

Numerical study on natural convection and entropy generation in squares and corrugated cavities

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Abstract. The present work deals with the study of the entropy generation in the natural convection process in both square cavities and square cavities with hot wavy walls through numerical simulations for different undulations and Rayleigh numbers, while keeping the Prandtl number constant. The results show that the heat transfer rate is notably affected by the shape of the hot wall geometry compared with the square one. It has been found in this investigation that the mean Nusselt number in the case of heat transfer in a cavity with wavy walls is lower, as compared to heat transfer in a cavity without undulations. Based on the obtained dimensionless velocity and temperature values, the distributions of the local entropy generation due to heat transfer and fluid friction, the local Bejan number, and the local entropy generation are determined and plotted for different undulations and Rayleigh number. The study is performed for Rayleigh number $10^5 < Ra < 10^7$ irreversibility coefficients $10^{-4} < \phi < 10^{-2}$. The total entropy generation is found to increase with increasing undulation number

1. Introduction

In the past few decades, many researchers have investigated convective heat transfer in different configurations for practical engineering and industrial applications such as, heat exchangers, environmental comfort, and electronic cooling. By investigating the physical, geometric and the total entropy generation of processes, researchers can design systems with minimum loss of available energy, and enhance the thermal performance. Computational fluid dynamics and numerical codes make the calculation and simulation of local and total entropy generation easy in the complicated thermal models. Heat transfer in the system results in thermodynamic irreversibility, which generates the entropy. The entropy generation minimization is the method of optimization of the thermodynamic imperfections and fluid flow irreversibilities. Consequently, investigation of entropy generation and minimization in the heating systems provides an insight view of the thermodynamic irreversibilities involved. Bejan [1,2] has spent much effort to determine the gap between thermodynamics, heat transfer, and fluid mechanics. He has applied the second law of thermodynamics to determine the entropy generations due to heat and flow transport and consequently minimize the entropy generation. Narusawa [3] studied convective pattern change of natural convection in a rectangular cavity utilizing the second law of thermodynamics. He found that entropy generation from the perturbed temperature and velocity fields depends on aspect ratio of the cavity, the critical Rayleigh number and the ratio of entropy generation by viscous friction to that of thermal transport. Magherbi et al. [4] analyzed the entropy generation for laminar natural convection in transient state. They showed the effect of Rayleigh number and the irreversibility factor on the maximum entropy generation and the entropy generation in steady state condition. Famouri and Hooman [5] investigated numerically the entropy generation of natural convection in a partitioned cavity, with adiabatic horizontal and isothermally cooled vertical walls. They reported that, while entropy production due to fluid friction has small contribution to total entropy generation, the heat transfer irreversibility increases monotonically with both the Nusselt number and the dimensionless temperature difference.. Kaluri and Basak [6] studied entropy generation inside square cavities with distributed heated sources at wide ranges of Pr and Ra, using finite element method. They obtained that the heat transfer irreversibility dominates during conduction regime while irreversibility due to the fluid friction dominates in the convection regime. It is



et al. [7] studied numerically the entropy generation in rectangular cavities with the same area but different aspect ratios. They found that for high value of Rayleigh number, with increasing aspect ratio the entropy generation due to fluid friction and the total entropy generation firstly increased, reaching to a maximum and then decreased. Bouabid et al. [8] investigated natural convection in an inclined rectangular cavity numerically. Their results showed that entropy generation tends towards asymptotic values for lower Grashof numbers, whereas it takes oscillation behaviour for higher values. Sheikhzadeh and Nikfar [9] investigated numerically laminar natural convection focused on entropy generation in a square cavity with a rectangular obstacle filled with either pure water or Cu-water nanofluid to simulate the effects of changing aspect ratio of the obstacle on fluid flow, heat transfer and entropy generation. Sabeur et al. [10] studied the influence of non-uniform boundary conditions on natural convection in inclined rectangular cavities differentially heated. The aim of the present work is to study the entropy generation in natural convection processes in a square cavity. The contribution of this work is the analysis of the variation of the entropy generation as a function of the Rayleigh number, undulation number, and irreversibility coefficient.

2. Mathematical Model

A two-dimensional heat transfer problem is considered in both cavities; the first one is square with plates walls while the second is square with wavy hot wall maintained at the constant temperature T_h . The cold wall is opposite to the latter and has a constant temperature T_c , while the other sides are insulated. The Rayleigh number is varied up to $Ra = 10^7$, while the Prandtl number is fixed to be $Pr = 0.71$. Figure 1 shows the geometry of the cavity under consideration and the coordinate system. The thermo-physical properties of the fluid in the flow model are assumed to be constant, except for density variations responsible for the body force term in the momentum equation. The Boussinesq approximation is assumed for the fluid properties related to the density and the temperature. The governing equations for the steady natural convection flow include the equations of conservation of mass, momentum, and energy:

$$\begin{aligned} \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= -\frac{\partial P}{\partial X} + Pr \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \\ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} &= -\frac{\partial P}{\partial Y} + Pr \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra \, Pr \, \theta \\ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} &= \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \end{aligned} \quad (1)$$

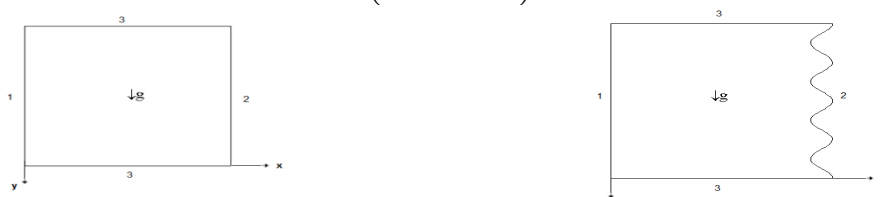


Figure 1. Geometry of the square cavity without and with undulation: (1) cold wall; (2) hot wall; (3) insulated walls.

. The shape of the wavy vertical wall is taken to be sinusoidal: $f(y) = [1 - Amp(1 - \cos 2\pi ny)]$ (2)

n and Amp are the number of undulations and the amplitude, respectively. The amplitude is fixed in this study at 0.05. The heat transfer rate due to convection in an enclosure is obtained from the Nusselt number calculation. On the wavy wall, the mean Nusselt number is expressed as follows:

$$\overline{Nu} = \frac{1}{s} \int_0^s \frac{\partial \theta}{\partial n} ds \quad (3)$$

The dimensionless local entropy generation due to heat transfer and the fluid friction (S_{LHT} and S_{LEF} respectively) for a two-dimensional heat and fluid flow in the Cartesian coordinates can be written as

$$S_{LHT} = \left(\frac{\partial \theta}{\partial X} \right)^2 + \left(\frac{\partial \theta}{\partial Y} \right)^2 \quad (4)$$

$$S_{LEF} = \phi \left\{ 2 \left[\left(\frac{\partial U}{\partial X} \right)^2 + \left(\frac{\partial V}{\partial Y} \right)^2 \right] + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 \right\} \quad (5)$$

The local entropy generation S_i is the sum of S_{LHT} and S_{LEF} . The local Bejan number indicates the strength of the entropy generation due to heat transfer irreversibility

3. Grid validation

3.1 Square Cavity

Validation was carried out with respect to the benchmark case in square cavity as well as to those benchmark quality results in the literature. The grid independence test is performed successively sized grids 40×40 , 50×50 , 67×67 and 83×83 . Table 1 indicates the average Nusselt numbers throughout the cavity for Rayleigh 10^5 . It is clearly seen that the agreement is excellent with the deviation of less than 1%. So the grid 67×67 (Figure 2) is chosen for further computations.

Table1. Validation of the Grid

Grid	$\overline{Nu}$
40×40	4.48
50×50	4.52
67×67	4.518
83×83	4.537
Davis [11]	4.519

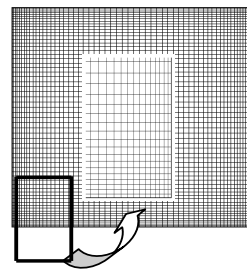


Figure 2. Mesh distribution in the square cavity

3.2 Cavity with the wavy wall

In order to study the precision of calculations; various grids were tested. Table 2 shows the various grids used for the Rayleigh number equal to 10^5 . The values of average Nusselt number are also presented. The choice of the grid 67×80 (Figure 3) which is used in the rest of the study is justified by the fact that the difference between the found values is lower than 4 %.

Table2. Validation of the Grid

Grid	$\overline{Nu}$
29×34	3.907
40×48	3.820
67×80	3.760
83×100	3.720

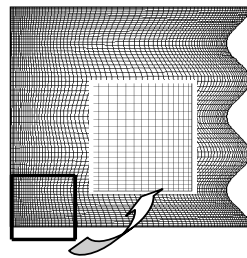


Figure3. Mesh distribution in the undulated cavity

4. Numerical Procedure

After transformation of coordinates, the Equations governing the flow and the energy are solved using a finite volume method with pressure-correction method as introduced by Patankar [12]. The detailed information about the numerical procedure and convergence criteria can be found in Ref [10]. From the known temperature and velocity fields given by solving equations (1), the local entropy generation for the entire cavity is easily obtained after knowing the temperature gradient at the walls of the cavity.

5. Results and Discussions

Due to the several cases studied, the work was carried out in this paragraph only for the square cavity and the square cavity with four undulations, Rayleigh number equal to 10^7 , and for $\phi = 0.0001$

Figure 4 illustrates the streamlines and isotherms in the square cavity. It was clearly seen that the thermal effects create a convection cell in the central region of the cavity turning it counterclockwise. It should be noted that the fluid remains a single cell by increasing the Rayleigh number; however stratification is obtained for high Rayleigh numbers. For low numbers of Rayleigh, the transfer mode is the dominant heat conduction. The increase of the Rayleigh intensifies air circulation loop in the

boundary layer close to vertical walls. The flow then passes from laminar ($10^3 < Ra < 10^6$) to turbulent ($Ra > 10^6$). It is noticed that, when the Rayleigh number values are above 10^6 , an important flow along the hot and cold walls which extends along the horizontal walls. The total section of the cavity is occupied by a recirculation zone of low intensity. As and when the Rayleigh number increases (predominantly convection) the main flow is concentrated in the active walls of the cavity, and at the same time promoting the shearing effect, which lead to the formation of more and more cells. Figure 5 shows the isotherms, streamlines, contours of the local entropy generation due to the heat transfer and fluid friction, contours of local entropy generation and contours of Bejan number for Rayleigh number equal to 10^7 . It is clearly seen that the undulated wall has an influence on the geometrical form taken by the cell as it is noticed on the different streamline patterns. In particular, near the hot wall, the streamlines converge toward the wall after each crest and diverge after each trough. This behavior has a repercussion on the heat transfer by convection near the wall. Indeed, when the normal velocity component approaches the streamline to the wall, the heat transfer increases. It was showed the thermo-convective flow accelerating more and is represented by the form of one diagonally stretched cell. Near the crest, the streamlines move near the wall just after the crest. The isotherm distribution shows the same feature near the latter region as presented in the figure (4). This observation proves the diminution of the thermal boundary layer thickness just after the crest. The flow stays stratified in the core region of the cavity. Near and along the hot wall, the boundary layer thickness increases then decreases under the effect of the undulation. It is clear that the undulated wall has an influence on the geometrical form taken by the cell as it is noticed on the different streamline patterns.

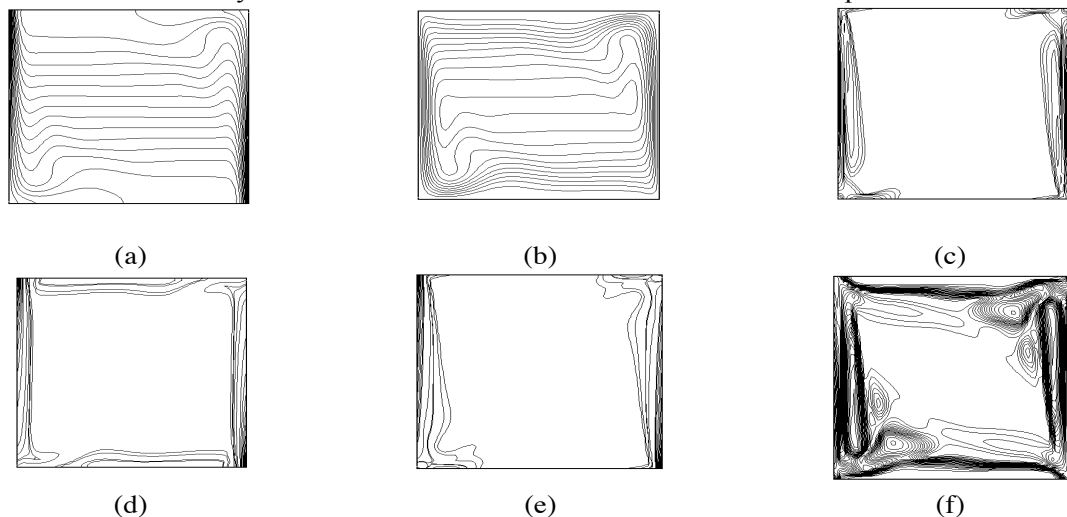
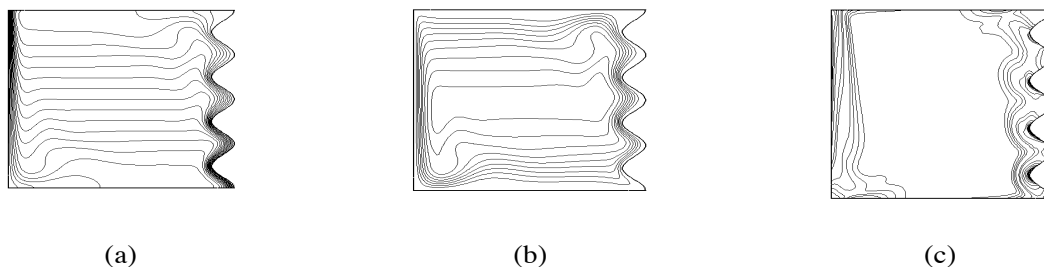


Figure 4. (a) Isotherms, (b) streamlines, (c) local entropy generation due to the heat transfer, (d) local entropy generation due to the fluid friction, (e) local entropy generation (f) Bejan Number in the square cavity ($Ra=10^7$) and $\phi = 0.0001$



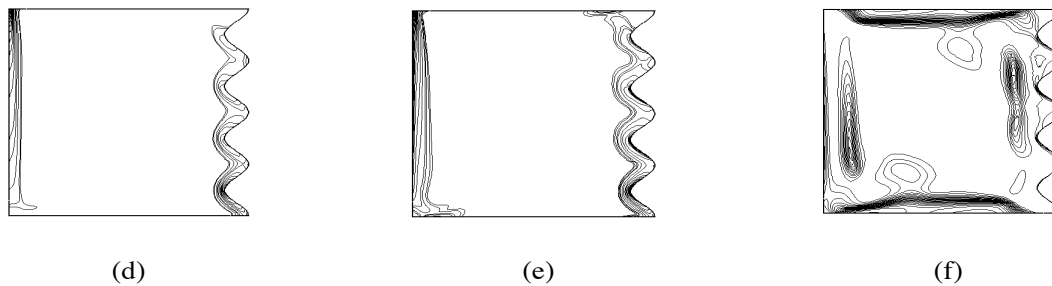


Figure 5. (a) Isotherms, (b) streamlines, (c) local entropy generation due to the heat transfer, (d) local entropy generation due to the fluid friction, (e) local entropy generation (f) Bejan Number in the undulated Cavity ($Ra=10^7$) and $\phi = 0.0001$

Figure 6 shows the variation of the total entropy generation with Rayleigh number for all cavities carried out and for $\phi=0.0001$. Distribution pattern of entropy generation are very much similar for all undulations and for the square cavity. It is clearly seen that the increasing of Rayleigh number induces a linear increasing of entropy generation value. It could be noticed that for increasing undulation number, the total entropy generation value also increases.

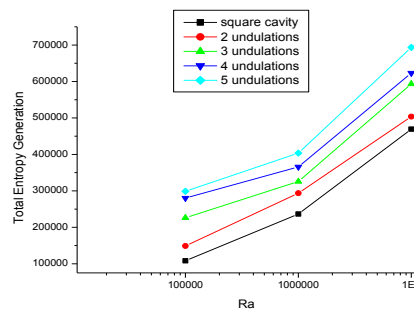


Figure 6. Variation of total entropy generation with Rayleigh number for all cavities and for $\phi = 0.0001$

6. Conclusion

This study has performed a numerical investigation into the natural convection heat transfer performance and entropy generation in two kinds of cavities, the first one is a square and the second one is a wavy wall square cavity filled with air. In modeling the cavity, it has been assumed that the left wall has a wavy surface and is heated along the wavy region only, while the right wall is flat and has a constant low temperature. In addition, it has been assumed that the upper and lower cavity walls are both flat and insulated. The simulations have examined the effects of Rayleigh number and wavy surface geometry parameters on the mean Nusselt number, total entropy generation and Bejan number, respectively. The numerical results support the following major conclusions

- The mean Nusselt number and total entropy generation both increase as the Rayleigh number increases.
- As the amplitude and wavelength of the partially-heated wavy surface increase, the mean Nusselt number reduces and the total entropy generation increases.
- The Bejan number increases as the amplitude and wavelength of the-heated wavy surface increase.
- The mean Nusselt number increases and the entropy generation reduces as the peak in the wavy surface approaches the horizontal center plane of the cavity

7. References

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