

Interaction of spatial solitons with a gapless stripe embedded into a Bragg-grating area

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Abstract. We introduce a model in which the grating is absent in a finite-width stripe in the waveguide, thus creating a gapless channel in the gapped medium. Two semi-infinite grating separated by the plain stripe may have a relative phase shift. This system modifies the Bragg bandgap, creating intra-gap defect modes (DFs) which are pinned to the gapless channel. A DF solution in the linear system is found analytically. Further, numerical analysis of the full nonlinear system demonstrates that the shape and stability of Bragg solitons are also strongly affected by the presence of the gapless channel, and by the possible phase shift between the two semi-infinite gratings. In particular, asymmetric and flat-top solitons appear.

1. Introduction and the model

Bragg gratings (BGs) are photonic structures produced by a periodic modulation of the refractive index along an optical fiber or waveguide. Gap solitons (GSs) in gratings [1] are supported through the balance between the BG-induced dispersion or diffraction, which incorporates the bandgap in the system's linear spectrum, and the Kerr nonlinearity of the fiber or waveguide. Analytical solutions for BG solitons in the standard BG model are well known [2, 3]. Slightly more than half of the soliton family is stable [4], [5]. GSs were created in the experiment, using a relatively short BG written in the cladding of an optical fiber [6]. GSs are one of fundamental species of solitons in optics, as well as in other nonlinear media.

An issue of great significance is the development of schemes for the creation of new varieties of BG solitons. One of them is *apodization* [7], i.e., making the Bragg reflectivity gradually varying along the fiber (or waveguide). Also available are BGs with periodic apodization, i.e., an effective *superlattice* built on top of the BG [8]. Analysis of the light propagation in the so developed *supergratings* was performed too [9, 10].

In this work, we address quite a simple scheme which, nevertheless, was not explored before: a structure formed by two semi-infinite BG areas, with possible phase shift α between them, which are separated by a flat (gapless) stripe of width L . The objective is to find the linear spectrum of this system and GSs in its nonlinear version.



The model is based in the following coupled-mode equations:

$$\begin{aligned} i\frac{\partial u}{\partial t} + i\frac{\partial u}{\partial x} + \kappa(x)v + \left(|v|^2 + \frac{1}{2}|u|^2\right)u &= 0, \\ i\frac{\partial v}{\partial t} - i\frac{\partial v}{\partial x} + \kappa^*(x)u + \left(|u|^2 + \frac{1}{2}|v|^2\right)v &= 0, \end{aligned} \quad (1)$$

which may also be realized in the spatial domain, replacing time t by propagation distance z . The local Bragg reflectivity is chosen as

$$\kappa(x) = \begin{cases} 1 & \text{at } x > 0, \\ 0 & \text{at } -L < x < 0, \\ \exp(i\alpha) & \text{at } x < -L, \end{cases} \quad (2)$$

where α takes values $0 \leq \alpha \leq \pi$, and κ^* is the complex conjugate of κ .

Stationary solutions with frequency ω are looked for as $\{u(x, t), v(x, t)\} = e^{-i\omega t} \{U(x), V(x)\}$, where complex functions U and V satisfy equations

$$\begin{aligned} +i\frac{dU}{dx} + \omega U + \kappa(x)V + \left[\left(|V|^2 + \frac{1}{2}|U|^2\right) \right] U &= 0, \\ -i\frac{dV}{dx} + \omega V + \kappa^*(x)U + \left[\left(|U|^2 + \frac{1}{2}|V|^2\right) \right] V &= 0, \end{aligned} \quad (3)$$

which are compatible with the relation that reduces the two stationary fields to one, $U^*(x) = -V(x)$. The stationary solutions are characterized by the total energy (or total power, in terms of the spatial-domain model), $E = \int_{-\infty}^{+\infty} (|U(x)|^2 + |V(x)|^2) dx$.

2. The linear spectrum and defect mode in the linear system

First, we explore the spectrum of the linearized version of system (1), the objective being to find the bandgap in which solitons may exist. The computation produces the bandgap which is shown, as a function of α , and for different values of L , in Fig. 1. It is seen that the bandgap shrinks with the increase of α , vanishing at $\alpha = \pi$. At $\alpha = 0$, the width of the bandgap is smaller than in the usual uniform Bragg grating (the one corresponding to $L = 0$), where the bandgap is $-1 < \omega < +1$.

The linear version of Eq. (3), subject to reduction $U^*(x) = -V(x)$, reduces to the single linear equation,

$$i\frac{dU}{dx} + \omega U - \kappa(x)U^* = 0. \quad (4)$$

Equation (4) gives rise an exact solution for a *defect mode* (DM) pinned to the central stripe where the grating is absent:

$$U(x) = \begin{cases} \exp\left(\frac{i}{2} \arccos \omega - \sqrt{1 - \omega^2} x\right), & \text{at } x \geq 0, \\ A \exp\left(i\omega\left(x + \frac{L}{2}\right)\right), & \text{at } -L \leq x \leq 0, \\ \exp\left(\frac{i}{2} \alpha - \frac{i}{2} \arccos \omega + \sqrt{1 - \omega^2} (x + L)\right), & \text{at } x \leq -L. \end{cases} \quad (5)$$

The eigenfrequency of this localized mode is determined by the condition of the continuity of $U(x)$ at $x = 0$ and $x = -L$, which yield $A = \exp(i\alpha/4 + i\pi n)$,

$$\omega_{\text{eigen}} L + \alpha/2 = \arccos(\omega_{\text{eigen}}) + 2\pi n, \quad (6)$$

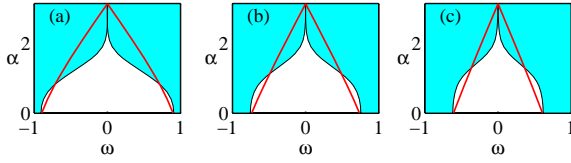


Figure 1. The bandgap structure found from the linearization of Eq. (1) for (a) $L = 0.5$, $L = 1.0$ and $L = 1.5$. Shaded areas are occupied by Bloch bands. The red solid lines correspond to the analytical solutions for the defect modes, see Eq. (6)

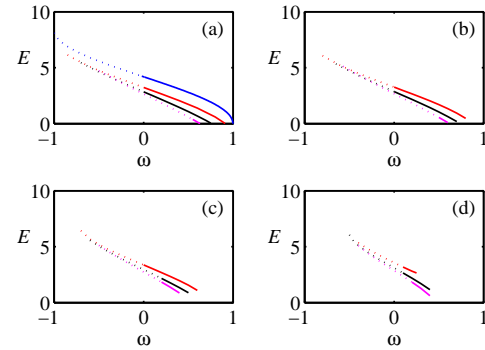


Figure 2. Total energy E of gap solitons versus ω for different L . (a) $\alpha = 0$ (b) $\alpha = \pi/8$ (c) $\alpha = \pi/4$ (d) $\alpha = \pi/3$. Continuous and dashed segments depict stable and unstable solitons, respectively. Different colors correspond to $L = 0$ (blue); $L = 0.5$ (red); $L = 1.0$ (black); $L = 1.5$ (magenta).

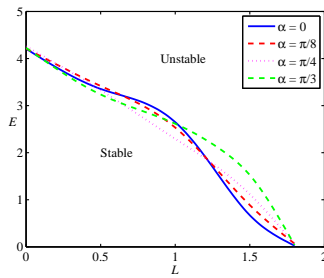


Figure 3. Stability regions for the solitons in the plane of (L, E) at different fixed values of α . In the case of $\alpha = 0$, the result for $L = 0$ pertains to the uniform nonlinear Bragg grating.

where n is an arbitrary integer. In particular, numerical solution of Eq. (6) for $\alpha = 0$ and $n = 0$ yields

$$\omega_{\text{eigen}}(L = 0.5) \approx 0.90, \omega_{\text{eigen}}(L = 1.0) \approx 0.74, \omega_{\text{eigen}}(L = 1.5) \approx 0.61, \quad (7)$$

which is close to the values at which the bandgap closes at $\alpha = 0$ in Fig. 1. The respective eigenvalues at negative frequencies correspond to the defect modes subject to reduction $U(x) = +V^*(x)$, instead of that given by $U^*(x) = -V(x)$.

It follows from Eq. (6) that, for any L , the eigenfrequency shifts to $\omega_{\text{eigen}} = 0$ at $\alpha = \pi$, which is consistent with the picture in Fig. 1. At large values of L , new DMs appear, corresponding to $n > 0$. In particular, the ones corresponding to $\omega = 1$ appear, for each $n \geq 1$, at $L_{\text{cr}}^{(n)} = 2\pi n - \alpha/2$. Therefore, for given large L , the total number of DMs is $n_{\text{DM}} = 1 + \left\lfloor \frac{L + (\alpha/2)}{2\pi} \right\rfloor$, where [...] stands for the integer part.

3. Numerical Results

The existence and stability of numerically found GS solutions of the full system (3) is summarized in Fig. 2, which displays typical dependences $E(\omega)$ for different L . The stability was identified by means of systematic simulations of Eqs. (1). The stability region is additionally shown in Fig 3 in the plane of (L, E) for different values of α . It is clearly seen that the region shrinks

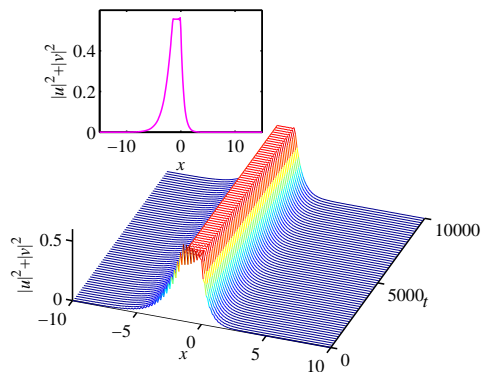


Figure 4. (a) An example of a stable “flat-top” soliton found close to the existence boundary (which is $\alpha = \pi/2.8$) at $\alpha = \pi/3, L = 1.5, \omega = 0.2$. The inset illustrates a the cross section of the final profile.

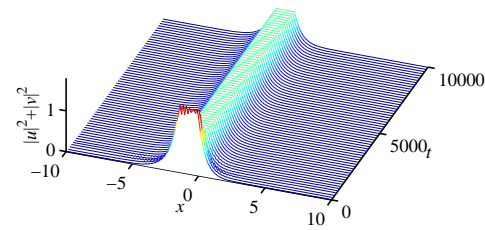


Figure 5. A typical example of the evolution of unstable solitons, for $\omega = -0.3, L = 1.5, \alpha = \pi/4$.

and vanishes at $L \approx 1.8$. Close to their existence boundary, the solitons, while remaining stable, acquire a specific flat-top shape, see an example in Fig. 4 for $\alpha = \pi/3$. The power profile shown in Fig. 4 is not only flat-top, but also features conspicuous spatial asymmetry, while the profiles of the two components, $|u|^2$ and $|v|^2$, remain mutually identical. A typical example of the evolution of unstable solitons is shown in Fig. 5. It is seen that unstable solitons are not destroyed. Instead, they shed off a part of their energy and thus transform into stable solitons with smaller energy.

4. Summary

We have introduced the model of a complex built of two nonlinear Bragg gratings separated by a flat segment of the waveguide. The linear defect mode was found in the exact form, while gap solitons were found numerically. The solitons feature asymmetry and the flat-top shape. It may also be possible to simulate collisions of moving solitons with the flat segment.

5. Acknowledgment

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6. References

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