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Propagation of the chirped microwave pulse in the long-distance sliding-mode plasma waveguide in air.

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Abstract. Propagation of the phase-modulated microwave pulse in the sliding-mode plasma waveguide in air is considered. It is shown that with the proper choice of the chirp parameter the peak intensity of the microwave pulse can be significantly enhanced with respect to the non-phase-modulated pulse thus compensating the attenuation by the pulse compression. Calculations show that the microwave pulse peak intensity at the end of plasma waveguide can be made larger than the initial peak intensity, even for the plasma waveguides of a few kilometres length.

1. Introduction

Impressive progress in creation of long ionized channels in atmosphere with the use of filamentation effect [1,2] has renewed interest in such promising applications as initiation of lightning and triggering of high-voltage discharges [3-6], microwave transport [7,8], remote sensing, biological and chemical detection [1,9]. At the present moment, a numerous schemes of microwave transport utilizing different plasma configurations have been studied theoretically, some of them have been tested experimentally. High-conductivity dense plasma hollow waveguide scheme has been studied theoretically in [10], the experimental test of which was performed with the 4.5 cm diameter waveguide formed in laboratory air by multiple filaments, and as a result, it has been demonstrated microwave (at λ=3 cm the wavelength, 10 GHz the frequency) transport on the distance of ~16 cm [7]. Using of periodically arranged filaments can provide somewhat longer propagation distances of microwaves [11, 12]. Prospects of microwave transmission lines based on Zernike-Zommerfeld surface wave excited at a single plasma filament have been studied in [13], the two-filament (i.e., analog of double-wire line) configuration has been considered in [14].

The most prospective for the long-distance (up to few kilometers) microwave transport is the concept of the hollow plasma sliding-mode waveguide which has been developed in [8, 15-17]. The concept is based on the effect of total reflection at the interface with an optically less dense medium (i.e., plasma wall of the waveguide): When the inner radius of tubular plasma waveguide is large enough with respect to the microwave wavelength $R>>\lambda$, the effective angle of incidence for the lowest-order modes exceeds the critical angle determined by the ratio of refractive indices of air and plasma. In contrast with [7, 10], high conductivity of the plasma is not necessary, and one can provide effective transport of sub-mm and mm-range radiation at low degree of air ionization, with the waveguide wall plasma density of $10^{11} \div 10^{14}$ cm^{-3} . This sliding-mode microwave transport scheme


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was demonstrated for the first time in the experiment [8, 15], in which the 35.5 GHz signal has been transported over 60 m distance with the use of plasma tubular waveguide of 10 cm diameter created in the laboratory air by KrF laser radiation. Theoretical calculations show [17] the characteristic propagation distance of the mm-range wave can be enlarged up to few kilometers with ~30 cm waveguide radius and plasma wall density of ~ 10^{12} cm^{-3}.

Theoretical analysis [16, 17] shows that sliding-mode plasma waveguides are characterized by rather strong dispersion. At the air plasma densities below 10^{14} cm^{-3} the dominant mechanism of the plasma decay is the three-body attachment of electrons, the characteristic time of which is ~10 ns [18], thus constraining the microwave pulse width. This fact means that such a rather short microwave pulses can exhibit dispersive growth in their pulse widths when propagates over large (kilometer-range) distances, which leads to an increase in pulse peak intensity with respect to the case of monochromatic signal.

In this paper we propose, in analogy with the conventional nonlinear optics (see, for example, [19]), the technique of phase-modulated microwave pulses to compensate this effect of dispersive spreading and even enlarge the peak intensity at the end of the sliding-mode waveguide.

2. Dispersion characteristics of the sliding mode plasma waveguide

To characterize the dispersion properties of the sliding-mode plasma waveguide, let us consider the dispersion equations for the lowest axial-symmetric modes E_{01} ($\chi = \varepsilon_p$) and H_{01} ($\chi = 1$),

$$\frac{1}{\kappa_1 R}\frac{J_1(\kappa_1 R)}{J_0(\kappa_1 R)} = \frac{\chi}{\kappa_2 R}\frac{H_1^{(1)}(\kappa_2 R)}{H_0^{(1)}(\kappa_2 R)}, \tag{1}$$

and for the lowest axial-asymmetric mode EH_{11},

$$\left[\left(\frac{1}{\kappa_1}\frac{J_0(\kappa_1 R)}{J_1(\kappa_1 R)} - \frac{\varepsilon_p}{\kappa_2 R}\frac{H_1^{(1)}(\kappa_2 R)}{H_0^{(1)}(\kappa_2 R)}\right) - \left(\frac{1}{\kappa_1^2 R} - \frac{\varepsilon_p}{\kappa_2^2 R}\right)\right]\times$$
$$\left[\left(\frac{1}{\kappa_1}\frac{J_0(\kappa_1 R)}{J_1(\kappa_1 R)} - \frac{1}{\kappa_2 R}\frac{H_1^{(1)}(\kappa_2 R)}{H_0^{(1)}(\kappa_2 R)}\right) - \left(\frac{1}{\kappa_1^2 R} - \frac{1}{\kappa_2^2 R}\right)\right] = \left(\frac{1}{\kappa_1^2 R} - \frac{\varepsilon_p}{\kappa_2^2 R}\right)\left(\frac{1}{\kappa_1^2 R} - \frac{1}{\kappa_2^2 R}\right) \tag{2}$$

Here, $\kappa_1^2 = \omega^2/c^2 - h^2$ and $\kappa_2^2 = \varepsilon_p\,\omega^2/c^2 - h^2$ are the transverse wavenumbers within the waveguide (the dielectric susceptibility of air is assumed to be equal 1) and the waveguide wall, respectively, h is the propagation constant of the microwave signal of the frequency ω, $J_n(x)$ and $H_n^{(1)}$ are the Bessel functions and the Hankel functions of the first kind. The wall plasma dielectric susceptibility is $\varepsilon_p = 1 - \Omega_p^2/(\omega(\omega + i\nu))$, $\Omega_p = (4\pi e^2 n_e/m)^{1/2}$ is the plasma frequency, ν is the electron transport collision rate, n_e is the plasma density. Dispersion equations (1) and (2) are written under assumption of the abrupt plasma density profile. A real plasma waveguide wall has a spread density profile which is caused by the UV laser intensity profile, diffraction, plasma diffusion etc. It has been shown in [17] that this density spreading doesn't effect on the sliding microwave mode dispersion characteristics if $\kappa_2 d \ll 1$, where d is the characteristic size of the plasma wall density spreading.

It has been shown in [8, 16, 17] that the sliding mode regime in the hollow plasma waveguide is possible when the parameter $\mu = \xi k_0 R > 1$, $\xi = \Omega_p/(\omega^2 + \nu^2)^{1/2}$. In the range of parameters $\mu \sim 0.5 \div 1$, the characteristic propagation length sharply decreases and vanishes with $\mu \to 0$.

At the limit $\mu >> 1$, $\nu/\omega >> 1$ the roots of dispersion equations (1), (2) can be approximated by the following analytic expressions [16, 17].

(a) At sufficiently low densities, when $|1 - \varepsilon_p| \ll k_0 R\sqrt{\xi\nu/\omega}$, we find for the microwave signal wavenumber:

$$h - k_0 \approx -\frac{\alpha^2}{2k_0R^2}\left(1 - i\frac{\chi}{\mu}\sqrt{\frac{2\omega_0}{\nu}}\right) \quad (3)$$

Here $\alpha = \alpha_1$ for the axially symmetric modes H_{01} ($\chi = 1$) and E_{01} ($\chi = \varepsilon_p$), $\alpha_1 \approx 3.83$ is the first root of the Bessel function $J_1(x) = 0$ and $\alpha = \alpha_0$ for the lowest asymmetric mode EH_{11}, $\alpha_0 \approx 2.405$ is the first root of $J_0(x) = 0$. The characteristic propagation length for the EH_{11} sliding mode is thus $(\alpha_1/\alpha_0)^2 \approx 2.54$ times larger than that for the axially-symmetric modes.

(b) At moderate densities, $k_0R >> \xi\sqrt{\nu/\omega} >> 1$, we find the solution for the H_{01} mode is given by the formula (3) whereas for $E_{01}$and EH_{11} modes wavenumbers are

$$h - k_0 \approx -\frac{\alpha^2}{2k_0R^2}\left(1 - i\frac{\xi}{k_0R}\sqrt{\frac{\nu}{2\omega_0}}\right) \quad (4)$$

with $\alpha = \alpha_1$ for the E_{01} mode and $\alpha = \alpha_0$ for the EH_{11}mode, respectively.

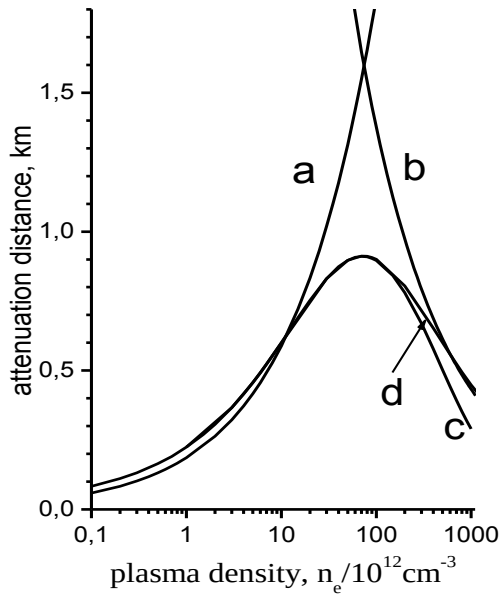


Figure 1a. Characteristic attenuation length of E_{01} mode vs plasma density calculated according analytical formulas (3) and (4) (curves a and b, respectively) and numerical solution of dispersion equation (1) with (c) and without (d) Coulomb collisions taking into account.

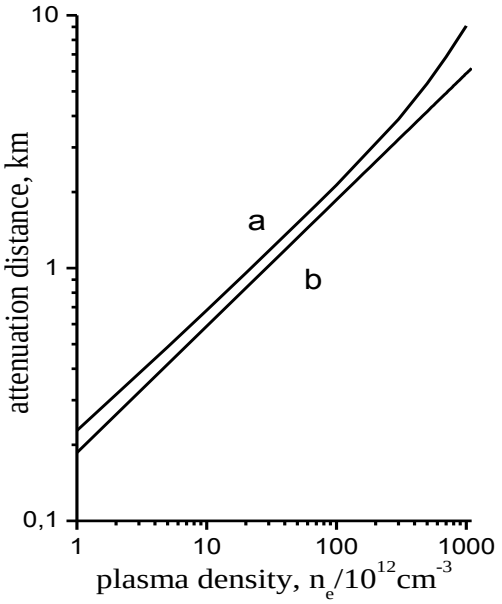


Figure 1b. Characteristic attenuation length of H_{01} mode vs plasma density calculated according analytical formula (3) (curve b) and numerical solution of dispersion equation (1) with Coulomb collisions taking into account (curve a).

At the figure 1 the characteristic propagation distance $(\mathrm{Im}\ h)^{-1}$ for the axially-symmetric sliding modes H_{01} and E_{01} is shown as a function of the plasma wall density. The wavelength of the microwave signal is λ = 8 mm and the inner waveguide radius is R = 30 cm. Calculations are made using the above analytical approximations along with the numerical solution of the dispersion equations (1), (2). The electron elastic transport collision rate is modeled as $\nu = \sqrt{\nu_T^2 + \nu_C^2}$, where

$\nu_T \approx 10^{12}$ s^{-1} is the characteristic electron – neutral collision rate in air and ν_C is the Coulomb electron – ion collision rate. Analytical relations (3), (4) describe the roots of the dispersion equations reasonably well, so one can use them to estimate the dispersion parameters of the sliding mode plasma waveguides.

3. Propagation of the phase-modulated microwave pulse in the waveguide

In analogy to conventional nonlinear optics [19], we expect that the initially phase-modulated microwave pulse will exhibit compression or stretching when propagates along the sliding mode plasma waveguide due to its dispersion properties. To demonstrate this pulse compression effect, we will develop a simplified approach assuming the transverse shape of the sliding mode is constant whereas the amplitude and phase of the microwave pulse are the slow-varying functions along the propagation distance. Within this SVAP (slow-varying amplitude and phase) approximation, the Maxwell equations lead in the conventional manner to the following equation which describes evolution of the pulse amplitude $A(t,z)$

$$\left(\frac{\partial}{\partial z}+\frac{u'}{|u|^2}\frac{\partial}{\partial t}-i\frac{k_2}{2}\frac{\partial^2}{\partial t^2}\right)A(t,z)-\left(i\delta\frac{\partial}{\partial t}\right)A(t,z)=0, \qquad (5)$$

here z is the co-ordinate along the waveguide's axis, $u=\left(\partial\omega/\partial h\right)_{|\omega_0}=u'+iu''$ and $k_2=\left(\partial^2 h/\partial\omega^2\right)_{|\omega_0}=k'_2+ik''_2$ are the complex parameters characterizing the dispersion properties of the waveguide, $\delta=u''/|u|^2$. We assume the initial condition of equation (5) is of the form $A(t,z=0)=C\exp\left[-(\tau_0^{-2}+i\alpha_0)t^2/2\right]$, τ_0 and α_0 are the initial pulse width and chirp parameters, respectively. The above consideration makes it possible to approximate the real waveguide's dispersion by the analytical solutions (3), (4). In the domain of plasma waveguide's parameters of interest, we find that the value of u'' is sufficiently small thus $\delta<<1$, and we can use the perturbation approach to solve the equation (5) assuming

$$A(t,z)=A_0(t,z)+\delta A_1(t,z)+\delta^2 A_2(t,z)+\ldots \qquad (6)$$

Restricting ourselves by the first order perturbation, we find from equation (6) the following system of equations

$$\left[\frac{\partial}{\partial z}+\frac{u'}{|u|^2}\frac{\partial}{\partial t}-i\frac{k_2}{2}\frac{\partial^2}{\partial t^2}\right]\begin{pmatrix}A_0\\A_1\end{pmatrix}=i\frac{\partial}{\partial t}\begin{pmatrix}0\\A_0\end{pmatrix} \qquad (7)$$

Introducing $\eta=t-z/v_g$ the pulse eigen time variable, $v_g=|u|^2/u'$ is the effective group velocity one can find the solution:

$$A_0(\eta,z)=Cf^{-1/2}\exp\left(-(\tau_0^{-2}+i\alpha_0)\eta^2/2f\right), \qquad (8.1)$$

$$A_1(\eta,z)=-i(\tau_0^{-2}+i\alpha_0)(z\eta/f)A_0(\eta,z). \qquad (8.2)$$

Here $f(z)=1+ik_2(\tau_0^{-2}+i\alpha_0)z$ is the microwave pulse envelope function. Solution (8) is valid until the first-order correction is small with respect to zero-order solution, and one can find the correspondent condition for the propagation distance:

$$\left(\frac{\sqrt{\tau_0^{-4} + \alpha_0^2}\,\delta^2 z^2}{e|f(z)|} \right)^{1/2} << 1 \qquad (9)$$

At the figure 2, the dependence of the peak intensity of the phase- modulated microwave pulse on the propagation distance is shown. It is assumed the long –distance sliding mode plasma waveguide is of the radius $R = 30\,\text{cm}$ and the plasma density is $n_e = 1.5\times10^{14}\,\text{cm}^{-3}$. For the microwave pulse of the wavelength $\lambda = 8\,\text{mm}$ we find from equals (3),(4) the real and imaginary parts of the group velocity dispersion coefficient. These parts are $k_2^{'} = \left(\partial^2 h'/\partial\omega^2\right)_{\omega_0} \approx -3.72*10^{-26}\,\text{s}^2/\text{cm}$ and $k_2^{''} = \left(\partial^2 h''/\partial\omega^2\right)_{\omega_0} \approx 1.4*10^{-27}\,\text{s}^2/\text{cm}$. The calculations are made for different pulse chirp parameters α_0 which are optimum for the particular waveguide's lengths of 1km, 1.5 km and 2 km, respectively. We have to point out that for the optimum chirp parameters the ratio $\sqrt{\alpha_0}/\omega_0 \approx 0.06 \div 0.08$ is small enough so that the SVAP approximation is valid.

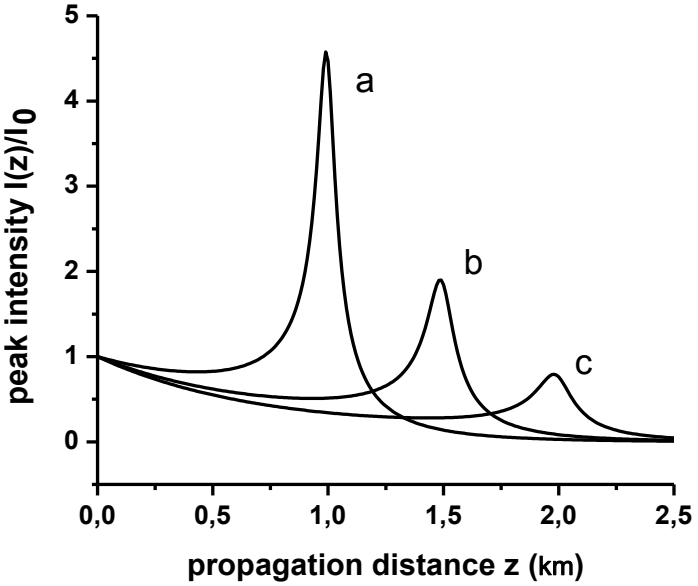


Figure 2. Peak intensity of the microwave pulse vs the propagation distance at different chirp parameters (a) $\alpha_0 = -2.7\times10^{20}\,\text{s}^{-2}$, (b) $\alpha_0 = -1.8\times10^{20}\,\text{s}^{-2}$, (c) $\alpha_0 = -1.35\times10^{20}\,\text{s}^{-2}$.

4.Conclusions

In conclusion, we have considered the propagation of the phase-modulated microwave pulse in the sliding-mode plasma waveguide in air. It is shown that with the proper choice of the chirp parameter the peak intensity of the microwave pulse can be significantly enhanced with respect to the non-phase-modulated pulse thus compensating the attenuation by the pulse compression. Calculations show that the microwave pulse peak intensity at the end of plasma waveguide can be made larger than the initial peak intensity, even for the plasma waveguides of a few kilometres length.

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