A quantum theory for the excitation spectrum of a rectangular Andreev billiard

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Abstract. We consider a $\mathcal{N} - \mathcal{S}$ box system consisting of a rectangular conductor coupled to a superconductor. The Green functions are constructed by solving the *Bogoliubov-de Gennes* equations at each side of the interface, with the pairing potential described by a step-like function. Taking into account the mismatch in the Fermi wave number and the effective masses of the normal metal - superconductor and the tunnel barrier at the interface, we use the quantum section method in order to find the exact energy Green function yielding accurate computed eigenvalues and the density of states. Furthermore, this procedure allow us to analyze in detail the *nontrivial* semiclassical limit and examine the range of applicability of the *Bohr-Sommerfeld* quantization method.

1. Introduction

There is an increasing technological interest in the study of normal-conductor($\mathcal{N}$) ballistic quantum dots attached to a superconductor($\mathcal{S}$), giving rise to the coherent scattering of electron into holes and conversely at the superconductor-conductor interface. This peculiar phenomena known as Andreev reflection is an important concept, necessary to understand the properties of nanostructures with $\mathcal{N} - \mathcal{S}$ hybrid structures, commonly called Andreev billiards[1]. In this direction, interesting experiments and theoretical works using graphene are still calling the attention of the scientific community [2, 3] .

The $\mathcal{N} - \mathcal{S}$ box system consists of a rectangular conductor $\mathcal{N}$ of height w and width a attached to a superconductor $\mathcal{S}$. The composed system is integrable and its density of states is gapless. The Bogoliubov-de-Gennes (BdeG) equation describes the physics of the system, solved under suitable boundary conditions enabling us to construct explicit Green functions at each side of the junction, the latter defined as the quantum section, as shown in Figure 1.a. The quantum section method is a tool useful to construct the full energy Green function, thus providing a method to compute the quantum spectrum, and to properly derive the semiclassical limit[4]. In section 2 we apply the quantum section method and obtain the energy Green function eigenvalue condition. Next in section 3, we quantify the role of classical orbits. In section 4 we obtain an expression for the density of states and discuss its leading order approximation. Finally, our conclusions are presented in section 5.

2. The $\mathcal{N}-\mathcal{S}$ Box System

Let us consider the BdeG equation: $\begin{pmatrix} \mathcal{H}_o & \triangle \\ \triangle^\star & -\mathcal{H}_o \end{pmatrix}\Psi = \mathcal{E}\Psi$, where $\mathcal{H}_o = \frac{\mathcal{P}^2}{2M} - \mathcal{E}_\mathcal{F}$. $\mathcal{P}$ is the particle momentum, M its mass and the chemical potential at any region is defined as equal to $\mathcal{E}_\mathcal{F}^\mathcal{N}$ for the $\mathcal{N}$ region and equal to $\mathcal{E}_\mathcal{F}^\mathcal{S}$ for the $\mathcal{S}$ region. In order to set up the wave functions, we construct the wave functions as a single layer potential density over the quantum section. Thus, considering $\chi_m(x_2)$ an eigenmode function along the section the integral $\int_\Sigma \mu_m \chi_m(x\prime_2) dx_2\prime$ is always a constant. The prescription $\mid \triangle \mid \equiv 0$ is imposed inside the normal region $\mathcal{N}$ and the wave function is written as:

$$\Psi^\mathcal{N} \;=\; \sum_m \left[a_m^e \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mathcal{G}_m^{e\mathcal{N}}(x_1, x\prime_1) + a_m^h \begin{pmatrix} 0 \\ 1 \end{pmatrix} \mathcal{G}_m^{h\mathcal{N}}(x_1, x\prime_1) \right] \chi_m(x_2)\,. \tag{1}$$

Similarly, for the superconductor region $\mathcal{S}$, $\mathcal{E} \;\leq\mid \triangle \mid \neq 0$ and

$$\Psi^\mathcal{S} \;=\; \sum_{m\prime} \left[c_{m\prime}^e \begin{pmatrix} \gamma_e \\ \frac{1}{\sqrt{2}} \end{pmatrix} \mathcal{G}_m^{e\mathcal{S}}(x_1, x\prime\prime_1) + c_{m\prime}^h \begin{pmatrix} \gamma_h \\ \frac{1}{\sqrt{2}} \end{pmatrix} \mathcal{G}_m^{h\mathcal{S}}(x_1, x\prime\prime_1) \right] \chi_{m\prime}(x_2)\,. \tag{2}$$

The choice $\chi_m(x_2) = \sqrt{\frac{2}{w}}\sin\left(\frac{m\pi}{w}x_2\right)$ satisfies Dirichlet boundary condition along x_1 for $x_2 = 0$ and w. Next, wave functions above (Eqs. (1)-(2)) are substituted into the BdeG equation to find the appropriated horizontal *Green-mode function*. Inside the $\mathcal{N}$-region, for the m-mode we define $\mathcal{K}_\mathcal{F}^\mathcal{N} = \sqrt{2\frac{M_\mathcal{N}}{\hbar^2}\mathcal{E}_\mathcal{F}^\mathcal{N}}$ and by exchanging $\mathcal{E}\prime \to \mathcal{E} - \frac{\hbar^2}{2\,M_\mathcal{N}}\left(\frac{m\pi}{w}\right)^2$, we write for electrons (e) $\left[\partial_x^2 + \frac{2\,M_\mathcal{N}}{\hbar^2}\left(\mathcal{E}_\mathcal{F}^\mathcal{N} + \mathcal{E}\prime\right)\right]\mathcal{G}_m^{e\mathcal{N}}$, similarly for holes ($h$) exchanging $\mathcal{E}\prime \to -\mathcal{E}\prime$. To satisfy Dirichlet's boundary conditions along the vertical axis at $x = a$, we choose $\mathcal{G}_m^{e(h)\mathcal{N}}(x, x\prime) = \frac{1}{k_m^{e(h)}} e^{\pm\imath k_m^{e(h)}(a-x\prime)} \sin k_m^{e(h)}(a-x)$ with $k_m^{e(h)} = \mathcal{K}_\mathcal{F}^\mathcal{N}\sqrt{1 \pm \frac{\mathcal{E}}{\mathcal{E}_\mathcal{F}^\mathcal{N}} - \left(\frac{\pi\, m}{\mathcal{K}_\mathcal{F}^\mathcal{N}\, w}\right)^2}$. Here, we can observe that these *scattering functions* describe two classical paths: a short path around the neighborhood of the section (from $x\prime$ to x). The other path is longer ($2a$), since it now leaves the section and bounces at the boundary of the conductor ($x = a$) and then goes back to the neighborhood of the section. Observe also that electrons and holes travel opposite classical paths, this classical picture will be of paramount importance later when we derive the semiclassical limit. Then we consider the superconductor region and similarly we test $\mathcal{G}_{m\prime}^{e(h)\mathcal{S}}(x, x\prime\prime) = \frac{1}{q_{m\prime}^{e(h)}} e^{\mp\imath\, q_{m\prime}^{e(h)}(x - x\prime\prime)}$, where amplitude damping results in *quasi-short-path* contributions, from the semiclassical point of view. Setting up $\mathsf{K}_\mp = \mathcal{K}_\mathcal{F}^\mathcal{S}\sqrt{\tilde{k}_{m\prime}^{e(h)\mathcal{S}}}$ and $\mathcal{K}_\mathcal{F}^\mathcal{S} = \sqrt{2\frac{M_\mathcal{S}}{\hbar^2}\mathcal{E}_\mathcal{F}^\mathcal{S}}$, for the $m\prime-$eigenmode, we now have

$$\begin{bmatrix} -\partial_x^2 - \mathsf{K}_+^2 & \frac{2\,M_\mathcal{S}}{\hbar^2}\triangle \\ \frac{2\,M_\mathcal{S}}{\hbar^2}\triangle^\star & -\left(-\partial_x^2 - \mathsf{K}_-^2\right) \end{bmatrix} \begin{bmatrix} \mathcal{G}_{m\prime}^{e\mathcal{S}} \\ \mathcal{G}_{m\prime}^{h\mathcal{S}} \end{bmatrix} \;=\; \begin{bmatrix} q_{m\prime}^{e(h)\,2} - \mathsf{K}_+^2 & \frac{2\,M_\mathcal{S}}{\hbar^2}\triangle \\ \frac{2\,M_\mathcal{S}}{\hbar^2}\triangle^\star & -\left(q_{m\prime}^{e(h)\,2} - \mathsf{K}_-^2\right) \end{bmatrix} \begin{bmatrix} \gamma_{e(h)} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \;=\; 0\,. \tag{3}$$

To find solutions for $\gamma_{e(h)}$, we compute the secular equation yielding (with $|\gamma_{e(h)}| = 1/\sqrt{2}$):

$$\begin{aligned} &\gamma_{e(h)} = \frac{1}{\sqrt{2}}\, e^{\pm\imath\arccos(\mathcal{E}/|\triangle|)}, \qquad q_m^{e(h)} = \mathcal{K}_\mathcal{F}^\mathcal{S}\sqrt{\tilde{\mathcal{Q}}_m^\mathcal{S}}\, e^{\frac{\imath}{2}(\mathsf{q}_m^{e(h)} + \frac{\theta}{2})}, \qquad \tilde{k}_m^{e(h)\mathcal{S}} = \left[1 - (+)\frac{\mathcal{E}}{\mathcal{E}_\mathcal{F}^\mathcal{S}} - \left(\frac{m\pi}{\mathcal{K}_\mathcal{F}^\mathcal{S}\, w}\right)^2\right], \\ &\tilde{\mathcal{Q}}_m^\mathcal{S} = \sqrt{\left(\frac{\triangle}{\mathcal{E}_\mathcal{F}^\mathcal{S}}\right)^2 + \tilde{k}_m^{e\mathcal{S}}\,\tilde{k}_m^{h\mathcal{S}}}, \quad \mathsf{q}_m^e = \arctan\left[\frac{\left(\frac{\triangle}{\mathcal{E}_\mathcal{F}^\mathcal{S}} - \tilde{k}_m^{e\mathcal{S}}\right)}{\left(\frac{\triangle}{\mathcal{E}_\mathcal{F}^\mathcal{S}} + \tilde{k}_m^{e\mathcal{S}}\right)}\tan\frac{\theta}{2}\right], \quad \mathsf{q}_m^h = \arctan\left[\frac{\left(\frac{\triangle}{\mathcal{E}_\mathcal{F}^\mathcal{S}} + \tilde{k}_m^{h\mathcal{S}}\right)}{\left(\frac{\triangle}{\mathcal{E}_\mathcal{F}^\mathcal{S}} - \tilde{k}_m^{h\mathcal{S}}\right)}\tan\frac{\theta}{2}\right] - \pi\,. \end{aligned} \tag{4}$$

Defining $\lambda = \frac{M_\mathcal{N}}{M_\mathcal{S}}$ and $\mathcal{Z} = 2\frac{M_\mathcal{N}}{\hbar^2}U_o$, we now search for eigenvalues imposing the continuity of the wave function and a step-like discontinuity of the wave function derivative, at the section,

$$\left[\Psi^\mathcal{N} - \Psi^\mathcal{S}\right]_\Sigma = 0 \quad , \quad \left[\partial_x\left(\Psi^\mathcal{N} - \lambda\,\Psi^\mathcal{S}\right) - \mathcal{Z}\,\Psi^{\mathcal{S}(\mathcal{N})}\right]_\Sigma = 0\,. \tag{5}$$

With $x\prime = x\prime\prime = 0$ on the section, we obtain the energy Green function $\mathcal{G}_m(\mathcal{E})$ imposing that at the eigenenergy:

$$\langle m|\mathcal{G}(\mathcal{E})|m\rangle = \frac{1}{\sqrt{2}}\sum_{mm\prime}\int_{\Sigma}\langle m|y\rangle\left[\gamma_e\,\mathcal{D}^{ee}_{mm}\,\mathcal{D}^{hh}_{m\prime m\prime} - \gamma_h\,\mathcal{D}^{he}_{m\prime m}\,\mathcal{D}^{eh}_{mm\prime}\right]\langle y|m\prime\rangle\,dy \equiv 0\,. \tag{6}$$

Since $\int_{\Sigma}\langle m|x_2\rangle\langle x_2|m\prime\rangle dx_2 = \delta_{mm\prime}$, with $\mathcal{D}^{\alpha\beta}_{mm} = \det\begin{vmatrix} \mathcal{G}^{\alpha\,\mathcal{N}}_m & \mathcal{G}^{\beta\,\mathcal{S}}_m \\ \mathcal{G}^{\alpha\,\mathcal{N}}_{m}\prime & (\mathcal{Z}\mathcal{G}^{\beta\,\mathcal{S}}_n + \lambda\mathcal{G}^{\beta\,\mathcal{S}}_m\prime) \end{vmatrix}$; for $\alpha = (e\,,\,h)$ and $\beta = (e\,,\,h)$. The next step consists to rewrite the energy Green function in terms of the scattering matrix. We will see that this strategy is useful when searching for a semiclassical description.

3. The Classical Orbits

We define $\mathcal{P}_m = \imath\frac{\lambda^2\sin\theta}{2\,K_m\Gamma^2_m}\,\mathcal{Z}^e_m\mathcal{Z}^h_m e^{\imath(k^-_m a+[\mathsf{z}^e_m+\mathsf{z}^h_m]-\theta)}\,[\cos(k^-_m a) - \cos(k^+_m a)]$ with $k^{\mp}_m = k^e_m \mp k^h_m$, $\mathsf{z}^{e(h)}_m = \arctan\left\{\left(\frac{\Gamma_m-\mathcal{Z}}{\Gamma_m+\mathcal{Z}}\right)\tan\frac{1}{4}(\mathsf{q}^{e(h)}_m + \frac{\theta}{2} - (+)\pi)\right\}$, $\mathcal{Z}^{e(h)}_m = \sqrt{\mathcal{Z}^2 + \Gamma^2_m + 2\mathcal{Z}\Gamma_m\cos\frac{1}{2}(\mathsf{q}^{e(h)}_m + \frac{\theta}{2} - (+)\pi)}$, $K_m = k^e_m k^h_m$, $\Gamma_m = \lambda\mathcal{K}^{\mathcal{S}}_{\mathcal{F}}\sqrt{\tilde{\mathcal{Q}}^{\mathcal{S}}_m}$, and rewrite Eq. (6) as a function of the scattering matrix's terms of reflection $(r_m, \tilde{r}_m)$ and transmission$(t_m, \tilde{t}_m)$ [5]

$$\mathcal{G}_m(\mathcal{E}) = \mathcal{P}_m\det|I - S_m| = \left[\mathcal{P}_m - \mathcal{P}_m\,(r_m + \tilde{r}_m) + \mathcal{P}_m\,r_m\,\tilde{r}_m - \mathcal{P}_m\,t_m\,\tilde{t}_m\right], \tag{7}$$

yielding $\mathsf{r}^{-(+)}_m = \arctan\left[\left(\frac{k^e_m+(-)\,k^h_m}{k^e_m-(+)\,k^h_m}\right)\tan\theta\right]$, $\mathcal{R}^{-(+)}_m = \sqrt{[k^e_m]^2 + [k^h_m]^2 - (+)\,2\,K_m\,\cos 2\theta}$,
$\mathsf{d}^{-(+)}_m = \arctan\left[\frac{\left(\Gamma_m\mathcal{R}^{-(+)}_m\cos(\frac{1}{4}[\mathsf{q}^h_m-\mathsf{q}^e_m]+\mathsf{r}^{-(+)}_m)-\mathcal{Z}\,k^{+(-)}_m\sin\theta\right)}{\left(\Gamma_m\mathcal{R}^{-(+)}_m\cos(\frac{1}{4}[\mathsf{q}^h_m-\mathsf{q}^e_m]+\mathsf{r}^{-(+)}_m)+\mathcal{Z}\,k^{+(-)}_m\sin\theta\right)}\tan\frac{\theta}{2}\right]$, $\varpi^{+(-)}_m = \mathcal{Z}\,k^{+(-)}_m\sin\theta$,
$\Xi^{-(+)}_m = \Gamma_m\,\mathcal{R}^{-(+)}_m\cos(\frac{1}{4}[\mathsf{q}^h_m - \mathsf{q}^e_m] + \mathsf{r}^{-(+)}_m)$, $\mathcal{D}^{-(+)}_m = \sqrt{\left(\varpi^{+(-)}_m\right)^2 + \left(\Xi^{-(+)}_m\right)^2 + 2\,\varpi^{+(-)}_m\,\Xi^{-(+)}_m\cos\theta}$,

$$\begin{aligned} -\mathcal{P}_m\,(r_m + \tilde{r}_m) &= \imath\frac{\lambda^2}{2\,K_m\,\Gamma^2_m}\left\{\mathcal{D}^-_m\,e^{\imath\,(k^-_m a+\mathsf{d}^-_m-\frac{3}{2}\theta)}\sin(k^+_m a) - \mathcal{D}^+_m\,e^{\imath\,(k^-_m a+\mathsf{d}^+_m-\frac{3}{2}\theta)}\sin(k^-_m a)\right\}, \\ \mathcal{P}_m\,r_m\,\tilde{r}_m - \mathcal{P}_m\,t_m\,\tilde{t}_m &= \imath\frac{\lambda^2\sin\theta}{2\,\Gamma^2_m}\,e^{\imath(k^-_m a-2\theta)}\,[\cos(k^-_m a) + \cos(k^+_m a)]\,. \end{aligned}$$

Here we found that the scattering reflection contributions are diminished by the scattering transmission contributions, taking account of the quantum capability of the wave function to tunnel.

4. Leading order approximations

We consider all the eigenmode contributions to compute the density of states,

$$\begin{aligned} \rho(\mathcal{E}) = -\tfrac{1}{\pi}\textstyle\sum_m\mathcal{G}_m(\mathcal{E}) &= -\tfrac{1}{\pi}\textstyle\sum_m\frac{\lambda^2\sin\theta}{4\,K_m\,\Gamma^2_m}\,\{\Phi_{1m}\cos(2\theta + \Theta_{1m}) - \Phi_{2m}\cos(2k^-_m a - 2\theta - \Theta_{2m}) \\ &- \Phi_{3m}\cos([k^-_m - k^+_m]a - 2\theta - \Theta_{3m}) + \Phi_{4m}\cos([k^-_m + k^+_m]a - 2\theta - \Theta_{4m})\}\,. \end{aligned} \tag{8}$$

We now analyze the phase of the respective contributions in the above expression. Even the short trajectory is affected by quantum tunneling, the phase 2θ appears in all contributing terms in $\rho(\mathcal{E})$. The second term is a long trajectory which is influenced by Andreev retroreflections $2k^-_m a\left(= 2[k^e_m - k^h_m]\,a\right)$, where both electron and hole particles travel the same length $(2a)$ in opposite directions while bouncing inside the conductor, see Figure 1.b. The last two terms are influenced by Andreed retroreflections and specular-reflections represented by $k^+_m a\left(= [k^e_m + k^h_m]\,a\right)$, respectively. In particular, the third term considers contributions of two bouncing electrons traveling the same length a; one in a clockwise and another in a counterclockwise fashion. Meanwhile the hole travels twice the width of the conductor, it is shown in Figure 1.c. Exchanging electron by hole the physical scenario presented before can be applied to the fourth term, as depicted in Figure 1.d. We find that every phase contribution is also affected by a term Θ_{im} taking into account the fact that the waves functions can go further into the superconductor region. In fact this is a correction applied to the usual tunneling term θ, since it depends on it and the superconductor material physical parameters. Finally, the amplitudes Φ_{im} are related to the particle's path length distribution[3, 6, 7],

Aiming at a better understanding of the semiclassical framework, we make the following approximations: $\frac{\tilde{\mathcal{A}}^{\pm}_{2m}\sin\theta}{\tilde{\mathcal{A}}^{\mp}_{2m}\cos(\theta-\tilde{\alpha}^{\mp}_{1m})} < 1$, $\frac{\tilde{\mathcal{B}}^{\pm}_{2m}\sin\theta}{\tilde{\mathcal{B}}^{\mp}_{2m}\cos(\theta-\tilde{\beta}^{\mp}_{1m})} < 1$ and $\mathcal{Z}\left(\frac{\triangle/\mathcal{E}^{\mathcal{S}}_{\mathcal{F}}}{\tilde{\mathcal{Q}}^{\mathcal{S}}_{m}}\right) < 1$, and obtain

$$\begin{aligned}
\tilde{\mathcal{A}}^{\mp}_{1m} &\approx \sqrt{\left(\Gamma_m k^{+}_{m}\right)^2 + \left(\mathcal{Z}k^{-}_{m} \mp 2\Gamma^2_m \sin^2\theta\right)^2}, \quad \tilde{\mathcal{A}}^{\pm}_{2m} \approx \Gamma_m k^{+}_{m}\cos\theta \pm \left[\left(\Gamma^2_m + \mathcal{Z}^2\right)\cos 2\theta + K_m\right] . \\
\tilde{\mathcal{B}}^{\mp}_{1m} &\approx \sqrt{\left(\Gamma_m k^{-}_{m}\right)^2 + \left(\mathcal{Z}k^{+}_{m} \mp 2\Gamma^2_m \sin^2\theta\right)^2}, \quad \tilde{\mathcal{B}}^{\pm}_{2m} \approx \Gamma_m k^{-}_{m}\cos\theta \pm \left[\left(\Gamma^2_m + \mathcal{Z}^2\right)\cos 2\theta - K_m\right] .
\end{aligned}$$

$$\tilde{\alpha}^{\mp}_{1m} \approx \arccos\left(\frac{\Gamma_m k^{+}_{m}}{\tilde{\mathcal{A}}^{\mp}_{1m}}\right) \quad , \quad \tilde{\beta}^{\mp}_{1m} \approx \arccos\left(\frac{\Gamma_m k^{-}_{m}}{\tilde{\mathcal{B}}^{\mp}_{1m}}\right).$$

$$\Phi_{1m} \approx \tilde{\mathcal{A}}^{-}_{1m}\cos(\theta-\tilde{\alpha}^{-}_{1m}), \Phi_{2m} \approx \tilde{\mathcal{A}}^{+}_{1m}\cos(\theta-\tilde{\alpha}^{+}_{1m}), \Phi_{3m} \approx \tilde{\mathcal{B}}^{-}_{1m}\cos(\theta-\tilde{\beta}^{-}_{1m}), \Phi_{4m} \approx \tilde{\mathcal{B}}^{+}_{1m}\cos(\theta-\tilde{\beta}^{+}_{1m}).$$

$$\Theta_{1m} \approx \arccos\left(\frac{\tilde{\mathcal{A}}^{+}_{2m}\sin\theta}{\Phi_{1m}}\right), \Theta_{2m} \approx \arccos\left(\frac{\tilde{\mathcal{A}}^{-}_{2m}\sin\theta}{\Phi_{2m}}\right), \Theta_{3m} \approx \arccos\left(\frac{\tilde{\mathcal{B}}^{+}_{2m}\sin\theta}{\Phi_{3m}}\right), \Theta_{4m} \approx \arccos\left(\frac{\tilde{\mathcal{B}}^{-}_{2m}\sin\theta}{\Phi_{4m}}\right).$$

In the strong limit $\mathcal{E} << \mathcal{E}_{\mathcal{F}}$, the phases and the amplitudes are approximately equal and the net contribution of the mixture of Andreev retroreflection and specular reflection are negligible, thus retroreflection constitute the most important contribution, and so the *Bohr-Sommerfeld* (*BS*) formula quantization rule becomes $2\left(k^{e}_{m} - k^{h}_{m}\right)a - 2\theta - \Theta_{2m} = (2n-1)\pi$, with m, n integers. The Θ_{2m} term is not present in the *BS* expression obtained in reference[6], where both the mass of the particles and the Fermi energy of the $\mathcal{N}-\mathcal{S}$ system are considered indistinguishable and clean.

5. Conclusions

We presented a full quantum theory for the $\mathcal{N}-\mathcal{S}$ system via the quantum section method. It enable us to properly obtain the semiclassical limit. In general, both Andreev retroreflection and Andreev specular reflections determine the spectrum of the system. Our formulation enable us to analyze the *B-S* quantization method. We found that in the strong limit $\mathcal{E} << \mathcal{E}_{\mathcal{F}}$ the *BS* formula applies, including the extra term Θ_{2m}. Numerical work in progress may corroborate Cserti *et al.*[6] conjecture, namely: third and fourth contributions in equation (8) cancel each other in the semiclassical limit.

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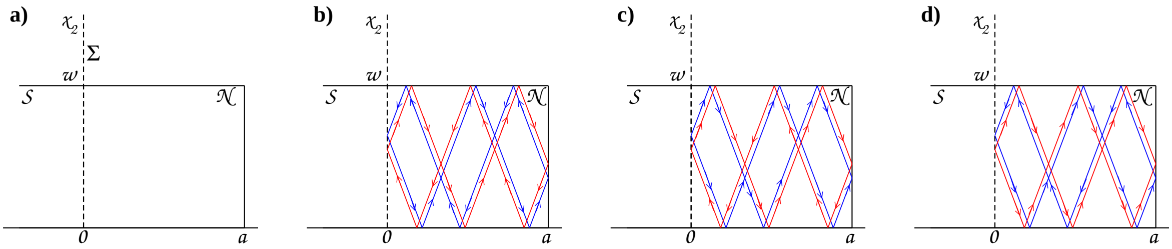

Figure 1. **a)** The Andreev billiard composed of two rectangular regions, one conducting and the another superconducting. The quantum section Σ is defined over the junction at $x_1 = 0$. Different phase contributions of an electron (blue-line) and hole (red-line) into the Andreev billiard. Andreev retroreflections is depicted in **b)**. In **c)**, two electrons traveling in clockwise and counterclockwise directions and one hole. In **d)**, two holes traveling in clockwise and counterclockwise directions and one electron. These two contributions are interpreted as a superposition of Andreev retroreflection $(k^e_m - k^h_m)$ and specular reflections $(k^e_m + k^h_m)$, respectively.