

Dynamic of HNLS Solitons using Compact Split Step Padé Scheme

Moussa Smadi¹ and Derradji Bahloul²

^{1,2}Département de Physique, Faculté des Sciences, Université de Batna
Laboratoire de Physique des Rayonnements et de leurs Interactions avec la Matière
LPRIM, 5000 Avenue Chahid Boukhrouf, 05000 Batna, Algeria

¹msmadi@univ-batna.dz

²bahloul@univ-batna.dz

Abstract: In this communication, we use a compact split step Padé scheme (CSSPS) to solve the scalar higher-order nonlinear Schrödinger equation (HNLS) with higher-order linear and nonlinear effects. The second part, consisting of two sections, the first section is dedicated to the study numerically the stabilization of high-order solitons dynamic in optical fibers by compensation or by the interplay of higher order nonlinearity - especially quintic nonlinearity- and the self-steepening. In the second section we study also numerically the propagation of conventional chirped or unchirped solitons in optical fibers with to the management of the non-linearity, dispersion and loss (gain).

Keywords: higher order optical solitons, compact split step Padé scheme, higher-order nonlinear Schrödinger equation (HNLS), quintic nonlinearity, dispersion managed.

1. Introduction:

Propagation of solitons in optical fibers has attracted a great attention of many authors [1-3]. The balance between nonlinear Kerr effect and chromatic dispersion is the clue of the stability of optical NLS solitons [2]. The propagation of ultra-short and ultra-intense optical solitons in optical fibers is not so obvious and requires efficient and fast numerical methods in order to be investigated. At high power higher order nonlinearities may alter the propagation of the solution especially with the interplay of higher order dispersion effects such as third or fourth ordered dispersion. Quintic nonlinearity [4] is the most important nonlinear effect due to the saturation of optical field [5] and must be taken into account in the study of ultra-intense optical solitons. In this communication we use a compact split step Padé scheme (CSSPS) [6] to solve the higher-order nonlinear Schrödinger (HNLS) equation with power law nonlinearity [7,8] and higher order dispersion effects, in order to study numerically the impact of the combined nonlinear effects, such self-steepening and quintic nonlinearity on the propagation of the higher-order soliton in optical fibers, and the evolution of conventional unchirped or chirped solitons in optical fibers with to the management of the non-linearity, dispersion and loss (gain).



2. Theory:

The higher-order nonlinear Schrödinger HNLS equation with power law nonlinearity and higher order dispersion:

$$\frac{\partial A(Z, T)}{\partial Z} = -\frac{\alpha}{2}A + i \sum_{n=2}^{N=4} \frac{i^n}{n!} \beta_n \frac{\partial^n A}{\partial T^n} + i\gamma \left[|A|^2 A + \gamma' |A|^{2m} A + \frac{i}{\omega_0} \frac{\partial}{\partial T} (|A|^2 A) - T_R A \frac{\partial |A|^2}{\partial T} \right] \quad (1)$$

Where $i = \sqrt{-1}$ and $A(Z, T)$ is the slowly varying envelope of the optical pulse and $T = t_{lab} - Z/v_g$ the temporal coordinate in retarded frame that moves at the group velocity v_g of the pulse and Z the spatial coordinate representing the distance of transmission $\beta_2, \beta_3, \beta_4$ and Represents dispersion parameters of the second, third and fourth orders. α is the attenuation factor. For silica fibers $\beta_2 \sim 20 \text{ps}^2/\text{km}$ at $\sim 1.55 \mu\text{m}$ [1] $\gamma = n_2 \omega_0 / c A_{eff}$ and γ' are respectively the nonlinear Kerr effect coefficient and the power law nonlinearity coefficient. n_2 is the nonlinear index coefficient, for silica fibers $n_2 = 2.2 \times 10^{-20} \text{m}^2/\text{W}$ at the same wave length [1]. The term proportional to $1/\omega_0$ governs the Kerr dispersion that is responsible of self-steepening and shock formation. The last term is responsible of self-frequency shift induced by intrapulse Raman scattering and T_R is related to the Raman response function at wave length $\lambda_p \sim 1.55 \mu\text{m}$ $T_R \approx 3 \text{fs}$. [1]

3. Numerical scheme

We first normalize equation (1) in the following way:

$$t = \frac{T}{T_0}, z = \frac{Z}{L_D}, L_D = \frac{T_0^2}{|\beta_2|}, L'_D = \frac{T_0^3}{|\beta_3|}, L''_D = \frac{T_0^4}{|\beta_4|}, L_{NL} = \frac{1}{\gamma P_0}, \kappa = \frac{1}{\gamma' P_0^{m-1}}, s = \frac{1}{\omega_0 T_0}, \tau_R = \frac{T_R}{T_0}$$

$$\text{and } u(z, t) = \frac{A(z, t)}{\sqrt{P_0}}$$

Where T_0 is the initial pulse width, L_D, L'_D, L''_D and L_{NL} are respectively the dispersion lengths for the different orders and the nonlinear length. The parameters s and τ_R govern the effects of self-steepening and intrapulse Raman scattering respectively. Both of these effects are quite small for picoseconds pulses but must be considered for ultra-short pulses with $T_0 < 0.1 \text{ps}$ [1]. In our simulation, we suppose the parameter ($\alpha = 0$) for lossless fibers. We choose to rewrite the equation (1) in the form (2) because it is more convenient for the numerical calculations.

$$\frac{\partial u}{\partial z} = i\alpha_2 \frac{\partial^2 u}{\partial t^2} + \alpha_3 \frac{\partial^3 u}{\partial t^3} + i\alpha_4 \frac{\partial^4 u}{\partial t^4} + i \left[\alpha_5 |u|^2 u + \alpha_6 |u|^{2m} u - i\alpha_7 \frac{\partial}{\partial t} (|u|^2 u) + \alpha_8 u \frac{\partial |u|^2}{\partial t} \right] \quad (2)$$

Where:

$$\alpha_2 = -\frac{\text{sgn}(\beta_2)}{2}, \quad \alpha_3 = \frac{\text{sgn}(\beta_3)}{6} \frac{L_D}{L'_D}, \alpha_4 = \frac{\text{sgn}(\beta_4)}{24} \frac{L_D}{L''_D}, \quad \alpha_5 = \frac{L_D}{L_{NL}}, \quad \alpha_6 = \frac{L_D}{L_{NL}} \kappa, \quad \alpha_7 = -s \frac{L_D}{L_{NL}},$$

$$\text{and } \alpha_8 = -\tau_R \frac{L_D}{L_{NL}}$$

This equation (2) can be written formally in the form:

$$\frac{\partial u}{\partial z} = (\hat{L} + \hat{N})u \quad (3)$$

Where \hat{L} is a linear operator that accounts for all the linear effects, and \hat{N} is a nonlinear operator that governs the effect of all the fiber nonlinearities.

$$\hat{L}u = \left(i\alpha_2 \frac{\partial^2}{\partial t^2} + \alpha_3 \frac{\partial^3}{\partial t^3} + i\alpha_4 \frac{\partial^4}{\partial t^4} \right) u \quad (3-a)$$

$$\hat{N}u = i\alpha_5 |u|^2 u + i\alpha_6 |u|^{2m} u + \alpha_7 \frac{\partial}{\partial t} (|u|^2 u) + i\alpha_8 u \frac{\partial |u|^2}{\partial t} \quad (3-b)$$

The exact solution is approximated by solving for the half step size $\Delta z/2$ separately and alternatively the purely linear equation

$$\frac{\partial u}{\partial z} = \hat{L}u \quad (3 - c)$$

And purely nonlinear equation

$$\frac{\partial u}{\partial z} = \hat{N}u \quad (3 - d)$$

The solution of one sub problem is employed as an initial condition for the next sub problem. The validity of this approximation is discussed in reference [1] using Baker–Hausdorff formula . The linear equation is solved using the compact Padé scheme algorithm an implicit scheme that is unconditionally stable and well adapted for the different derivative orders for more detail see for example [6]. Whereas the nonlinear equation is solved using a fourth order Runge-Kutta scheme (RK4) that satisfies the CFL condition.

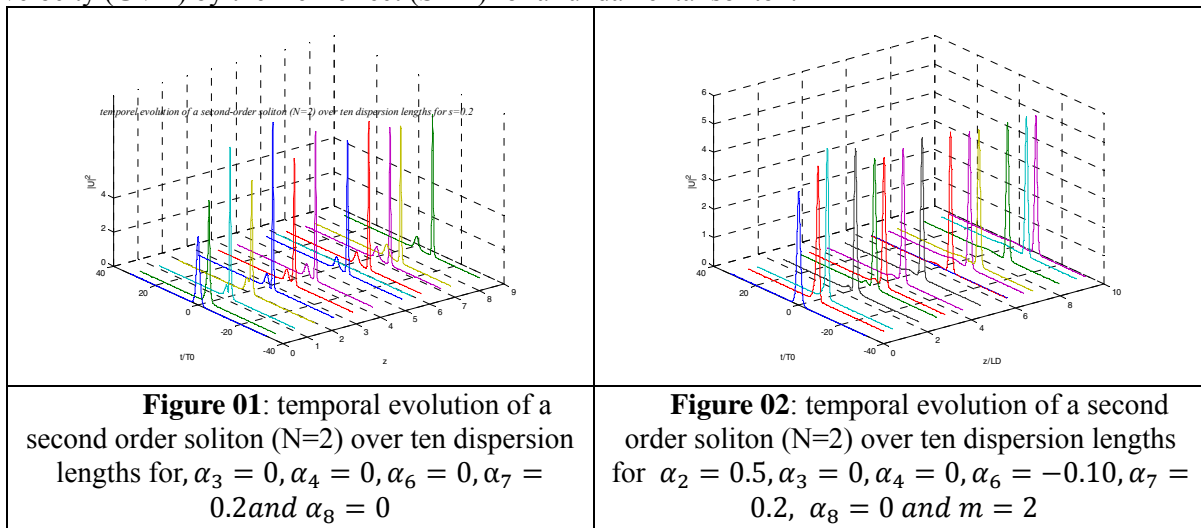
4. Simulation and discussion

4.1. Study of the combined effects self-steepening and quintic nonlinearity on the propagation of high order soliton in optical fiber.

The remarkable effect of the self-steepening on the propagation of higher order-solitons in optical fibers is the breakup of such solitons into their constituents, a phenomenon referred to as soliton fission [9]. In the first case we study numerically by means of (CSSPS) method the impact of the self-steepening governed by the parameter α_7 in the equation (2). In the second case we study the impact of combined effects the self-steepening and the quintic effect governed by the parameter α_6 in the equation (2) on the propagation of higher order-solitons. Pulse evolution inside fibers is then governed by equation (4):

$$\frac{\partial u}{\partial z} = i\alpha_2 \frac{\partial^2 u}{\partial t^2} + i \left[\alpha_5 |u|^2 u + \alpha_6 |u|^{2m} u - i\alpha_7 \frac{\partial}{\partial t} (|u|^2 u) \right] \quad (4)$$

Figure 1, shows this comportment for a second order soliton with $\alpha_7 = 0.2$. We note that both solitons propagate at different speeds, as a result, they separate from each other, and the separation increases in a linear manner as a function of distance this result is in good agreement by comparison with literature [10]. Figure 2, shows the temporal and amplitude evolution of a second order soliton ($N = 2$) over ten dispersion lengths. One can conclude that the simultaneous presence of the self-steepening effect and quintic effect allows the high-order soliton to keep its shape longer without significant breaking or deformation on a longer propagation distance, when the quintic parameter increases. It can be concluded as that the existence of the quintic effect plays an important role in the removal of the breakup caused by the self-steepening effect. This is almost similar to the phenomenon of cancellation effects caused by the dispersion of the group velocity (GVD) by the Kerr effect (SPM) for a fundamental soliton.



4.2 Study of conventional unchirped or chirped soliton propagation in optical fiber in the presence of management of dispersion, nonlinearity and gain (loss).

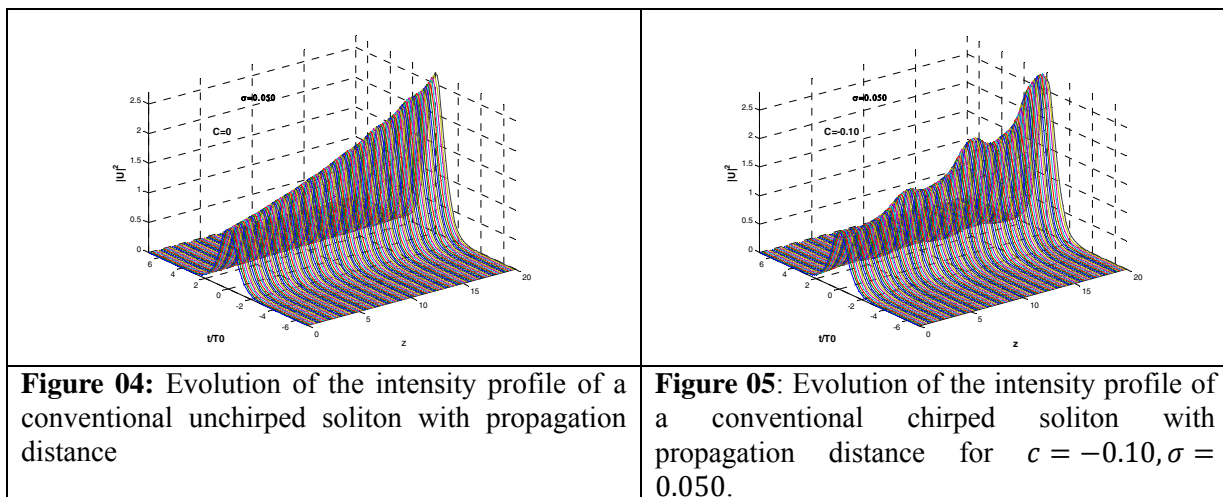
The concept of dispersion management technique is to compensate these effects by using a periodic dispersion map of combining fibers with different characteristics [1]. However in real soliton application systems, the dispersion, nonlinearity, gain and loss are generally varied with the propagation distance. In the following we present the results of our study obtained by application of the (CSSPS) in the case of the propagation of conventional unchirped or chirped soliton in an optical fiber when the dispersion, non-linearity, gain and loss are generally varied with the propagation distance. In many references [11-12-13-14] intensive studies have been devoted to this situation. The Schrödinger equation model which governs the dynamics of solitons in this type of fiber is given by the equation (5):

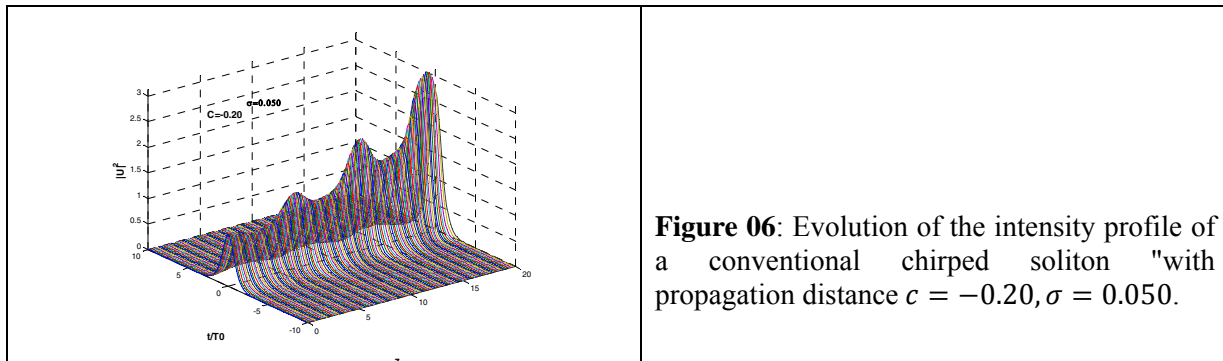
$$i \frac{\partial A(Z, T)}{\partial Z} + \frac{1}{2} D(z) \frac{\partial^2 A}{\partial T^2} + \gamma(z) |A|^2 A = i \alpha(z) A \quad (5)$$

Where $D(z)$ represents the variation of the dispersion of the group velocity (GVD) as a function of the distance, $\gamma(z)$ represents the variation of the Kerr non-linearity parameter and $\alpha(z)$ represent the variation of the gain (loss) as a function of propagation distance also. Note that the nonlinear Schrödinger equation with variable coefficients (6) is not integrable except for a few very special cases of combinations of variables carefully chosen $D(z)$, $\gamma(z)$ and $\alpha(z)$ [15]. In our numerical simulations we considered that the variation of the dispersion of the group velocity (GVD) is given by [16].

$$D(z) = \frac{1}{D_0} \exp(\sigma z) \gamma(z), \quad \gamma(z) = \gamma_0 + \gamma_1 \sin(g \cdot z) \quad , \quad \alpha(z) = \frac{\sigma}{2} \quad (5-1)$$

The peak power of the initial pulse takes into account by the D_0 factor. γ_0 , γ_1 and g parameters represents the Kerr nonlinearity. We take $\sigma < 0$ for a system having a dispersion decreasing and loss (absorption), $\sigma > 0$ for the reverse case. The initial pulse injected in the form of a conventional chirped soliton given by equation [1]: $u(0, T) = \text{sech}(T) \exp\left(-\frac{i C T^2}{2}\right)$ (5-2), Where C is the chirp parameter. And for the others parameters we took the following values [14]: $\gamma_0 = 0, D_0 = 1; \gamma_1 = 1; g = 1$ (5-3), The results obtained are shown by curves (4) (5) and (6) below:





We noticed that the amplitude of a conventional unchirped solitons have an increase in a linear manner as a function of distance when the parameter σ is positive, in the opposite case σ negative the amplitude of a conventional unchirped solitons have an decrease in a linear manner as a function of distance. While the amplitude of the conventional chirped soliton oscillates (increase / decrease) depending on the module of the "chirped" parameter and it can be seen that the chirped soliton is obviously and periodically compressed and broadening with the increase of propagation distance. These results are in good agreement with the model presented in reference [17]

5-Conclusion

The compact split step Padé Scheme has been implemented to investigate the propagation of ultra-intense and ultra-short optical solitons under the effect of power law nonlinearity and high order dispersion effect. This scheme is more efficient, rapid and takes less memory space. It is also well adapted to the higher order derivatives. We applied this method for the case of combined effects such self-steepening quintic nonlinearity and concluded that the quintic effect plays an important role in the removal of the breakup caused by the self-steepening effect on the propagation of optical second-order solitons. In the presence of management of dispersion, nonlinearity and gain (loss) the amplitude of a conventional unchirped solitons have a decrease in a linear manner as a function of distance of propagation. While the amplitude of the conventional chirped soliton oscillates (increase / decrease) depending on the module of the "chirped" parameter.

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