

The numerical calculation of shear zone refraction's law crossover the different stickiness sand

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Abstract. The refraction's law of sheared region near the boundary of two kinds of sand with the different stickiness is investigated by a simulation method. Our experimental apparatus for sand system is one of the simplest sets in order to obtain shear pattern, called linear split-bottom cell. In this paper we give an analysis based on a classical numerical calculation for the above-mentioned system. Although a sand particle has a variety of forms from sphere to needle in general, but here we assume an ensemble of the simplest shape such as cubes in a rectangular parallel pipe which is split into two halves. One of the halves can be moved along the split quasi-statically. We obtain a refraction pattern like an equipotential surface across the interface of different dielectric constants shown in the textbook. Various quantities obtained from our calculation were compared with experiments in the preceding conference. In this conference, we will show the refraction's law at the shear zone boundary crossing over two different sand systems with unequal stickiness.

1. Introduction

Properties in sand system have recently been studied from elementary processes [1]-[3] and also actual applications to technological fields [4]. As is well-known, sand systems consist of sand particles. Generally, forms of sand particles diverge from sphere to needle. Ensemble of such particles forms an example of typical complex system. When stress is given at a site, along a line, along a surface and/or to bulk body, stress and strain propagates and complicate pattern of stress region is formed. It is also important to investigate how the quasi-static propagation of stress and strain in the sand system progresses.

Therefore we built up an experimental apparatus to study how the shear propagates and is formed as simple as possible which is shown in Fig.1. Our results of sheared width which depend on height from the bottom in homogeneous sand system have been shown in Ref. [5]. Reference [6] is improved the insufficient parts of the preceding conference results [5].



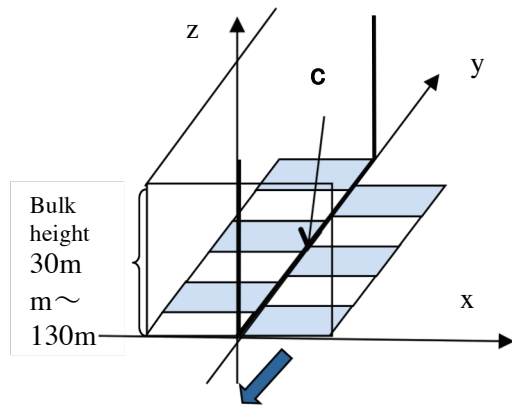


Fig.1 Experiment apparatus which is made of acrylic box with a split along the longest edge of the box. One side of the box is movable with a quasi-static velocity(blue thick arrow).

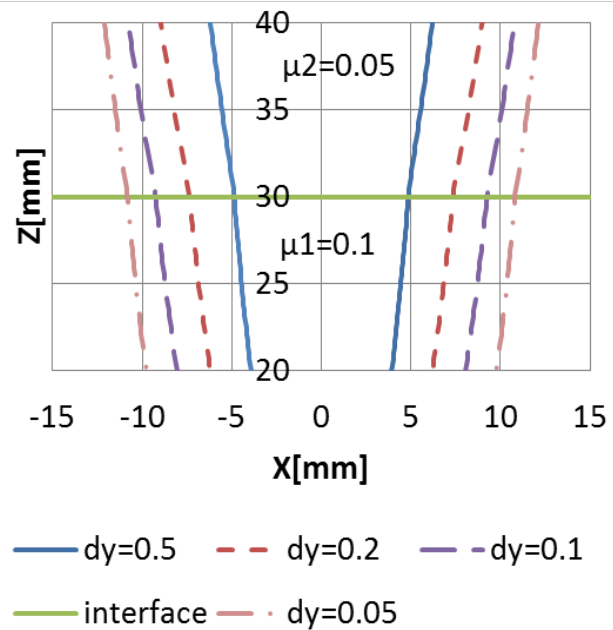


Fig.2 Shear lines across the horizontal interface of two sand layers with two different stickiness.

In Fig.1, the right hand side of box can be moved quasi-statically, where stress propagates from bottom to top. Although a sand particle has a variety of forms from sphere to needle in general, but here we assume an ensemble of the simplest shape such as cubes in a rectangular parallel-pipe split into two halves. One of the halves is moved along the split quasi-statically shown in Fig.1. Figure 2 shows the boundary of shear region which across the horizontal interface of different kinds of sand in our numerical calculation. The longitudinal lines are shear lines which sliding distances from initial positions are 0.5[mm], 0.2[mm], 0.1[mm] and 0.05[mm], respectively. But all incidence angles to the interface are so small that we can't confirm whether the refraction's law is followed Snell's law known for geometric optics [7]. In this paper we give an analysis based on numerical calculation to get the large incident angle for the above-mentioned system. Then the interface of difference sand size is leaned 45 degrees from the level.

2. A simulation corresponding the present experiment

We assume that sand particles are approximated by small cubes and our sand ensemble is regularly piled 340×680 cubes which approximates a random piling. When one side of the box is under the force which is shown by an arrow in Fig.1, the sand particles at the bottom move together with the bottom plate and many particles at the top share the slip between the right and left half boxes. By assuming the friction force between two particles we have obtained the following equation.

$$\begin{aligned}
& [\mu_x(f(x, z) - f(x-1, z)) - d_{shr}] + [\mu_x(f(x, z) - f(x+1, z)) - d_{shr}] \\
& + [\mu_z(f(x, z) - f(x, z+1))(z_{top} - z) - d_{shr}] \\
& + [\mu_z(f(x, z) - f(x, z-1))(z_{top} - z + 1) - d_{shr}] = 0
\end{aligned} \quad (1)$$

Here, $f(x, z)$ is displacement of a particle at x and z , $\mu_x(f(x, z) - f(x-1, z))$ is friction between the particles at x, z and $x-1, z$ and $\mu_z(f(x, z) - f(x, z+1))(z_{top} - z)$ is friction between the particles at x, z and $x, z+1$ where μ_x and μ_z are the friction coefficients, as shown in Fig.3. A factor $(z_{top} - z)$ is consideration about the effect of gravity. Then, $[G - d_{shr}]$ means $G - d_{shr}$ if $G - d_{shr} > 0$, and 0 if $G - d_{shr} < 0$. Once we know the numeric values of four neighbour sites of $f(x, z)$, we can obtain the value of $f(x, z)$ by using this equation.

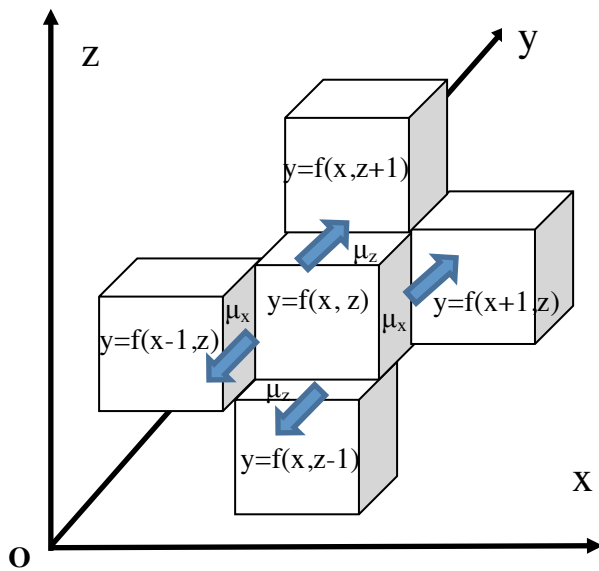


Fig.3. A sand particle is under the force from four nearest neighbour cubes.

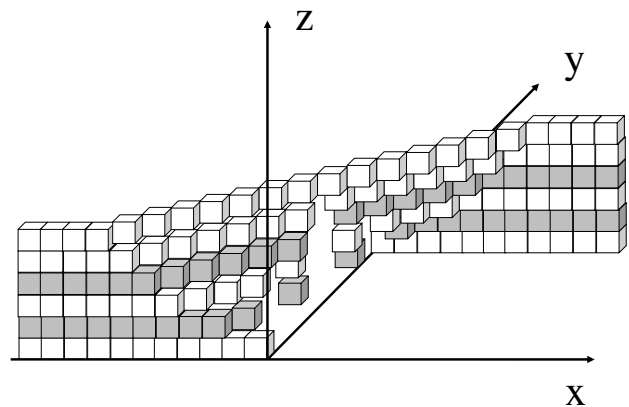


Fig. 4. A sketch of sheared portion.

3. Some results of calculation in heterogeneous sand system

Figure 4 shows a movement of particles at the central area schematically. Here we adopt that a sliding distance is 7mm as experiment where a side box is moved in a quasi-static manner. The particles at the bottom move by a little together with the plate, but next upper particles near the sliding plate move a little less distance. In this heterogeneous sand simulation, the inclination of the interface of different sand size is 45° from the horizon. The graph shown in Fig.5 is an intersection by x-z plane by adopting $\mu_1=0.1$, $\mu_2=0.05$ and $d_{shr}=0.01$. The straight line of 45° inclination is the interface of different sand size. The curves extended in the lengthwise direction of both sides in Fig.5 are shear lines whose sliding distance is 1.5 mm. The central lines show the position of the maximum slid. In order to confirm the refraction's law, the figure which expanded near a part ① of Fig. 5 is Fig. 6. The incidence angle θ_1 is 32.3° and the refraction angle θ_2 is 50.7° . Therefore $\mu_1 \tan\theta_1 = 0.0447$, $\mu_2 \tan\theta_2 = 0.0433$ and $\mu_1 \sin\theta_1 = 0.0378$, $\mu_2 \sin\theta_2 = 0.0274$. As a result of this, the refraction's law doesn't obey Snell's law known for

geometric optics but obeys the refraction law like the equipotential surface across the interface of different dielectric constants.

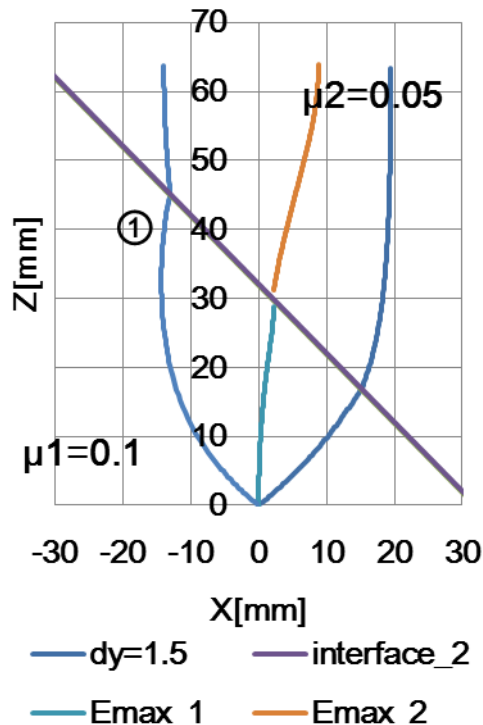


Fig.5. The maximum shear line and the line of the mean value of shear when $\mu_1 > \mu_2$.

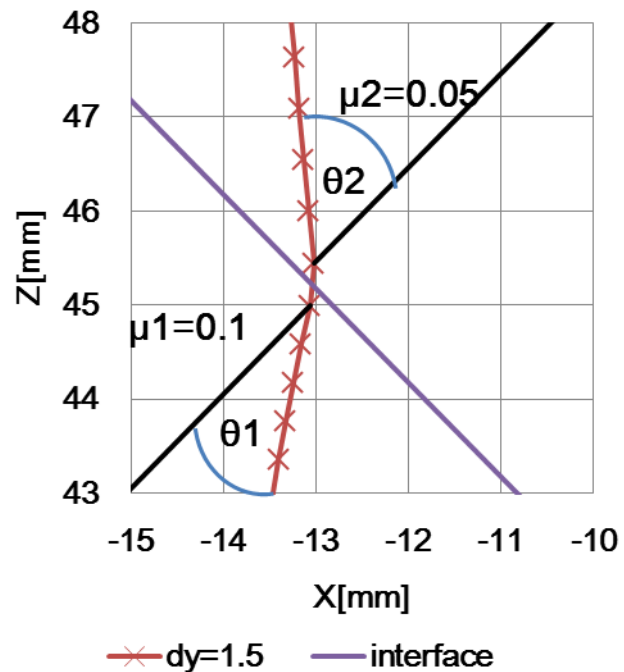


Fig.6. The enlargement shear line near ① shown in Fig.5.

4. Concluding remarks

Shear pattern in the sand system is examined experimentally or numerically by many scientists and here is a trial of simulation for the experiment. Some results of experiment are understood by the present simulation, but some are not. The present calculation shows that the refraction's law of shear line obeys the refraction law like the equipotential surface across the interface of different dielectric constants. The maximum slid line shows a complicated feature. Probably, our model will still be insufficient.

References

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