Actuator fault diagnosis and fault-tolerant control: Application to the quadruple-tank process

Mariusz Buciakowski, Michał de Rozprza-Faygel, Joanna Ochałek, Marcin Witczak

Institute of Control and Computation Engineering, University of Zielona Góra, Poland

E-mail: {m.buciakowski, m.derozprza-faygel, j.ochalek, m.witczak}@issi.uz.zgora.pl

Abstract. The paper focuses on an important problem related to the modern control systems, which is the robust fault-tolerant control. In particular, the problem is oriented towards a practical application to quadruple-tank process. The proposed approach starts with a general description of the system and fault-tolerant strategy, which is composed of a suitably integrated fault estimator and robust controller. The subsequent part of the paper is concerned with the design of robust controller as well as the proposed fault-tolerant control scheme. To confirm the effectiveness of the proposed approach, the final part of the paper presents experimental results for considered the quadruple-tank process.

1. Introduction

Fault-Tolerant Control (FTC) systems can be divided into two of distinct classes [24], i.e., passive and active. Regarding the passive FTC [10, 11, 16, 3] systems, there is no need for fault diagnosis, owing to the fact that they are robust to a set of predefined faults. Although, the presented approach usually degrades the overall performance. Contrarily, the above-mentioned passive FTC scheme, active one, reacts to faults actively. To maintain the system stability and acceptable performance, controller reconfigures the control actions. The control system relies on Fault Detection and Isolation (FDI) [9, 18, 17, 20, 23, 14, 13, 19] as well as an accommodation technique [1] to achieve a challenging problem. As can be observed, the problems of FDI and FTC are treated separately by most of the works, that exist in the literature. According to the fact that perfect FDI and fault identification are impossible to achieve, there is always an inaccuracy related to this process. Hence that, there is a need for an integrated FDI and FTC schemes. In last decade, a number of books were focused on the emerging problem of the FTC. In particular, the work[7] is mainly concentrated on fault diagnosis. Moreover, it provided general rules for the hardware-redundancy-based FTC systems. The book [12] introduces the concepts of FTC divided into active and passive structures. It also investigates the problem of stability and performance of the FTC under imperfect (imprecise and delayed) fault diagnosis. In particular, the authors consider (under a chain of some, not necessarily easy to satisfy assumptions) the effect of an imperfect fault identification and a delayed fault detection. The proposed approach deals with the fault diagnosis scheme separately, during the design, excluding real integration of the fault diagnosis and the FTC. Plenty of practical case studies of FTC are treated in [15], i.e., a winding machine, a three-tank system, and an active suspension system. Unfortunately, the FTC integrated with the fault diagnosis is not studied, in spite of the incontestable appeal of the proposed approach. The paper is organised as follows. Section 2 introduces a general scheme of the proposed framework. Whilst section 3 describes the concept of the fault compensation mechanism. The subsequent section 4 presents the robust control framework, while the final part of the paper is concerned with an illustrative example.

2. A general description of system and fault-tolerant strategy

Let us consider a linear discrete-time system:

$$\boldsymbol{x}_{f,k+1} = \boldsymbol{A}\boldsymbol{x}_{f,k} + \boldsymbol{B}\boldsymbol{u}_{f,k} + \boldsymbol{B}\boldsymbol{f}_k + \boldsymbol{W}\boldsymbol{w}_k, \tag{1}$$

where $\boldsymbol{x}_{f,k} \in \mathbb{X} \subset \mathbb{R}^n$ is the state vector, $\boldsymbol{u}_{f,k} \in \mathbb{U} \subset \mathbb{R}^r$ stands for the input, $\boldsymbol{f}_k \in \mathbb{R}^r$ is the actuator fault, and $\boldsymbol{w}_k \in l_2$ denotes an exogenous disturbance vector, where:

$$l_2 = \{\mathbf{w} \in \mathbb{R}^n | \ \|\mathbf{w}\|_{l_2} < +\infty\}, \tag{2}$$

$$\|\mathbf{w}\|_{l_2} = \left(\sum_{k=0}^{\infty} \|\boldsymbol{w}_k\|^2\right)^{\frac{1}{2}}. \tag{3}$$

For the purpose of further deliberations, let us formulate the following condition [21]:

$$\mathrm{rank}(\boldsymbol{B}) = r, \tag{4}$$

$$\boldsymbol{f}_k = \boldsymbol{f}_{k-1} + \bar{\boldsymbol{v}}_k, \quad \bar{\boldsymbol{v}}_k \in l_2. \tag{5}$$

The proposed control scheme is a combination of the fault compensation [22] and a classical control scheme:

$$\boldsymbol{u}_{f,k} = -\boldsymbol{K}\boldsymbol{x}_k - \hat{\boldsymbol{f}}_{k-1}, \tag{6}$$

where:

- $\boldsymbol{K}$ is the $\mathcal{H}_\infty$ controller designed to achieve robustness,
- $\hat{\boldsymbol{f}}_{k-1}$ is the fault estimate, which compensates the effect of a fault,

The control strategy design boils down to solving a two set of problems:

- to design a robust controller $\boldsymbol{K}$,
- to estimate the fault $\boldsymbol{f}_k$.

The proposed approach, described by (6) is presented in Fig. 1.

3. Fault estimation

The general objective of this section is to present the fault estimation technique. Thus, following [6, 18], by computing

$$\boldsymbol{H} = \boldsymbol{B}^+ = \left[\boldsymbol{B}^T\boldsymbol{B}\right]^{-1}\boldsymbol{B}^T, \tag{7}$$

and then multiplying (1) by $\boldsymbol{H}$, it can be shown that:

$$\boldsymbol{f}_k = \boldsymbol{H}\boldsymbol{x}_{f,k+1} - \boldsymbol{H}\boldsymbol{A}\boldsymbol{x}_{f,k} - \boldsymbol{u}_{f,k} - \boldsymbol{H}\boldsymbol{W}\boldsymbol{w}_k, \tag{8}$$

while its estimate can be given as:

$$\hat{\boldsymbol{f}}_k = \boldsymbol{H}\boldsymbol{x}_{f,k+1} - \boldsymbol{H}\boldsymbol{A}\boldsymbol{x}_{f,k} - \boldsymbol{u}_{f,k}, \tag{9}$$

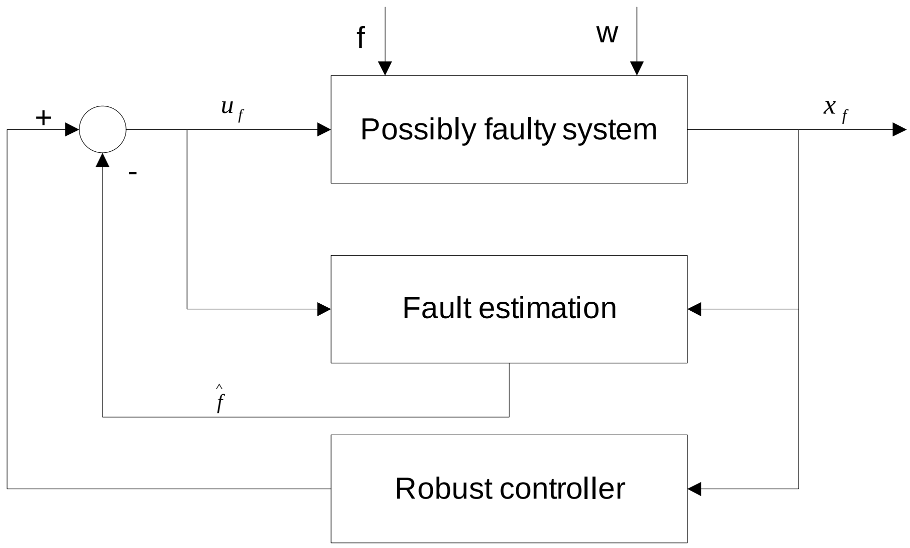


Figure 1. Scheme of the proposed robust predictive FTC

with the associated fault estimation error

$$\boldsymbol{\varepsilon}_{f,k} = \boldsymbol{f}_k - \hat{\boldsymbol{f}}_k = -\boldsymbol{HW}\boldsymbol{w}_k. \tag{10}$$

Since the general framework for computing the fault estimate (9) is given, then its computational feasibility can be verified. To obtain $\hat{\boldsymbol{f}}_k$, it is necessary to have $\boldsymbol{x}_{f,k+1}$. Thus, the only choice to compensate $\boldsymbol{f}_k$ in (1) is to use $\hat{\boldsymbol{f}}_{k-1}$. This determines the above-proposed control strategy:

$$\boldsymbol{u}_{f,k} = -\hat{\boldsymbol{f}}_{k-1} - \boldsymbol{K}\boldsymbol{x}_{f,k}. \tag{11}$$

Taking into account (10) and (5), it can be shown that

$$\hat{\boldsymbol{f}}_k = \hat{\boldsymbol{f}}_{k-1} + \boldsymbol{HW}[\boldsymbol{w}_k - \boldsymbol{w}_{k-1}] + \bar{\boldsymbol{v}}_k. \tag{12}$$

Bearing in mind that all faults present in the real systems have a finite value and knowing that $\boldsymbol{w}_k, \bar{\boldsymbol{v}}_k \in l_2$ it is evident that there exists $\boldsymbol{v}_k$ such that:

$$\hat{\boldsymbol{f}}_k = \hat{\boldsymbol{f}}_{k-1} + \boldsymbol{v}_k, \quad \boldsymbol{v}_k \in l_2. \tag{13}$$

Thus, (11) can be written in an equivalent form, which will be used for further deliberations

$$\boldsymbol{u}_{f,k} = -\hat{\boldsymbol{f}}_k + \boldsymbol{v}_k - \boldsymbol{K}\boldsymbol{x}_{f,k}. \tag{14}$$

4. Controller design

The main objective of this section is to present the design procedure of the robust controller. Substituting (14) into (1) gives

$$\boldsymbol{x}_{f,k+1} = \boldsymbol{A}_1\boldsymbol{x}_{f,k} + [\boldsymbol{I} - \boldsymbol{BH}]\,\boldsymbol{W}\boldsymbol{w}_k + \boldsymbol{B}\boldsymbol{v}_k, \tag{15}$$

with

$$\boldsymbol{A}_1 = \boldsymbol{A} - \boldsymbol{BK}. \tag{16}$$

Analysing (15), and in particular

$$\left[\boldsymbol{I} - \boldsymbol{B}\left[\boldsymbol{B}^T\boldsymbol{B}\right]^{-1}\boldsymbol{B}^T\right]\boldsymbol{W}\boldsymbol{w}_k = \qquad (17)$$
$$\boldsymbol{W}\boldsymbol{w}_k - \boldsymbol{B}\left[\boldsymbol{B}^T\boldsymbol{B}\right]^{-1}\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{w}_k,$$

along with the fact that any vector $\boldsymbol{W}\boldsymbol{w}_k \in \text{col}(\boldsymbol{B})$, where $\text{col}(\boldsymbol{B}) = \{\boldsymbol{\alpha} \in \mathbb{R}^n : \boldsymbol{\alpha} = \boldsymbol{B}\boldsymbol{\beta}\}$ for some $\boldsymbol{\beta} \in \mathbb{R}^r$ can be written as $\boldsymbol{W}\boldsymbol{w}_k = \boldsymbol{B}\bar{\boldsymbol{w}}_k$ for some non-zero $\bar{\boldsymbol{w}}_k$, leads (17) to

$$\boldsymbol{B}\bar{\boldsymbol{w}}_k - \boldsymbol{B}\left[\boldsymbol{B}^T\boldsymbol{B}\right]^{-1}\boldsymbol{B}^T\boldsymbol{B}\bar{\boldsymbol{w}}_k = \boldsymbol{0}. \qquad (18)$$

This significant simplification of (15) yields its new form:

$$\boldsymbol{x}_{f,k+1} = \boldsymbol{A}_1\boldsymbol{x}_{f,k} + \boldsymbol{B}\boldsymbol{v}_k \qquad (19)$$

The general framework for designing $\mathcal{H}_\infty$ robust controller is to determine the gain matrix $\boldsymbol{K}$ such that

$$\lim_{k\to\infty}\boldsymbol{x}_{f,k} = \boldsymbol{0} \quad \text{for } \boldsymbol{v}_k = \boldsymbol{0} \qquad (20)$$
$$\|\boldsymbol{x}_f\|_{l_2} \le \mu\|\boldsymbol{v}_k\|_{l_2} \quad \text{for } \boldsymbol{v}_k \ne \boldsymbol{0},\ \boldsymbol{x}_0 = \boldsymbol{0}. \qquad (21)$$

Thus, the problem is to find a Lyapunov function V_k such that:

$$\Delta V_k + \boldsymbol{x}_{f,k}^T\boldsymbol{x}_{f,k} - \mu^2\boldsymbol{v}_k^T\boldsymbol{v}_k < 0,\ k = 0,\ldots\infty, \qquad (22)$$

where

$$\Delta V_k = V_{k+1} - V_k, \qquad (23)$$

and

$$V_k = \boldsymbol{x}_{f,k}^T\boldsymbol{P}\boldsymbol{x}_{f,k}. \qquad (24)$$

Indeed, if $\boldsymbol{v}_k = \boldsymbol{0}$ then (22) boils down to

$$\Delta V_k + \boldsymbol{x}_{f,k}^T\boldsymbol{x}_{f,k} < 0,\ k = 0,\ldots\infty, \qquad (25)$$

and hence $\Delta V_k < 0$, which leads to (20). If $\boldsymbol{v}_k \ne \boldsymbol{0}$ then (22) yields

$$J = \sum_{k=0}^{\infty}\left(\Delta V_k + \boldsymbol{x}_{f,k}^T\boldsymbol{x}_{f,k} - \mu^2\boldsymbol{v}_k^T\boldsymbol{v}_k\right) < 0, \qquad (26)$$

which can be written as

$$J = -V_0 + \sum_{k=0}^{\infty}\boldsymbol{x}_{f,k}^T\boldsymbol{x}_{f,k} - \sum_{k=0}^{\infty}\mu^2\boldsymbol{v}_k^T\boldsymbol{v}_k < 0, \qquad (27)$$

Knowing that $V_0 = 0$ for $\boldsymbol{x}_{f,0} = \boldsymbol{0}$, equation (27) leads to (21). For (24) the inequality (22) is

$$\Delta V + \boldsymbol{x}_{f,k}^T\boldsymbol{x}_{f,k} - \mu^2\boldsymbol{v}_k^T\boldsymbol{v}_k < 0, \qquad (28)$$

with

$$\begin{aligned}\Delta V =& \boldsymbol{x}_{f,k}^T\left[\boldsymbol{A}_1^T\boldsymbol{P}\boldsymbol{A}_1 - \boldsymbol{P}\right]\boldsymbol{x}_{f,k} + \\ & \boldsymbol{x}_{f,k}^T\left[\boldsymbol{A}_1^T\boldsymbol{P}\boldsymbol{B}\right]\boldsymbol{v}_k + \\ & \boldsymbol{v}_k^T\left[\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{A}_1\right]\boldsymbol{x}_{f,k} + \\ & \boldsymbol{v}_k^T\left[\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{B}\right]\boldsymbol{v}_k. \end{aligned} \qquad (29)$$

Thus, it can be shown that (28) is equivalent to

$$\begin{bmatrix} \boldsymbol{A}_1^T\boldsymbol{P}\boldsymbol{A}_1 + \boldsymbol{I} - \boldsymbol{P} & \boldsymbol{A}_1^T\boldsymbol{P}\boldsymbol{B} \\ \boldsymbol{B}^T\boldsymbol{P}\boldsymbol{A}_1 & \boldsymbol{B}^T\boldsymbol{P}\boldsymbol{B} - \mu^2\boldsymbol{I} \end{bmatrix} \prec \boldsymbol{0}. \tag{30}$$

Lemma 1. *The following statements are equivalent [4, 2, 5]*

(i) There exists $\boldsymbol{X} \succ 0$ *such that*

$$\boldsymbol{V}^T\boldsymbol{X}\boldsymbol{V} - \boldsymbol{W} \prec 0 \tag{31}$$

(ii) There exists $\boldsymbol{X} \succ 0$ *such that*

$$\begin{bmatrix} -\boldsymbol{W} & \boldsymbol{V}^T\boldsymbol{U}^T \\ \boldsymbol{U}\boldsymbol{V} & \boldsymbol{X} - \boldsymbol{U} - \boldsymbol{U}^T \end{bmatrix} \prec 0. \tag{32}$$

The following theorem presents the main tool of this section

Theorem 1. *For a prescribed fault estimation uncertainty and disturbances attenuation level* $\mu > 0$ *for the* $\boldsymbol{x}_{f,k}$*, the* $\mathcal{H}_\infty$ *controller design problem for the system* (1) *is solvable if there exist* $\boldsymbol{U}$*,* $\boldsymbol{N}$ *and* $\boldsymbol{P} \succ \boldsymbol{0}$ *such that the following condition is satisfied:*

$$\begin{bmatrix} \boldsymbol{I} - \boldsymbol{P} & \boldsymbol{0} & \boldsymbol{A}\boldsymbol{U} - \boldsymbol{B}\boldsymbol{N} \\ \boldsymbol{0} & -\mu^2\boldsymbol{I} & \boldsymbol{U}^T\boldsymbol{B}^T \\ \boldsymbol{U}^T\boldsymbol{A}^T - \boldsymbol{N}^T\boldsymbol{B}^T & \boldsymbol{U}\boldsymbol{B} & \boldsymbol{P} - \boldsymbol{U} - \boldsymbol{U}^T \end{bmatrix} \prec \boldsymbol{0}, \tag{33}$$

with $\boldsymbol{N} = \boldsymbol{K}\boldsymbol{U}$.

Finally, the design procedure boils down to solving (33) with respect to $\boldsymbol{U}$, $\boldsymbol{N}$ and $\boldsymbol{P}$, subsequently gain matrix of the controlled can be calculated as follow:

$$\boldsymbol{K} = \boldsymbol{N}\boldsymbol{U}^{-1}. \tag{34}$$

5. Case study

To verify the proposed approach, it was implemented for the quadruple-tank process. A schematic diagram of the process is presented in Fig. 2. The goal is to control the water level in tank 1, 2 and to estimate the faults for pump 1 and pump 2, respectively. The system can be described by following equations [8]:

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \tag{35}$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \tag{36}$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \tag{37}$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1 \tag{38}$$

$$\tag{39}$$

where A_i is cross–sectional area of the tank, a_i stands for cross–section of the outlet orifice, h_i is water level, v_i is voltage applied to pump, γ_i are determined from how the valves are set prior to an experiment, g is the acceleration of gravity. The numerical values of above parameters are as follows: $A_1 = A_3 = 28[cm^2]$, $A_2 = A_4 = 32[cm^2]$, $a_1 = a_3 = 0.071[cm^2]$, $a_2 = a_4 = 0.057[cm^2]$, $g = 981.0[cm/s^2]$. The model was linearised around the steady-state

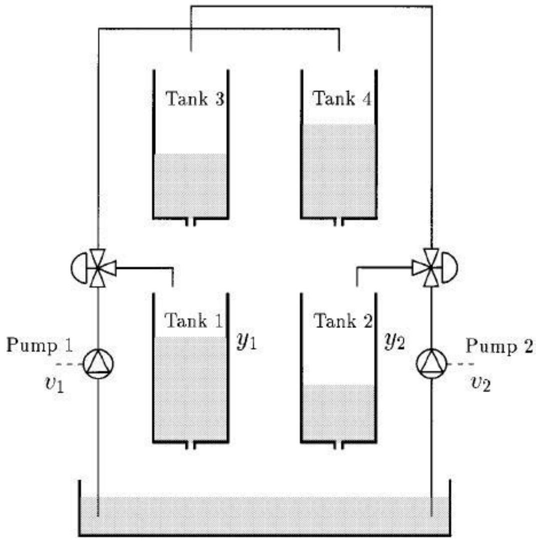


Figure 2. The quadruple-tank process [8]

$h_1^0 = 12.4[cm]$, $h_2^0 = 12.7cm$, $h_3^0 = 1.8cm$, $h_4^0 = 1.4cm$, $k_1 = 3.33[cm^3/Vs]$, $k_2 = 3.35[cm^3/Vs]$, $\gamma_1 = 0.7[cm^3/Vs]$, $\gamma_2 = 0.6[cm^3/Vs]$ and discretized by using Euler method with the sampling time $T_s = 0.1s$. The discrete time system is given by:

$$\boldsymbol{x}_{f,k+1} = \boldsymbol{A}\boldsymbol{x}_{f,k} + \boldsymbol{B}\boldsymbol{u}_{f,k} + \boldsymbol{B}\boldsymbol{f}_k + \boldsymbol{W}\boldsymbol{w}_k, \tag{40}$$

with

$$\boldsymbol{A} = \begin{bmatrix} 0.9984 & 0 & 0.0042 & 0 \\ 0 & 0.9989 & 0 & 0.0033 \\ 0 & 0 & 0.9958 & 0 \\ 0 & 0 & 0 & 0.9967 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0.0083 & 0 \\ 0 & 0.0062 \\ 0 & 0.0048 \\ 0.0031 & 0 \end{bmatrix},$$

$$\boldsymbol{W} = \begin{bmatrix} 0.0500 & 0 & 0 & 0 \\ 0 & 0.0500 & 0 & 0 \\ 0 & 0 & 0.0500 & 0 \\ 0 & 0 & 0 & 0.0500 \end{bmatrix},$$

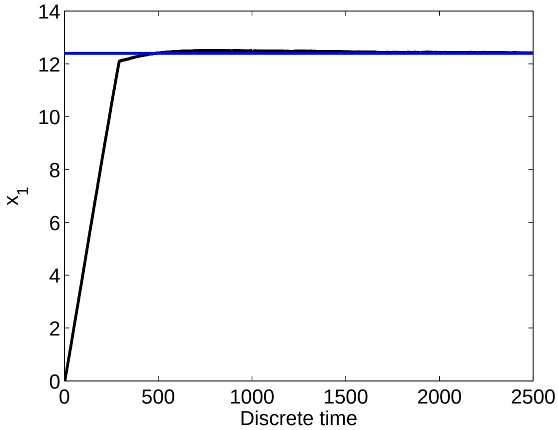
and the exogenous disturbance input

$$\boldsymbol{w}_k \sim \mathcal{N}(\boldsymbol{0}, 0.1^2\boldsymbol{I}). \tag{41}$$

The robust controller was designed with (33) and $\mu = 0.9$. The obtained robust controller $\boldsymbol{K}$ is as follows:

$$\boldsymbol{K} = \begin{bmatrix} 106.2134 & 3.5115 & -4.1539 & 36.5174 \\ -0.0436 & 112.2489 & 62.4471 & 0.4812 \end{bmatrix}$$

Figure 3 presents the system evolution for the fault-free case. It can be seen that the state converges to the required setpoint for the following initial conditions $\boldsymbol{x}_{f,0} = [0.03, 0.04, 0.01, 0.05]$. For further comparative study, two control strategies are employed:



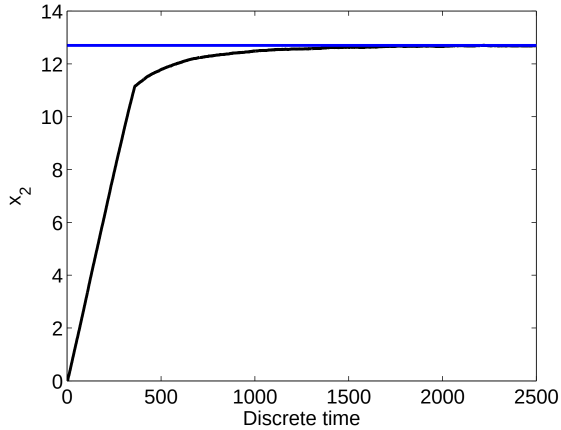


Figure 3. Set point, state $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ (dash line) (for $k = 0, \ldots, 2500$)

Without FTC

$$\boldsymbol{u}_{f,k} = -\boldsymbol{K}\boldsymbol{x}_{f,k}. \tag{42}$$

With FTC

$$\boldsymbol{u}_{f,k} = -\hat{\boldsymbol{f}}_{k-1} - \boldsymbol{K}\boldsymbol{x}_{f,k}. \tag{43}$$

Moreover, three fault scenarios are considered

S1: 5% performance decrease of the first actuator:

$$\boldsymbol{f}_{k,1} = \begin{cases} -0.05\boldsymbol{u}_{f,k} & 1500 \leq k \leq 1600, \\ 0 & \text{otherwise} \end{cases}$$
$$\boldsymbol{f}_{k,2} = 0.$$

S2: 10% performance decrease of the first actuator:

$$\boldsymbol{f}_{k,1} = \begin{cases} -0.1\boldsymbol{u}_{f,k} & 1500 \leq k \leq 1600, \\ 0 & \text{otherwise} \end{cases}$$
$$\boldsymbol{f}_{k,2} = 0.$$

S3: 5% and 10% performance decrease of the first and second actuator, respectively:

$$\boldsymbol{f}_{k,1} = \begin{cases} -0.05\boldsymbol{u}_{f,k,1} & 1500 \leq k \leq 1600, \\ 0 & \text{otherwise} \end{cases}$$
$$\boldsymbol{f}_{k,2} = \begin{cases} -0.1\boldsymbol{u}_{f,k,2} & 1500 \leq k \leq 1600, \\ 0 & \text{otherwise} \end{cases}$$

The results presented in Figs. 4–7, show the system performance with and without FTC strategy. Fault scenarios S1–S3 show the operation of the system for different actuator fault values. The control strategy without FTC provides the worst results for all considered fault scenarios S1–S3. The control strategy without FTC, which being a robust control only, does not consider any information about faults. Thus it is impossible to be realise the suitable recovery actions, while the control strategy with FTC gives better significantly better performance of the system.

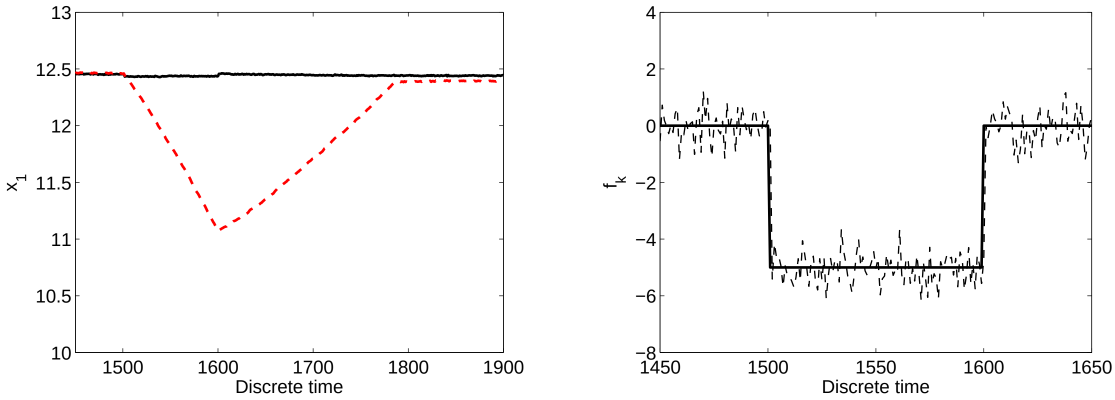


Figure 4. Performance of the system with (solid line) and without FTC (dashed line) (left) and fault (solid line) and its estimate (dashed line) (right) for scenario S1

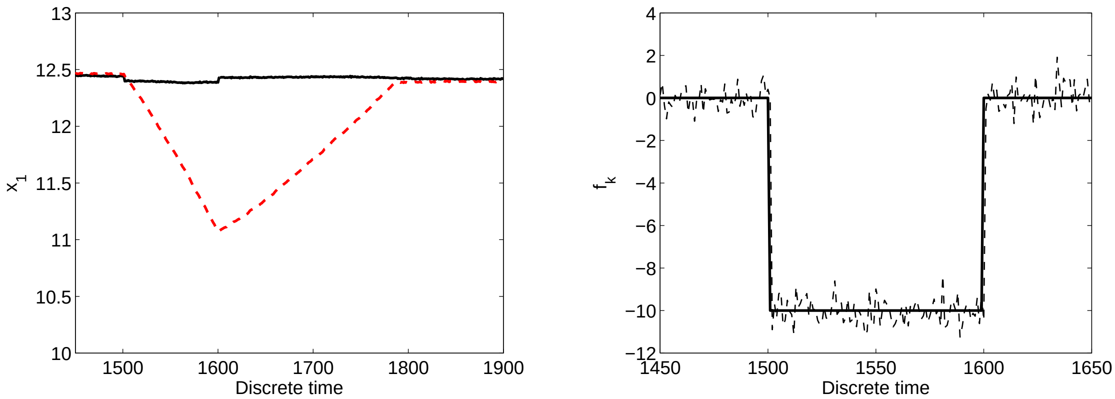


Figure 5. Performance of the system with (solid line) and without FTC (dashed line) (left) and fault (solid line) and its estimate (dashed line) (right) for scenario S2

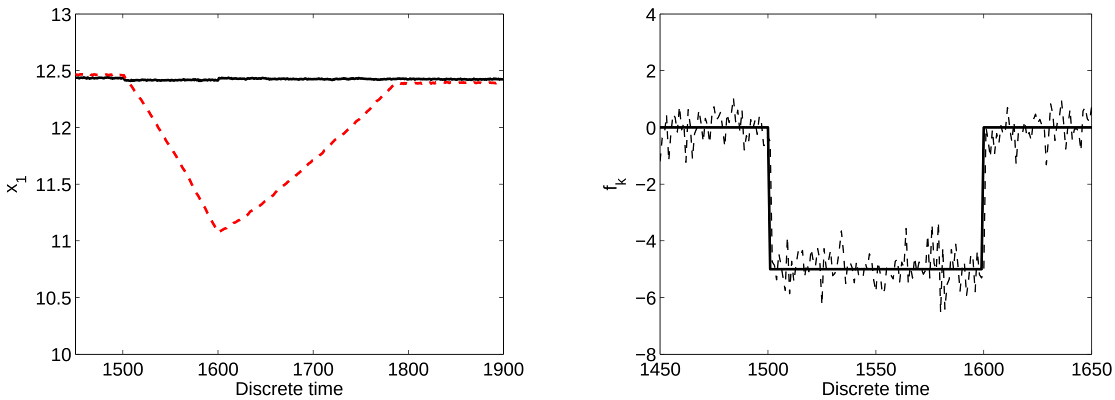


Figure 6. Performance of the system with (solid line) and without FTC (dashed line) (left) and fault (solid line) and its estimate (dashed line) (right) for scenario S3 (first actuator)

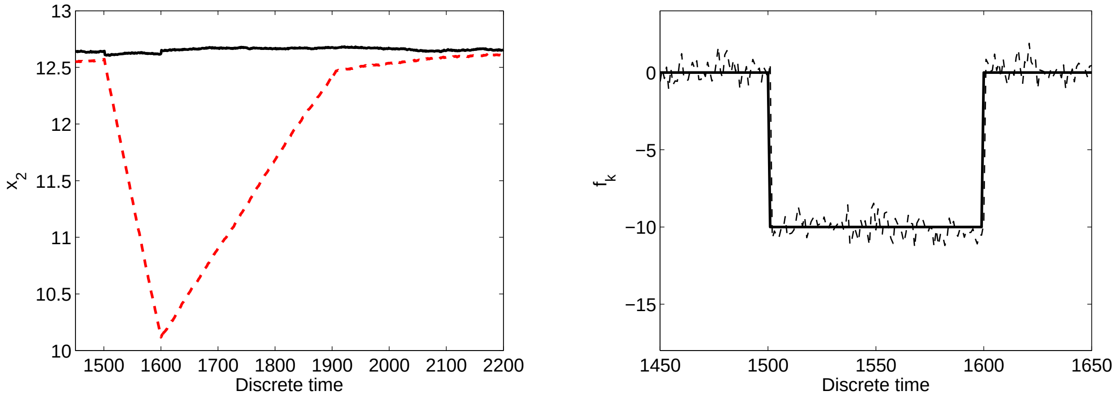


Figure 7. Performance of the system with (solid line) and without FTC (dashed line) (left) and fault (solid line) and its estimate (dashed line) (right) for scenario S3 (second actuator)

6. Conclusions

The main objective of the paper is to propose a robust fault-tolerant control scheme for a non-linear discrete-time systems. The achieved results were divide into the following points:

- minimisation the effects of exogenous disturbances and present fault estimation strategy, through the $\mathcal{H}_\infty$ approach,
- development the procedure for: fault estimation, fault compensation with robust controller,
- verification the proposed approach on the quadruple-tank process.

The proposed approach can be efficiently implemented to real-time system. The offline computations boil down to solving linear matrix inequalities. While the on-line computation boils down to calculate the fault and control signal. The proposed approach was applied to the quadruple-tank process. The achieved results clearly exhibit the high performance of the proposed scheme.

Acknowledgments

The work was supported by the National Science Centre of Poland under grant: 2014-2017

[1] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki. *Diagnosis and Fault-Tolerant Control.* Springer-Verlag, New York, 2003.

[2] J. Daafouz, P. Riedinger, and C. Iung. Stability analysis and control synthesis for switched systems: a switched lyapunov function approach. *IEEE Transactions on Automatic Control*, 47(11):1883–1887, 2002.

[3] S. de Oca, V. Puig, M. Witczak, and L. Dziekan. Fault-tolerant control strategy for actuator faults using lpv techniques: application to a two degree of freedom helicopter. *International Journal of Applied Mathematics and Computer Science*, 22(1):161–171, 2012.

[4] M.C. de Oliveira, J. Bernussou, and J.C. Geromel. A new discrete-time robust stability condition. *Systems and Control Letters*, 37(4):261–265, 1999.

[5] F. Delmotte, T.M. Guerra, and M. Ksantini. Continuous takagi–sugeno's models: reduction of the number of lmi conditions in various fuzzy control design technics. *IEEE Transactions on Fuzzy Systems*, 15(3):426–438, 2007.

[6] S. Gillijns and B. De Moor. Unbiased minimum-variance input and state estimation for linear discrete-time systems. *Automatica*, 43:111–116, 2007.

[7] R. Isermann. *Fault Diagnosis Applications: Model Based Condition Monitoring, Actuators, Drives, Machinery, Plants, Sensors, and Fault-tolerant Systems.* Springer-Verlag, Berlin, 2011.

[8] Johansson K.H. The quadruple-tank process: A multivariable laboratory process with an adjustable zero. *IEEE Transactions on Control Systems Technolofy*, 8(2):456–465, 2000.

[9] J. Korbicz, J. Kościelny, Z. Kowalczuk, and W. Cholewa (Eds.). *Fault diagnosis. Models, Artificial Intelligence, Applications.* Springer-Verlag, Berlin, 2004.

[10] Y. Liang, D. Liaw, and T. Lee. Reliable control of nonlinear systems. *IEEE Trans. Automatic Control*, 45(4):706–710, 2000.

[11] F. Liao, J. Wang, and G. Yang. Reliable robust flight tracking control: an lmi approach. *IEEE Trans. Control Systems Technology*, 10(1):76–89, 2000.

[12] M. Mahmoud, J. Jiang, and Y. Zhang. *Active Fault Tolerant Control Systems: Stochastic Analysis and Synthesis*. Springer-Verlag, Berlin, 2003.

[13] M. Mrugalski and M. Witczak. State-space gmdh neural networks for actuator robust fault diagnosis. *Advances in Electrical and Computer Engineering*, 12(3):65–72, 2012.

[14] M. Mrugalski, M. Witczak, and J. Korbicz. Confidence estimation of the multi-layer perceptron and its application in fault detection systems. *Engineering Applications of Artificial Intelligence*, 21(8):895–906, 2008.

[15] H. Noura, D. Theilliol, J. Ponsart, and A. Chamseddine. *Fault-tolerant Control Systems: Design and Practical Applications*. Springer-Verlag, Berlin, 2003.

[16] Z. Qu, C. M. Ihlefeld, J. Yufang, and A. Saengdeejing. Robust fault-tolerant self-recovering control of nonlinear uncertain systems. *Automatica*, 39(10):1763–1771, 2003.

[17] M. Witczak. Toward the training of feed-forward neural networks with the D-optimum input sequence. *IEEE Transactions on Neural Networks*, 17(2):357–373, 2006.

[18] M. Witczak. *Modelling and Estimation Strategies for Fault Diagnosis of Non-linear Systems*. Springer-Verlag, Berlin, 2007.

[19] M. Witczak, J. Korbicz, and M. Luzar. A lmi-based strategy for $\mathcal{H}_\infty$ fault estimation of non-linear systems: Application to the multi-tank system. In *Proceedings of the 13th European Control Conference, Strasbourg, France*, pages 270–275, 2014.

[20] M. Witczak, A. Obuchowicz, and J. Korbicz. Genetic programming based approaches to identification and fault diagnosis of non-linear dynamic systems. *International Journal of Control*, 75(13):1012–1031, 2002.

[21] M. Witczak and P. Pretki. Design of an extended unknown input observer with stochastic robustness techniques and evolutionary algorithms. *International Journal of Control*, 80(5):749–762, 2007.

[22] M. Witczak, V. Puig, and S. de Oca. A fault-tolerant control strategy for non-linear discrete-time systems: application to the twin-rotor system. *International Journal of Control*, 86(10):1788–1799, 2013.

[23] M. Witczak, R. Rybski, and J. Kaczmarek. Impedance measurement with the d-optimum experimental conditions. *IEEE Transactions on Instrumentation and Measurement*, 58(8):2535–2543, 2009.

[24] Y. Zhang and J. Jiang. Bibliographical review on reconfigurable fault-tolerant control systems. In *IFAC Symposium Fault Detection Supervision and Safety of Technical Processes, SAFEPROCESS*, pages 265–276, Washington, D.C., USA, 2003.