]^{K_1}. \tag{9}$$

The compensator [2] is the cascade of compensators $G_1(z)$ and $G_2(z)$:

$$G(z)= G_1(z)\, G_2(z), \tag{10}$$

where $G_1(z)$ and $G_2(z)$ are given in (8) and (9), respectively, and the values of parameters $b$ are given in [2].

The compensated comb is the cascade of comb and compensator, given by:

$$H_c(z) = H(z)G(z^M), \tag{11}$$

where $H(z)$ is given in (2) and the compensator $G(z)$ is given in (10).

Example 1:

In this example we illustrate the method [2] taking $K$=3, and $M$=21. The parameters of compensator [2] are: $K_1$=2 and $b$=3. Figure 3 shows the corresponding overall magnitude responses of comb and compensated comb filters along with the passband zooms.

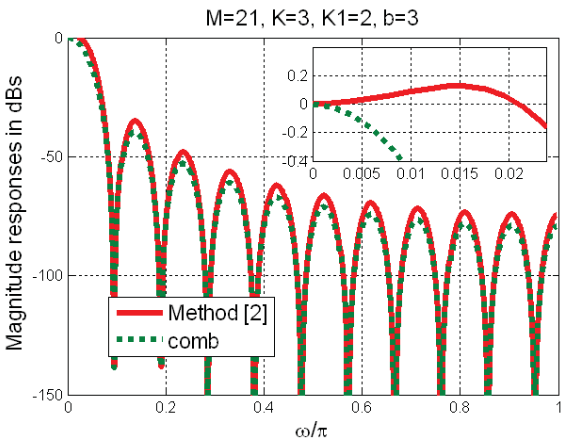


**Figure 3.** Magnitude responses of comb and compensated comb filters.

Note that the comb passband droop is significantly reduced. However, our goal here is to decrease the passband droop even more and practically obtain the flat passband characteristic. To this end we propose to use sharpening technique which is briefly described in the continuation.

### 3.2. Sharpening technique

The sharpening technique [9] uses the amplitude change function (ACF) which is a polynomial relationship of the form $H_{sh} = f(H)$ between the amplitudes of the sharpened and the prototype

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filters, $H_{sh}$ and $H$, respectively, both the pass-band and stop-band of a linear-phase FIR filter. The improvement in the gain response near the passband edge $H$=1, or near the stopband edge $H$=0, depends on the order of tangencies $m$ and $n$ of the ACF at $H$=1, or at $H$=0, respectively.

The expressions for the $m^{th}$ and $n^{th}$ order tangencies of the ACF at $H$=1 and $H$=0, respectively, are given as, [9]

$$H_{sh} = H^{n+1}\sum_{s=0}^{m}\frac{(n+s)!}{n!\,s!}(1-H)^s = H^{n+1}\sum_{s=0}^{m}C(n+s,s)(1-H)^s, \qquad (12)$$

where $C(n+s,s)$ is the binomial coefficient.

The most simple polynomial for the passband improvement, ($m$=1, $n$=0) is in the form $H_{sh}=2H-H^2$.

## 4. Sharpened compensated filter

We propose to use simple sharpening polynomial $H_{sh}=2H-H^2$ to improve the compensated comb passband characteristic.

The compensated comb is defined by,

$$H_{sh}(z) = 2H_c(z) - H_c^2(z), \qquad (13)$$

where $H_c(z)$ is given in (11).

Example 2:

In this example we compare magnitude responses of the compensated comb from Example 1, the sharpened compensated comb, and the comb filters. Figure 4 shows the overall magnitude responses and the passband zooms.

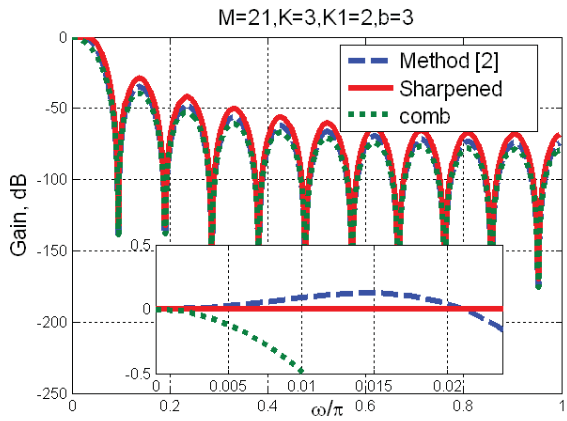


**Figure 4.** Magnitude responses of comb, compensated comb, and sharpened compensated comb filters.

Note that the passband characteristic of the sharpened compensated comb is practically flat. However, the sharpening is performed at high input rate which results in a higher complexity and the higher power consumption. In order to solve this problem we consider a two-stage structure in next session.

## 5. Two-stage sharpened compensated filter

Consider that the decimation factor can be factorized in the form $M=M_1M_2$. In this case we can split decimation into two stages, where in the first stage is the comb filter,

$$H_1(z) = \left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^K, \qquad (14)$$

decimated by $M_1$, while in the second stage is the comb filter,

$$H_2(z) = \left[\frac{1}{M_2}\frac{1-z^{-M_2}}{1-z^{-1}}\right]^K, \tag{15}$$

decimated by $M_2$.
The compensator (10) and the sharpened section $H_{sh2}(z)$:

$$H_{sh2}(z) = 2H_{c2}(z) - H_{c2}^2(z), \tag{16}$$

where

$$H_{c2}(z) = H_2(z)G(z^{M_2}), \tag{17}$$

are at the second stage.
The corresponding structure is given in Figure 5.

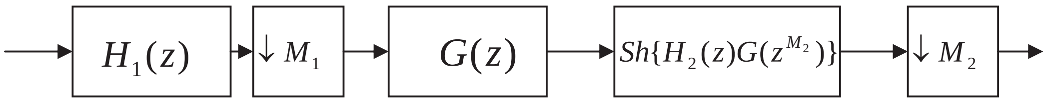


**Figure 5**. Two-stage sharpening structure.

Knowing that the sharpening is at the second stage, the choice of $M_1$ and $M_2$ has to satisfy

$$M_1 < M_2. \tag{18}$$

The choice of the parameters $b$ and $K_1$ is obtained by MATLAB simulation:

$$K_1 = K-1, \text{ for } K=2,\ldots,6. \tag{19}$$

$$b = \begin{cases} 3 & for \quad K = 2,3,4 \\ 4 & for \quad K = 6 \end{cases}. \tag{20}$$

The method is illustrated in the following example.
Example 3:
Consider $M$=32 and $K$=4. According to (18) we chose $M_1$=4 and $M_2$=8. From (19) and (20) we get the parameters $K_1$=3, and $b$=3, respectively.
Figure 6 compares magnitude responses of method [5], sharpened one stage structure, and the sharpened two-stage structure.

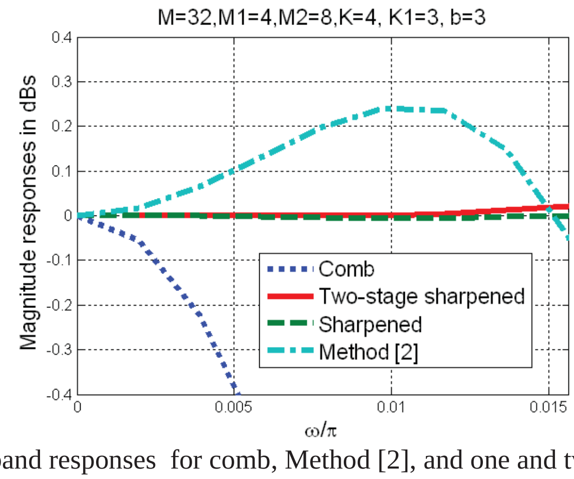


**Figure 6.** Passband responses for comb, Method [2], and one and two-stage sharpened.

Note that there is a small deviation compared with the one-stage sharpening. The deviation decreases if $M_1$<<$M_2$, as shown in Fig 7, for $M_1$=2 and $M_2$=16. However, in this case the sharpening section works at the rate which is only two times less than the high input rate. Consequently, the choice of $M_1$ and $M_2$ is a trade-off between the complexity and less deviation in the compensated comb passband.

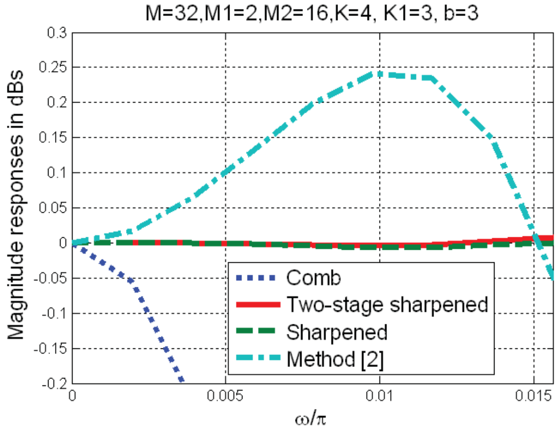


**Figure 7.** Passband responses for comb, method [2], and one and two-stage sharpened structures.

## 6. Concluding remarks

This paper presents the use of sharpening technique to obtain a flat passband comb magnitude characteristic. We consider the cascade of comb filter with narrowband and wideband compensators [2], which results in a passband deviation of approximately 0.25 dB. Applying the simple sharpening polynomial to the compensated comb, the flat passband comb magnitude characteristic is obtained. In order to decrease the complexity introduced by sharpening, we also proposed a two-stage structure, in which the sharpening is realized in the second stage. In this case the decimation factor must be presented as a product of two integers. As a result, the magnitude characteristic in the passband has only a non-significant deviation, in comparison with the passband response of the sharpened one-stage structure. The choice of the decimation factors is a trade-off between the decreased complexity and the increased passband deviation compared with the one-stage sharpened structure.

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