

# Critical phenomena in communication/computation networks with various topologies and suboptimal to optimal resource allocation

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**Abstract.** We generalize previous studies on critical phenomena in communication networks [1,2] by adding computational capabilities to the nodes. In our model, a set of tasks with random origin, destination and computational structure is distributed on a computational network, modeled as a graph. By varying the temperature of a Metropolis Montecarlo, we explore the global latency for an optimal to suboptimal resource assignment at a given time instant. By computing the two-point correlation function for the local overload, we study the behavior of the correlation distance (both for links and nodes) while approaching the congested phase: a transition from peaked to spread  $g(r)$  is seen above a critical (Montecarlo) temperature  $T_c$ . The average latency trend of the system is predicted by averaging over several network traffic realizations while maintaining a spatially detailed information for each node: a sharp decrease of performance is found over  $T_c$  independently of the workload. The globally optimized computational resource allocation and network routing defines a baseline for a future comparison of the transition behavior with respect to existing routing strategies [3,4] for different network topologies.

## 1. Introduction

Critical phenomena in random networks are an active research field not confined to theoretical physics or mathematics but their application to computer science, biology, traffic engineering and social sciences is increasingly common [5–7]. In this work the problem of congestion in computer networks is addressed, but it is easy to recognize the analogies with traffic flow, container logistics and energy distribution, just to name a few [5].

Ohira and Sawatari [1] first studied the onset of a phase transition within a simple model of communication network under different rates of traffic generation. Their model is defined by a set of nodes on a two dimensional lattice and controlled by a set of parameters such as packet generation rate and routing rules. The expected phase transition is observed by means of the average latency by which packets arrive at their destination during a dynamical simulation over time. In 2001, Solè and Valverde [2] slightly extended the model and studied the time-series dynamics of the number of packets at individual routers, and found a set of power laws at the critical point.

The present model tries to build on the previous attempts, in which only the network communication process has been considered, by (i) seamlessly including the computation time (information processing in general) into the total latency and (ii) avoiding the explicit temporal



evolution by considering an average traffic over a given instant. In this way the typical problem of the queue size of dynamic simulations is entirely avoided [8] and the focus is set on the “instantaneous” behavior of the system. In a sense, the model computes the finite-difference load derivative during the time window for each computational node and network link. The model allows an extremely fast evaluation of the latency and this unfolds the possibility of finding the network’s optimal resource allocation. The optimization is performed after setting the network topology and the random traffic distribution [9,10]. Here we try to set a baseline for comparing any routing algorithm with the best solution found by a Simulated Annealing (SA) process. Moreover, by varying the temperature in the Metropolis Montecarlo (MC), we estimate the sensitivity of the system performance with respect to the “amount of non-optimality” in the allocation of resources (both routing and processing).

We present the behavior of the average latency with respect to load and resource optimization for two different topologies: a two-dimensional lattice and a Barabási-Albert (BA) random network. We find a transition to the overloaded phase both for a critical load and for a critical temperature  $T_c$ . Finally, the two-point correlation function of the local overload for nodes and links is used to show how the system behaves while going from a free to a congested regime: the  $g(r)$  for the nodes is found to sharply change its shape from localized to spread when the (MC) temperature is raised.

## 2. The Model

The model considers three basic components: (i) the physical support for the communication/computation process, (ii) the discrete tasks that are transported/executed, and (iii) the limited capability of the links/nodes to handle each subtask. The communication network is mapped onto a graph where nodes mimic the communicating agents (i.e. routers and servers in a computer network), and the links between them represent communication lines. Each node is characterized by its computational power and each link has its own nominal latency and bandwidth. The computational capability is measured by the number of executable task requests allowable in unit time. This should be intended in two ways: each node is characterized by its own “speed” and by the maximum number of concurrent unitary tasks which can be executed at nominal speed. With respect to the network, each link transports the information at nominal speed when its bandwidth is larger than or equal to the total requests of the tasks running on it (this is equivalent to the betweenness centrality measure of the link). In both cases of networking and computation, whenever the load surpasses the available capabilities, a linear slowdown/overload factor is introduced to take into account the increased time of execution of each subtask. Each link is modeled to be able to support several concurrent communications without overloading, while each computational node can execute more than one subtask at the same time.

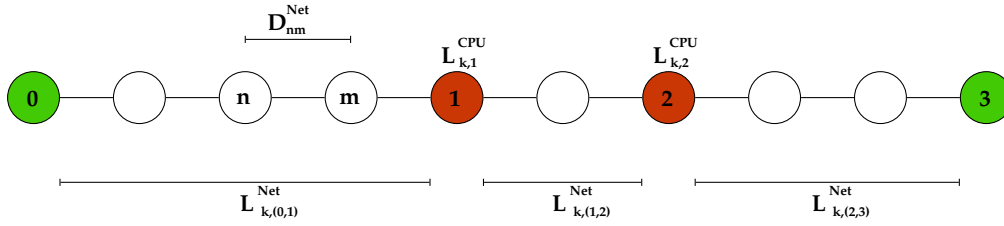
We consider two topologies: a simple 2D lattice and a Barabási-Albert random network, with the same number of nodes, but different connectivity.

From the initial node through each stage and until the final node, the information is passed to the next step via a sequence of network links (see Fig.1): the length of each sequence is, in principle, bounded only by the total number of links in the network (no loops are permitted).

The time (latency) needed to complete the  $k$ -th task  $T^k$  is the sum of two components:

$$L_k^{total} = L_k^{CPU} + L_k^{net},$$

- the network latency  $L_k^{net}$  is the sum of the delays  $L_{k,(n,m)}^{net}$  introduced by the direct hops  $n \Leftrightarrow m$  (with nominal delay  $D_{nm}^{Net}$ ) within the paths connecting each pair of subtask;
- the computation latency  $L_k^{CPU}$  is the sum of the delays  $L_{k,n=1,2}^{CPU}$  due to all subtasks.



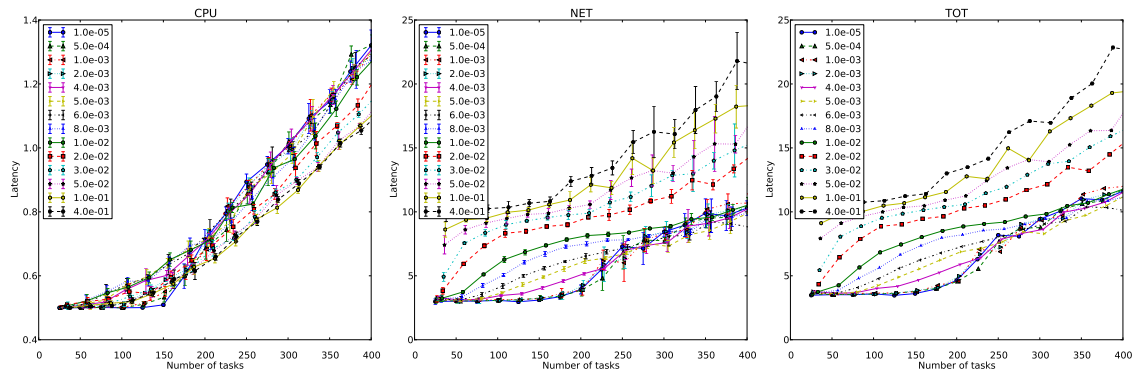
**Figure 1.** Task structure. Green circles are assigned as the input and output of the task. Red circles represent the processing stages. White circles are nodes used as simple routers between elementary network hops. The basic symbols used to compute the latency are depicted.

The optimizer tries to find the best possible combined performance for the tasks. After initializing the topology of the network and its capabilities, a random, uniform, distribution of traffic is generated. The SA procedure starts from a random allocation of the tasks over the network and typically has a poor performance since it is very unlikely that the computing nodes and the links will have a “good” load distribution (this could be easily estimated since it is equivalent to a balls-in-bins statistical problem).

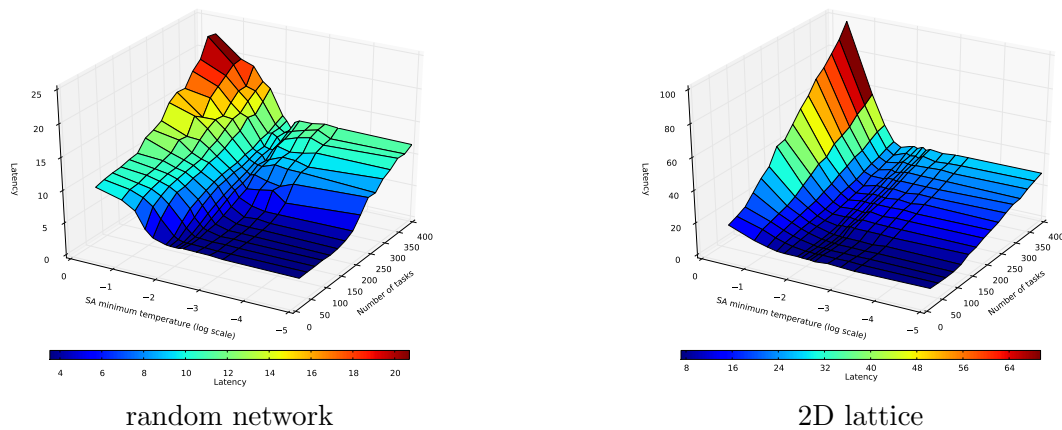
Being able to simulate the system at constant temperature ( $T$ ) is equivalent, in a sense, to having a parameter with which to set the amount of “errors” in the total resource allocation. At  $T \approx 0$ , the simulation ideally leads to an optimal routing and node allocation. Slowly increasing  $T$  drives the system through more realistically accessible states and finally to totally random resource allocations at high  $T$ .

## Results

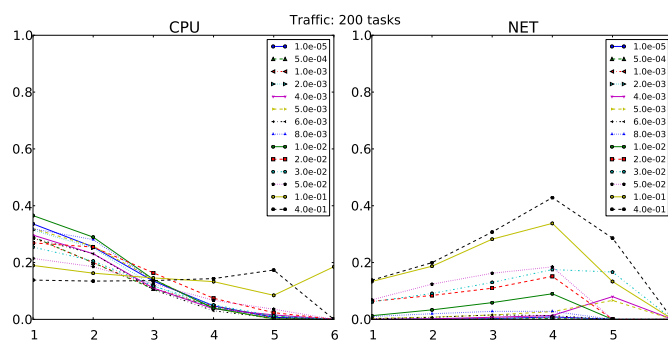
Simulations were run using several instances of BA networks and 2D lattices with 100 nodes and random traffic. Each node is assigned the same unit power. Each link capability is proportional to the sum of the node degrees. Tasks are modeled with 2 processing stages, each with a computational request of one fourth of unit power. This ratio sets the theoretical maximum computational requests before CPU overload to 200 tasks. Each point on the graphs is obtained by averaging over 10 runs. As shown in Fig.2 (left panel, CPU), the curve labeled with  $T = 10^{-5}$  represents the optimal latency at all loads and the transition sharply occurs with a load of  $nt = 150$  tasks (note that the maximum theoretical CPU capacity would allow  $nt = 200$  with no overload), then a linear latency growth follows with no saturation at high loads. In the central panel (NET), the curve labeled with  $T = 10^{-5}$  (optimal), starts a smooth transition at  $nt = 175$ . then the slope increases up to  $nt = 225$  and finally saturates for  $nt > 250$ . Up to  $T = 2.0 \cdot 10^{-3}$  nothing changes, then (solid violet curve) the latency worsens for low loads and grows from about  $nt = 75$ . The very high load region  $nt > 250$  does not suffer at all from resource allocation errors up to  $T = 2.0 \cdot 10^{-2}$ . Another transition is observed for  $T \geq 2.0 \cdot 10^{-2}$  (red curve): the whole curve grows (low and high loads) and has a steep slope at  $nt = 25$ . For  $T > 2.0 \cdot 10^{-2}$  the latency curve shows an increasing vertical offset, even for  $nt = 25$ . Summarizing, the critical load for both topologies (see Fig.3) starts well before the theoretical maximum CPU load of 200 tasks, but the average latency for the lattice is much larger than for the BA even though both share the same connectivity: The BA is able to resist higher loads than the lattice since the average path is much shorter, thus the network has a lesser impact on the global latency. The resource allocation quality ( $T$ ) on network performance has a very strong impact near the critical load ( $nt \approx 150$ ). Finally, the  $g(r)$  of the CPU overload at  $nt \approx 200$  (Fig.4), is a power law for  $T < T_c$  but becomes uniformly spread at high values, while for the network starts at zero everywhere and at high  $T$  rises especially for distant links. In general for high  $T$  we observe an exponential growth of the average number of internal task hops that is only limited by the network size.



**Figure 2.** Barabasi random network: Average latency (right) and its components (computation on the left and networking in the center box) dependence on the load computed for different Montecarlo temperatures ( $10^{-5}$ ,  $4 \cdot 10^{-3}$ ,  $10^{-2}$ ,  $10^{-1}$ ).



**Figure 3.** 3D representation of how the global average latency depends on the traffic and on the Montecarlo temperature: The deep blue basin with low latency can be reached with low traffic and moderately bad routing or with high traffic but optimal routing/resource allocation



**Figure 4.** BA: Two-point correlation of the overload (nodes and links) with a traffic of 200 tasks for several temperatures. Once  $T > 5 \cdot 10^{-2}$ , the CPU overload spreads over all the network.

## References

- [1] Ohira T and Sawatari R 1998 *Physical Review E* **58** 193–195 URL <http://journals.aps.org/pre/abstract/10.1103/PhysRevE.58.193>
- [2] Solé R and Valverde S 2001 *Physica A: Statistical Mechanics and its Applications* **289** 595–605 URL <http://www.sciencedirect.com/science/article/pii/S0378437100005367>
- [3] Wang W, Wang B, Yin C, Xie Y and Zhou T 2006 *Physical Review E* 1–7 URL <http://journals.aps.org/pre/abstract/10.1103/PhysRevE.73.026111>
- [4] Yan G, Zhou T, Hu B, Fu Z and Wang B 2006 *Physical Review E* 1–5 (Preprint 0505366v2) URL <http://journals.aps.org/pre/abstract/10.1103/PhysRevE.73.046108>
- [5] Dorogovtsev S N, Goltsev A V and Mendes J F F 2008 *Rev. Mod. Phys.* **80**(4) 1275–1335 URL <http://link.aps.org/doi/10.1103/RevModPhys.80.1275>
- [6] Barabási A L, Albert R and Jeong H 2000 *Physica A: Statistical Mechanics and its Applications* **281** 69–77 ISSN 03784371 URL <http://linkinghub.elsevier.com/retrieve/pii/S0378437100000182>
- [7] Barabási A L 2009 *Science (New York, N.Y.)* **325** 412–3 ISSN 1095-9203 URL <http://www.ncbi.nlm.nih.gov/pubmed/19628854>
- [8] Arrowsmith D K, Woolf M and Monfragon R J 2004 *Complex Dynamics in Communication Networks: Data Traffic, Topology and Congestion*
- [9] Danila B, Sun Y and Bassler K 2009 *Physical Review E* 1–7 (Preprint arXiv:0902.2415v2) URL <http://journals.aps.org/pre/abstract/10.1103/PhysRevE.80.066116>
- [10] Guimerà R, Díaz-Guilera A, Vega-Redondo F, Cabrales A and Arenas A 2002 *Phys. Rev. Lett.* **89**(24) 248701 URL <http://link.aps.org/doi/10.1103/PhysRevLett.89.248701>