

# One dark matter mystery: halos in the cosmic web

**Jose Gaite**

Instituto de Microgravedad IDR, ETSIAE,  
Universidad Politécnica de Madrid, E-28040 Madrid, Spain

E-mail: [jose.gaite@upm.es](mailto:jose.gaite@upm.es)

**Abstract.** The current cold dark matter cosmological model explains the large scale cosmic web structure but is challenged by the observation of a relatively smooth distribution of matter in galactic clusters. We consider various aspects of modeling the dark matter around galaxies as distributed in smooth halos and, especially, the smoothness of the dark matter halos seen in  $N$ -body cosmological simulations. We conclude that the problems of the cold dark matter cosmology on small scales are more serious than normally admitted.

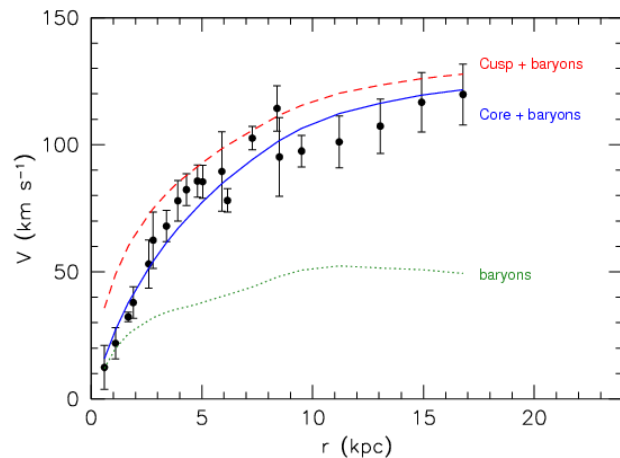
## 1. Introduction

Cold dark matter, which consists of weakly interacting particles whose velocity dispersion in the early universe was not sufficient to prevent the formation of structure on galactic and sub-galactic scales, is fundamental in modern cosmology, since the early 1980s. Naturally, the full standard cosmological model, namely, the  $\Lambda$ CDM model, presently includes a cosmological constant and inflationary initial conditions. However, just the cold dark matter (CDM) hypothesis, with the consequent richness of structure on very small scales, does not fit well the indirect observations of dark matter. To be precise, the results of  $N$ -body simulations of the evolution of collisionless CDM show that too much structure is produced on galactic scales. This excess of small-scale structure manifests itself in two problems: the core-cusp problem [1] and the missing satellites problem (or dwarf galaxy problem) [2, 3].

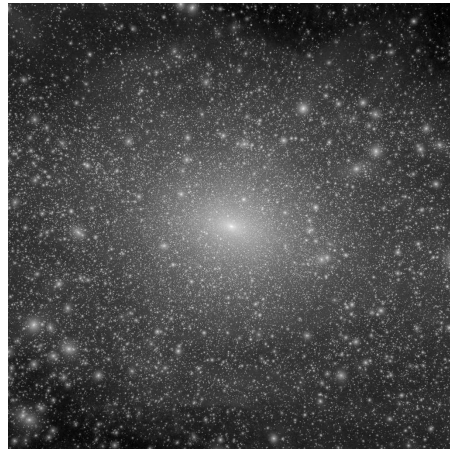
Observations of galactic rotation velocities seem to indicate an approximately constant dark matter density in the inner parts of galaxies, whereas cosmological  $N$ -body simulations indicate that the dark matter radial density profile has a singularity of power-law type. This discrepancy has been named “the core-cusp problem”. It is illustrated in Fig. 1, which shows the results of Kuzio de Naray et al [4] for a particular galaxy (F568-3): the points with error bars are rotation velocity measurements and are well fitted by the solid curve, which corresponds to an isothermal dark matter halo with a constant density core. In contrast, the dashed curve, corresponding to a Navarro-Frenk-White (NFW) singular density profile, clearly overpredicts the rotation velocity near the center. Any other power-law like profile would also induce too high rotation velocities near the center.

The “missing satellites” problem refers to the appearance of a large number of subhalos (satellites) close to every single halo in cosmological  $N$ -body simulations, without observational counterparts. Put in other words, the number of dwarf satellite galaxies around a normal-sized galaxy is much smaller than the number predicted by collisionless CDM  $N$ -body simulations. For example, only about 11 dwarf galaxies orbiting the Milky Way have been observed, whereas the number of dark matter subhalos produced in, for example, the “Via Lactea” cosmological





**Figure 1.** Measured rotation curve of a galaxy (points with error bars) compared to model fits assuming a cored dark matter halo (solid curve) or a cuspy dark matter halo with an NFW profile (dashed curve). The dotted curve shows the contribution of baryons (stars+gas). From Kuzio de Naray et al.



**Figure 2.** The dark matter distribution from the “Via Lactea II” cosmological simulation of a Milky Way-mass cold dark matter halo, from Diemand et al. (<http://www.ics.uzh.ch/~diemand/vl>). The cube has a size of 800 kpc.

simulation of a Milky Way-mass cold dark matter halo is much larger (the simulation has been made by Diemand et al [3]). This is easily perceived in Fig. 2. Naturally, this problem is intrinsic to the CDM model, since cold dark matter clusters hierarchically from the smallest scales (from the bottom up). If dark matter is composed of neutralinos, as is likely, the smallest clumps of dark matter can range from  $10^{-11} M_{\odot}$  to  $10^{-3} M_{\odot}$  [5].

Both problems suggest that the real dark matter distribution, on small scales, is smoother than the one produced by collisionless CDM  $N$ -body simulations: the real distribution is less grainy and has no density singularities. On the other hand, on medium or large scales, the results of  $N$ -body simulations agree with observations: the large scale structure of the Universe is arranged as a “cosmic web” constituted by matter sheets, filaments, and nodes, plus the cosmic voids that these objects leave in between. This type of structure was initially proposed in connection with a simplified model of the cosmic dynamics, namely, the adhesion model [6], and both galaxy surveys and cosmological  $N$ -body simulations have confirmed it.

On scales smaller than those corresponding to the cosmic-web structure in  $N$ -body simulations, the dark matter distribution transforms into a distribution of relatively smooth dark matter clusters or *halos* that have a limited range of sizes. These dark matter halos are to be identified with the ones surrounding galaxies, detected by rotation velocity measurements. Halo models of the large scale structure of matter [7] have become very popular. The distribution of dark matter inside an  $N$ -body halo is smooth, save for a concentration at its center, presumably constituting a density singularity, and for the presence of subhalos close to it. The existence of a density singularity and of an excess of subhalos are the already mentioned problems of the CDM model on small scales.

Although the existence of smooth halos is rarely questioned, they really are not part of the CDM model. In fact, their presence in  $N$ -body simulations can be a consequence of numerical artifacts [8]. From a theoretical viewpoint, the continuum CDM dynamics is ruled by the Vlasov-Poisson equations, which give rise to the same type of gravitational clustering on ever decreasing scales. Therefore, assuming that the cosmic web geometry is the attractor solution of

those equations, there is no reason why this solution should become relatively smooth from some galactic or super-galactic scale downwards. On the contrary, it would be natural that the cosmic web structure in collisionless  $N$ -body simulations were a self-similar multifractal, displaying the same features, namely, sheets, filaments, nodes, and cosmic voids, on ever decreasing scales, down to the scale of the smallest dark matter clumps (with mass  $\ll M_\odot$ ). For example, the cosmic web produced by the adhesion model [6] is of that type [9].

With the above in mind, we study the smoothness of halos in CDM  $N$ -body simulations, to determine the range of scales where the distribution of matter is smooth and the connection of halos with the non-smooth cosmic web structure [10]. We also consider how the abundance of halos depends on the mass resolution and on the discreteness parameters pertaining to  $N$  bodies. Our conclusions question the consistency of the CDM-halo model with observations much more seriously than the well-known problems, namely, the core-cusp problem and the missing satellites problem.

## 2. Smoothness of halos in $N$ -body simulations

A dark matter halo consists of a spherical like distribution of dark matter particles with a density that is generally smooth but singular at the center [7]. Most models of formation of halos are motivated by the spherical collapse model and its variants. Naturally, in a halo, smoothness and isotropy tend to diminish away from the center. If we imagine a spherical surface with origin on the halo center and with increasing radius, initially the dark matter distribution over the surface is mildly anisotropic (triaxial) but, as the radius increases, the anisotropy increases and, eventually, the spherical surface intersects a distribution that is essentially indistinguishable from a non-smooth cosmic web structure.

To measure anisotropy and non-smoothness, we need to define them in mathematical terms [10]. In mathematics, a function is smooth if it has an infinite number of derivatives. However, we cannot just use derivatives of the density field, on account of the density singularities at halo centers. Therefore, a more useful distinguishing feature is the change from mild to strong anisotropy on increasing scales: on small scales, namely, close to halo centers, the anisotropy is caused by triaxiality and, possibly, subhalos, whereas, on larger scales, the anisotropy patterns are much more complex. If we consider a spherical shell centered on a halo, the density in it must be fairly smooth for a small radius (except for subhalo singularities), but the density must become increasingly non-smooth as the radius grows.

To find how uniform a distribution of  $k$  particles in a spherical shell is, we can compare the angular spherical coordinates of the particles,  $\{\phi_i, \theta_i\}_{i=1}^k$ , with the coordinates corresponding to a uniform distribution of the particles. To do this, we must consider that the variables that must be uniformly distributed are not  $\phi$  and  $\theta$  but  $\phi$  and  $\cos \theta$ . For convenience, we define coordinates  $\phi/(2\pi)$  and  $(1 + \cos \theta)/2$  over the unit square  $[0, 1] \times [0, 1]$ , corresponding to a cylindrical equal-area projection on it. To analyze if the particles are uniformly distributed, we can use the Rényi entropies

$$S_q(\{p_i\}) = \frac{\log_2(\sum_{i=1}^M p_i^q)}{1 - q}, \quad q \neq 1, \quad (1)$$

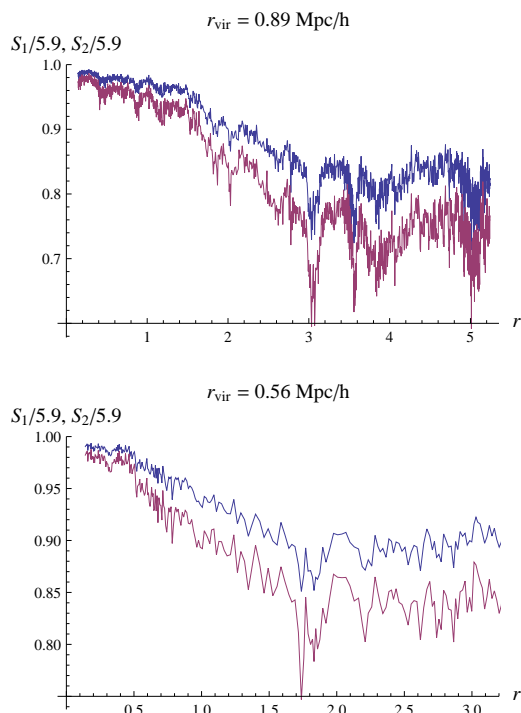
where  $\{p_i = n_i/N\}_{i=1}^M$ , with  $N = \sum_{i=1}^M n_i$ , and  $\{n_i\}_{i=1}^M$  are the counts-in-cells corresponding to the distribution of  $N$  particles in  $M$  spherical cells (set in the given coordinates). The limit of  $S_q$  as  $q \rightarrow 1$  yields the standard Boltzmann-Gibbs-Shannon entropy. The Rényi entropies with  $q \geq 0$  fulfill  $0 \leq S_q(\{p_i\}) \leq \log_2 M$ , so we divide them by  $\log_2 M$  to obtain numbers between 0 and 1: zero corresponds to maximum order (or dispersion of counts-in-cells) and one to minimum order. The entropic coefficients  $S_q(\{p_i\})/\log_2 M$  are just coarse Rényi dimensions divided by the dimension of the ambient space, which is, in the present case, a two-dimensional spherical surface.

When we observe how the  $q \geq 1$  entropic coefficients of a shell in a given halo change with the radius of the shell, we expect that they start at values close to one and have a generally decreasing trend, in accord with the transition from small-scale smoothness to large-scale graininess.

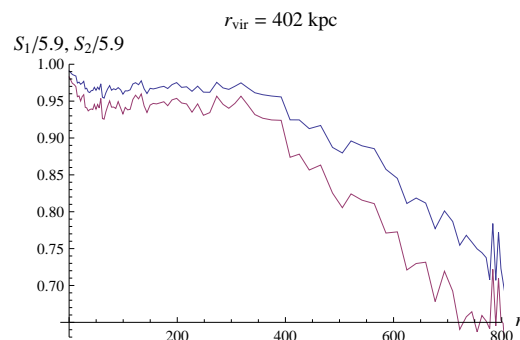
### 2.1. Halos in the Bolshoi and Via Lactea II simulations

The Bolshoi simulation [11] assumes cosmological parameters  $\Omega_\Lambda = 0.73$ ,  $\Omega_M = 0.27$ , Hubble parameter  $h = 0.70$ , and initial spectral index  $n = 0.95$ , while the “Via Lactea II” (VL2) simulation [3] takes  $\Omega_\Lambda = 0.76$ ,  $\Omega_M = 0.24$ , Hubble parameter  $h = 0.73$ , and initial spectral index  $n = 0.95$ . The comoving size of the simulation boxes are  $250 h^{-1}$  Mpc and  $40 h^{-1}$  Mpc, respectively. The number of Bolshoi particles is  $N = 2048^3$ , yielding a mass resolution of  $1.35 \cdot 10^8 h^{-1} M_\odot$ . The VL2 simulation focuses on the formation of a single, Milky-Way size CDM halo, using the method of refinement. The VL2 dark matter halo is refined with more than  $10^9$  high-resolution particles, yielding a mass resolution of  $4098 M_\odot$ .

Our procedure consists in adjusting spherical shells to have 1024 particles and then setting a  $8 \times 8$  mesh in the unit square corresponding to every shell (in our coordinates), obtaining  $M = 64$  cells per shell. Since subhalo singularities strongly affect smoothness measures, we avoid them by removing the four most populated cells of every shell, reducing  $M$  to 60. This operation hardly alters the overall smoothness properties. In consequence, the entropic coefficients are given by  $S_q / \log_2 60 = S_q / 5.9$ . The variation of the  $q = 1$  and  $q = 2$  coefficients with radius for two Bolshoi halos, on the one hand, and the VL2 halo, on the other, are plotted in Fig. 3 and Fig. 4, respectively.



**Figure 3.** Smoothness measured by two Rényi entropies in two Bolshoi halos, showing the decrease with radius (in  $h^{-1}$  Mpc).



**Figure 4.** The variation with radius of the same two Rényi entropies in the “Via Lactea II” halo. The radius is measured in kpc.

We see in all the plots the progressive decrease of smoothness with increasing radius, from values close to one to values that approximately tend to the asymptotic values corresponding

to the multifractal cosmic-web structure, namely, 0.8 and 0.6, for  $q = 1$  and  $q = 2$ , respectively [12, 13]. The considerable fluctuations superposed on the decreasing trend are due to subhalos, whose effect is not totally eliminated in our procedure. Remarkably, nothing special happens at the virial radius.

## 2.2. Subhalo abundance

The excellent mass resolution of the VL2 simulation reflects on the large number of subhalos, easily perceived in Fig. 2. Actually, there are more than 9000 subhalos within the virial radius of the Milky Way VL2 halo; or a few thousands, at any rate, if we suitably restrict the minimal halo size. In contrast, the halos in the Bolshoi simulation contain much fewer subhalos. For example, the Bolshoi simulation halo with  $r_{\text{vir}} = 0.56$  Mpc/h and whose entropies are plotted at the bottom of Fig. 3 contains only 22 subhalos within the virial radius (with more than 50 particles each).

This remarkable difference is a consequence of the different mass resolution of both simulations: the particle mass ratio between the two simulations is 33000, approximately. An improved resolution must bring out unresolved substructure. Presumably, as suggested by Diemand et al [3], “at infinite resolution one would find a long nested series of halos within halos within halos etc.”

However, a ratio of 33000 is considerably larger than the ratio between the numbers of subhalos of the respective halos. In fact, the definitions of subhalo in the two simulations do not coincide, but this is only a part of the cause for the discrepancy: the influence of the gravity softening on smoothness properties must be taken into account as well. We discuss both the discreteness parameters next.

## 3. Influence of discreteness on smoothness

The nature of halos and, in particular, their smoothness are influenced by the discreteness parameters, namely, total number of particles  $N$  and softening length  $\epsilon$ . The optimal choice of softening is still a debated issue [14, 15]. In a CDM simulation with  $N$  particles that produces objects of size  $R$ , namely, halos of size  $R$ , the optimal softening length  $\epsilon$  has to be determined in terms of  $N$  and  $R$ . Here,  $R$  is the average radius of halos, for a given simulation, where the radius of an individual halo is defined as its smoothness radius, in accord to the preceding section. Since the size of the simulation box  $L$  and, hence, the total number of particles  $N$ , must not have any effect on the nature and size of individual halos, we can express the softening length in terms of local quantities, namely, the discretization length,  $\ell = LN^{-1/3}$ , and  $R$ . In principle, the function  $\epsilon(\ell, R)$  can be arbitrary. Besides, considering that  $\ell$  and  $\epsilon$  are discreteness parameters, fixed at the start of the cosmological simulation, whereas the sizes of halos belong to the result of the simulation, it is preferable to write the relation among the three quantities as  $R(\ell, \epsilon)$ .

The evidence suggests that the function  $R(\ell, \epsilon)$  greatly depends on the variables  $\ell$  and  $\epsilon$  [8]. In this regard, we would like to know the separate dependence of  $R$  on  $\ell$  and  $\epsilon$ . While a finite  $\ell$  (a finite  $N$ ) introduces corrections to the mean-field Vlasov-Poisson dynamics, partially remedied by the gravity softening, this softening also perturbs the Vlasov-Poisson dynamics. Joyce et al [15] point out that the rigorous Vlasov-Poisson limit,  $\ell \rightarrow 0$ , is to be taken at *fixed* finite  $\epsilon$ , obtaining a smoothed version of the Vlasov-Poisson equations. One may ask what  $\lim_{\ell \rightarrow 0} R(\ell, \epsilon)$  is: dimensional analysis demands that  $\lim_{\ell \rightarrow 0} R(\ell, \epsilon) \sim \epsilon$ . In words, the softening on a scale  $\epsilon$  must produce a smoothing on the same scale. Therefore, if halo sizes are determined by smoothness, these sizes must be of the order of  $\epsilon$ . The subsequent  $\epsilon \rightarrow 0$  limit that leads to the actual Vlasov-Poisson equations makes the halo sizes vanish and the halo number density diverge. This implies that the relevant solutions of these equations correspond to a self-similar cosmic web structure that is present on ever decreasing scales. Naturally, the limit  $\ell \rightarrow 0$  at

*fixed* finite  $\epsilon$  or, in other words, the domain of discreteness parameters such that  $\epsilon \gg \ell$ , is not studied by cosmological  $N$ -body simulations. Nevertheless, its effect on halos can be understood by analogy with the adhesion model [9].

#### 4. Conclusions

The often mentioned problems of the CDM model on small scales, namely, the core-cusp problem and the missing satellites problem, reflect just the bottom-up formation of structure in the CDM model, which necessarily gives rise to a considerable amount of structure on galactic scales. The amount of structure produced on small scales is usually inferred from the results of collisionless CDM  $N$ -body simulations, which actually show that the distribution of matter on those scales can be described as a distribution of *smooth* halos with a limited range of sizes. The core-cusp problem arises because the halos in simulations are singular at the center, whereas the observations of galactic rotation velocities are better fitted by totally smooth halos, namely, with soft cores. The missing satellites problem arises because there are too many small halos and subhalos in simulations. In summary, the dark matter distribution on small scales in  $N$ -body simulations is a distribution of halos, which are relatively smooth but not as smooth as required by indirect observations of the dark matter distribution.

However, as we have seen in this paper, the existence of halos in the cosmic web is surely an aspect of the small-scale smoothening produced by  $N$ -body discreteness effects. Our analysis of the smoothness and abundance of halos corroborates this conclusion. The analysis of smoothness is based on the application of robust entropic measures to halos in two recent CDM  $N$ -body simulations with different resolutions, finding a progressive and essentially monotonic growth of halo graininess or anisotropy with radius, which stops at some radius, for every halo. The radius at which the limit graininess or anisotropy are attained can be defined as the final end of the smooth halo and the beginning of the cosmic web structure, which merge seamlessly. The removal of discreteness parameters should make all the smoothness radii vanish and therefore remove the halos and their relative smoothness altogether. This makes the problems of the CDM hypothesis on small scales somewhat different and much more serious than they are currently known to be.

#### References

- [1] de Blok W J G 2010 The Core-Cusp Problem, *Advances in Astronomy* 2010, article id. 789293
- [2] Moore B, Ghigna S, Governato F, Lake G, Quinn T, Stadel J and Tozzi P 1999 *Astrophys. J. Lett.* **524** L19-L22
- [3] Diemand J, Kuhlen M, Madau P, Zemp M, Moore B, Potter D and Stadel J 2008 *Nature* **454** 735–38
- [4] Kuzio de Naray R, McGaugh S S, de Blok W J G 2008 *Astrophys. J.* **676** 920–43
- [5] Berezhinsky V S, Dokuchaev V I and Eroshenko Yu N 2014 *Phys.-Usp.* **57** 1–36
- [6] Gurbatov S.N., Saichev A.I. and Shandarin S.F. 2012 *Phys.-Usp.* **55** 223–49.
- [7] Cooray A and Sheth R 2002 *Phys. Rep.* **372** 1–129.
- [8] Gaite J. 2013 Halo Models of Large Scale Structure and Reliability of Cosmological N-Body Simulations, *Galaxies* **1** 31–43.
- [9] Vergassola M., Dubrulle B., Frisch U. and Noullez A. 1994 *Astron. & Astrophys.* **289** 325–56
- [10] Gaite J 2014 Smooth halos in the cosmic web *Preprint* arXiv:1407.6197
- [11] Klypin A A, Trujillo-Gomez S and Primack J, 2011 *Astrophys. J.* **740** 102
- [12] Gaite J. 2007 *Astrophys. J.* **658** 11–24
- [13] Gaite J 2010 Fractal analysis of the dark matter and gas distributions in the Mare-Nostrum universe *J. Cosmol. Astropart. Phys.* **03** 006
- [14] Splinter R J, Melott A L, Shandarin S F and Suto Y 1998 *Astrophys. J.* **497** 38–61.
- [15] Joyce M, Marcos B and Baertschiger T 2009 *MNRAS* **394** 751–773