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Approximate active fault detection and control

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Abstract. This paper deals with approximate active fault detection and control for nonlinear discrete-time stochastic systems over an infinite time horizon. Multiple model framework is used to represent fault-free and finitely many faulty models. An imperfect state information problem is reformulated using a hyper-state and dynamic programming is applied to solve the problem numerically. The proposed active fault detector and controller is illustrated in a numerical example of an air handling unit.

1. Introduction

Automatic control is a well-established research field that aims at automatic process control without direct human intervention. In consequence energy can be saved, costs can be reduced, and quality can be improved. In recent decades, the problem of automatic fault detection (FD) has become a topical issue in many technical and nontechnical areas where increased reliability and efficiency is demanded. Safety critical applications in aircraft, trains, cars, or power plants must be essentially accurate, reliable, robust, etc. FD in such applications prevents casualties and unwanted damage. The purpose of FD may be different for home and commercial appliances. One of the contemporary trends aims at reducing energy consumption of buildings.

Two main approaches to FD exist: passive FD and active FD. Passive FD is the most widely used [1, 2]. A passive detector generates decisions about possible faults based on input-output data obtained from a system. However, unreliable decisions may be generated due to insufficient information in the input-output data. To alleviate this issue active fault detection utilizes a feedback to the system to improve the quality of decisions [3]. In addition, active fault detection and control was considered in [4] to simultaneously fulfill detection and control aims. A unified formulation of active fault detection and control was presented in [5]. A designer of an active fault detector and controller (AFDC) can compromise between controlling the system and exciting it to improve the quality of fault detection.

The existing literature [6–8] considers active FD mainly for linear systems and a finite time horizon. An active fault detector that generates a sufficiently informative input signal for a nonlinear system over an infinite time horizon was proposed in [9]. Although this active fault detector is powerful for detecting faults in a system, potential control aims must be considered separately. A goal of this paper is to design an AFDC for a class of nonlinear stochastic systems over an infinite time horizon, where the optimal compromise between fault detection and control should be achieved by considering detection and control aims simultaneously.

The paper is organized as follows. In Section 2 the problem of active fault detection and control is formulated. A multiple model approach is employed to represent a fault-free and faulty behaviors of a system. Section 3 presents a solution to the stated problem which employs various approximations. The optimization problem is reformulated such that numerical methods of dynamic programming can be used. The proposed approach is demonstrated in a numerical example of an air-handling unit in Section 4.

2. Problem formulation

In control systems, there are typically several types of possible faults such as actuator, sensor, and component faults. A common problem is comprised of abrupt change detection. A suitable model of the system behavior is achieved by a multiple model framework. Subsequently, the detection and control aims are expressed by a design criterion that evaluates costs of states, inputs, and decisions.

Let us assume an observed and controlled system that can be described at a time step $k \in \mathcal{T} = \{0, 1, \dots\}$ by the following discrete-time nonlinear model

$$\mathbf{x}_{k+1} = \mathbf{f}\left(\mathbf{x}_k, \mu_k, \mathbf{u}_k\right) + \mathbf{w}_k, \tag{1}$$

where $\mathbf{x}_k^{\mathrm{a}} = \left[\mathbf{x}_k^{\mathrm{T}}, \mu_k\right]^{\mathrm{T}} \in \mathbb{R}^{n_x} \times \mathcal{M}$ represents a hybrid state of the system, $\mathbf{x}_k \in \mathbb{R}^{n_x}$ is a fully measurable common part of the state, $\mu_k \in \mathcal{M} = \{1, 2, \dots, N\}$ is an unknown index of the fault-free or faulty model, $\mathbf{u}_k \in \mathcal{U} \subset \mathbb{R}^{n_u}$ is the input, $\mathcal{U} = \{\bar{\mathbf{u}}^1, \bar{\mathbf{u}}^2, \dots, \bar{\mathbf{u}}^M\}$ is a discrete set of admissible inputs, and $\mathbf{w}_k \in \mathbb{R}^{n_x}$ is the state noise with the known conditional probability density function (pdf) $p_{\mathbf{w}}\left(\mathbf{w}_k | \mathbf{x}_k^{\mathrm{a}}\right)$. The mean value $\mathbf{m}_{\mathbf{w}}(\mathbf{x}_k^{\mathrm{a}})$ and the covariance matrix $\mathbf{P}_{\mathbf{w}}(\mathbf{x}_k^{\mathrm{a}})$ of the state noise are known and depend on the hybrid state $\mathbf{x}_k^{\mathrm{a}}$. The system is characterized by a known vector function $\mathbf{f} : \mathbb{R}^{n_x} \times \mathcal{M} \times \mathcal{U} \mapsto \mathbb{R}^{n_x}$. The behavior of the system depends on μ_k which determines the nonlinear function $\mathbf{f}_{\mu_k} : \mathbb{R}^{n_x} \times \mathcal{U} \mapsto \mathbb{R}^{n_x}$

$$\mathbf{f}\left(\mathbf{x}_k, \mu_k, \mathbf{u}_k\right) = \begin{cases} \mathbf{f}_1\left(\mathbf{x}_k, \mathbf{u}_k\right) & \text{if } \mu_k = 1, \\ \mathbf{f}_2\left(\mathbf{x}_k, \mathbf{u}_k\right) & \text{if } \mu_k = 2, \\ \quad\vdots & \\ \mathbf{f}_N\left(\mathbf{x}_k, \mathbf{u}_k\right) & \text{if } \mu_k = N. \end{cases} \tag{2}$$

It is assumed that $\mathbf{f}_1$ represents the fault-free behavior of the system and the faulty behaviors are defined by functions $\mathbf{f}_i$ for $i \in \{2, 3, \dots, N\}$. The switching between the fault-free and faulty models is defined by a stationary finite-state Markov chain with the known transition probabilities $P_{i,j} = P\left(\mu_{k+1} = j | \mu_k = i\right)$. The pdf $p(\mathbf{x}_0)$ and the probability $P(\mu_0)$ of mutually independent initial conditions $\mathbf{x}_0$ and μ_0 are known. Note that the fault occurrence in real systems is always unknown and the transition probabilities are to be estimated.

The active fault detector and controller generates the decision $d_k \in \mathcal{M}$ about the possible faults and the input signal $\mathbf{u}_k$ that is injected into the system. The AFDC can be described as

$$\begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \boldsymbol{\rho}_k\left(\begin{bmatrix} \mathbf{x}_0^k \\ \mathbf{u}_0^{k-1} \end{bmatrix}\right), \tag{3}$$

where $\boldsymbol{\rho}_k : \mathbb{R}^{n_x(k+1)} \times \mathcal{U}^k \mapsto \mathcal{M} \times \mathcal{U}$ is an unknown nonlinear vector function. The sequence $\boldsymbol{\rho} = \{\boldsymbol{\rho}_0, \boldsymbol{\rho}_1, \dots\}$ is called a policy. Note that $\mathbf{x}_i^j = \left[\mathbf{x}_i^{\mathrm{T}}, \mathbf{x}_{i+1}^{\mathrm{T}}, \dots, \mathbf{x}_j^{\mathrm{T}}\right]^{\mathrm{T}}$ denotes the vector of stacked variables. It is supposed that for all μ_k exists a stabilizing policy $\boldsymbol{\rho}$.

The goal is to find a suitable detection and control law represented by a policy $\boldsymbol{\rho}$ such that a chosen design criterion $J(\boldsymbol{\rho})$ respecting detection and control aims is minimized. The design criterion has the following form

$$J\left(\boldsymbol{\rho}\right) = \lim_{F \to +\infty} \mathbb{E}\left\{\sum_{k=0}^{F} \lambda^k L\left(\mu_k, d_k, \mathbf{x}_k, \mathbf{u}_k\right)\right\}, \tag{4}$$

where $\lambda \in (0,1)$ is a discount factor, $\mathbb{E}\{\cdot\}$ is the expectation operator. The cost function $L : \mathcal{M} \times \mathcal{M} \times \mathbb{R}^{n_x} \times \mathcal{U} \mapsto \mathbb{R}^+$ is defined as

$$L(\mu_k, d_k, \mathbf{x}_k, \mathbf{u}_k) = \alpha L^{\mathrm{d}}(\mu_k, d_k) + (1-\alpha) L^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k), \tag{5}$$

where $\alpha \in [0,1]$ is a weighting factor, $L^{\mathrm{d}} : \mathcal{M} \times \mathcal{M} \mapsto \mathbb{R}^+$ is a detection cost function, $L^{\mathrm{c}} : \mathbb{R}^{n_x} \times \mathcal{U} \mapsto \mathbb{R}^+$ is a control cost function and $\mathbb{R}^+$ denotes a set of non-negative real numbers. It is assumed that the cost functions L^{d}, L^{c}, and L are bounded which makes the criterion well defined. The cost functions L^{d} together with L^{c} are chosen individually to satisfy requirements of a given problem.

Note that the problem formulation allows a designer to choose between three special cases. For $\alpha = 0$, the problem can be seen as optimal stochastic control, whereas $\alpha = 1$ represents a problem of active fault detection. Finally, $\alpha \in (0,1)$ compromises between the detection and control aims.

3. Active fault detector and controller

The design of an AFDC is divided into several steps. The presented problem formulation is a dynamic optimization problem that belongs to a class of imperfect state information problems. To use standard methods of dynamic programming, a reformulation as a perfect state information problem is employed. The optimal AFDC is obtained as a function of a state by solving the Bellman equation [10]. The Bellman equation is solved numerically by various approximation methods [11, 12]. An approximate solution to the Bellman equation is obtained by a quantization of the hyper-state space and using an iterative algorithm known as the value iteration (VI). The approximate AFDC is designed by yielding all partial results.

3.1. Problem reformulation

The problem reformulation is achieved by introducing a hyper-state $\mathbf{s}_k$ that includes a common state $\mathbf{x}_k$ of the system and a sufficient statistics for the unknown index μ_k. Since details of the problem reformulation using the hyper-state are presented in [9], only the result is provided in the following text.

The system, AFDC, and design criterion are reformulated using the hyper-state $\mathbf{s}_k$ which is defined as

$$\mathbf{s}_k = \begin{bmatrix} \mathbf{x}_k \\ \mathbf{b}_k \end{bmatrix} \in \mathcal{S} = (\mathbb{R}^{n_x} \times \mathcal{B}) \subset \mathbb{R}^{n_s}, \tag{6}$$

where $\mathbf{b}_k \in \mathcal{B}$, $\mathcal{B} = \{\mathbf{b} \in \mathbb{R}^{N-1} : \mathbf{b} \geq 0,\ \mathbf{1}^{\mathrm{T}}\mathbf{b} \leq 1\}$ is a belief state with components $b_{k,i} = P\left(\mu_k = i | \mathbf{x}_0^k, \mathbf{u}_0^{k-1}\right)$ referring to the unknown scalar variable μ_k. The belief state $\mathbf{b}_k$ can be understood as a conditional distribution of μ_k. The original model (1) of the system is replaced by a new time-invariant model

$$\mathbf{s}_{k+1} = \boldsymbol{\varphi}(\mathbf{s}_k, \mathbf{u}_k, \mathbf{x}_{k+1}), \tag{7}$$

with a nonlinear vector function $\boldsymbol{\varphi} : \mathcal{S} \times \mathcal{U} \times \mathbb{R}^{n_x} \mapsto \mathcal{S}$. Note that besides random variable $\mathbf{x}_{k+1}$ all arguments of the function $\boldsymbol{\varphi}$ are known or can be computed. The time-variant AFDC (3) is reformulated using stationary policies $\sigma : \mathcal{S} \mapsto \mathcal{M}$ and $\boldsymbol{\gamma} : \mathcal{S} \mapsto \mathcal{U}$ as follows

$$\begin{bmatrix} d_k \\ \mathbf{u}_k \end{bmatrix} = \begin{bmatrix} \sigma(\mathbf{s}_k) \\ \boldsymbol{\gamma}(\mathbf{s}_k) \end{bmatrix} = \bar{\boldsymbol{\rho}}(\mathbf{s}_k). \tag{8}$$

Lastly, the optimality discounted criterion (4) is reformulated as

$$\bar{J}\left(\bar{\boldsymbol{\rho}}\right) = \lim_{F\to+\infty} \mathbb{E}\left\{\sum_{k=0}^{F} \lambda^k \bar{L}\left(d_k, \mathbf{s}_k, \mathbf{u}_k\right)|\mathbf{s}_0\right\}, \tag{9}$$

where $\bar{L}\left(d_k, \mathbf{s}_k, \mathbf{u}_k\right) = \alpha\bar{L}^{\mathrm{d}}\left(d_k, \mathbf{s}_k\right) + (1-\alpha)\,\bar{L}^{\mathrm{c}}\left(\mathbf{s}_k, \mathbf{u}_k\right)$, $\bar{L}^{\mathrm{d}}\left(d_k, \mathbf{s}_k\right)$ is a detection cost function equivalent to $L^{\mathrm{d}}\left(\mu_k, d_k\right)$ and $\bar{L}^{\mathrm{c}}\left(\mathbf{s}_k, \mathbf{u}_k\right)$ is a control cost function equivalent to $L^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right)$. These functions are given as

$$\bar{L}^{\mathrm{d}}\left(d_k, \mathbf{s}_k\right) = \mathbb{E}\left\{L^{\mathrm{d}}\left(\mu_k, d_k\right)|d_k, \mathbf{x}_0^k, \mathbf{u}_0^{k-1}\right\}, \tag{10}$$

$$\bar{L}^{\mathrm{c}}\left(\mathbf{s}_k, \mathbf{u}_k\right) = L^{\mathrm{c}}\left(\left[s_{k,1}, \ldots, s_{k,n_x}\right]^{\mathrm{T}}, \mathbf{u}_k\right). \tag{11}$$

3.2. Approximate active fault detector and controller design

The optimal AFDC is based on a solution to the Bellman equation. The Bellman function V^* represents the optimal expected costs incurred from given time step to infinity and satisfies the Bellman equation

$$V^*\left(\mathbf{s}_k\right) = \min_{d_k\in\mathcal{M},\mathbf{u}_k\in\mathcal{U}} \mathbb{E}\left\{\bar{L}\left(d_k, \mathbf{s}_k, \mathbf{u}_k\right) + \lambda V^*\left(\mathbf{s}_{k+1}\right)|d_k, \mathbf{s}_k, \mathbf{u}_k\right\}. \tag{12}$$

The optimal detector σ^* and the optimal controller $\boldsymbol{\gamma}^*$ are given as

$$d_k^* = \sigma^*\left(\mathbf{s}_k\right) = \arg\min_{d_k\in\mathcal{M}} \alpha\bar{L}^{\mathrm{d}}\left(d_k, \mathbf{s}_k\right), \tag{13}$$

$$\mathbf{u}_k^* = \boldsymbol{\gamma}^*\left(\mathbf{s}_k\right) = \arg\min_{\mathbf{u}_k\in\mathcal{U}} \mathbb{E}\left\{(1-\alpha)\,\bar{L}^{\mathrm{c}}\left(\mathbf{s}_k, \mathbf{u}_k\right) + \lambda V^*\left(\mathbf{s}_{k+1}\right)|\mathbf{s}_k, \mathbf{u}_k\right\}. \tag{14}$$

The Bellman function V^* is computed off-line by solving (12) and then the AFDC is implemented to provide on-line detection and control by means of (13) and (14). However, the analytical solution to the Bellman equation (12) is almost impossible to find. Since the AFDC is designed for a nonlinear system, numerical methods are employed to find an approximate solution to the Bellman equation.

It is necessary to reduce an infinite number of states. The approximate solution is based on hyper-state space quantization. A uniform grid of points $\bar{\mathbf{s}} \in \mathbb{R}^{n_s}$ defined by the Cartesian product $\mathcal{S}^{\mathrm{g}} \equiv \mathcal{S}_1^{\mathrm{g}} \times \mathcal{S}_2^{\mathrm{g}} \times \ldots \times \mathcal{S}_{n_s}^{\mathrm{g}}$ of discrete sets $\mathcal{S}_j^{\mathrm{g}}$ is specified over a region with non-negligible probabilities of hyper-state trajectories. A non-grid hyper-state is projected to the grid using an aggregation function such that the Euclidean distance between the non-grid hyper-state and the grid point is minimized. In the paper it is supposed that the Bellman function V^* is approximated by a piecewise constant function $\bar{V}$. Note that the control space is assumed to be discrete with a reasonable number of controls.

An approximation of the unknown Bellman function V^* can be determined by the VI algorithm [13]. Following the general theoretical results, the VI algorithm consists in iterative computation of the contraction mapping

$$\bar{V}^{(i+1)}\left(\bar{\mathbf{s}}_k\right) = \min_{d_k\in\mathcal{M},\mathbf{u}_k\in\mathcal{U}} \mathbb{E}\left\{\bar{L}\left(d_k, \bar{\mathbf{s}}_k, \mathbf{u}_k\right) + \lambda\bar{V}^{(i)}\left(\bar{\mathbf{s}}_{k+1}\right)|d_k, \bar{\mathbf{s}}_k, \mathbf{u}_k\right\}, \tag{15}$$

where $i = 0, 1, 2, \ldots$ is an iteration index. Given any initial Bellman function $\bar{V}^{(0)}$ the VI algorithm converges to the fixed point. In practice, the iteration process is terminated after a fixed number of iterations.

In the iterative computation (15), it is necessary to compute the conditional expected value of the Bellman function. Due to nonlinearity of the system, the conditional expected value is computed approximately using the unscented transform with a parameter κ [14].

4. Numerical example

Control of temperature in a lecture hall and active fault detection in a corresponding air handling unit (AHU) are considered in this numerical example. The AHU mixes the ambient air and indoor air together with ratio proportional to the damper position. The mixed air is heated or cooled by a heating or cooling coil before it is supplied to the lecture hall. The AFDC can be focused on active fault detection of a stuck damper or it can be aimed at the temperature control of the lecture hall. The AHU and lecture hall temperature model can be described by the following continuous-time nonlinear model [15]

$$\begin{aligned} C^{\mathrm{i}}\dot{T}^{\mathrm{i}}(t) &= \dot{m}^{\mathrm{s}}c^{\mathrm{s}}\left(T^{\mathrm{s}}(t)-T^{\mathrm{i}}(t)\right)+\frac{1}{R^{\mathrm{ow}}}\left(T^{\mathrm{a}}(t)-T^{\mathrm{i}}(t)\right)+P^{w}(t),\\ T^{\mathrm{s}}(t) &= \Delta(t)T^{\mathrm{i}}(t)+\left(1-\Delta(t)\right)T^{\mathrm{a}}(t)+T^{\mathrm{hc}}(t),\\ T^{\mathrm{hc}}(t) &= s^{\mathrm{hc}}(t)\frac{P^{\mathrm{hc}}\eta^{\mathrm{hc}}}{\dot{m}^{\mathrm{s}}c^{\mathrm{s}}}, \end{aligned} \tag{16}$$

where $T^{\mathrm{i}}(t)$ [°C] is the indoor air temperature, $T^{\mathrm{a}}(t)$ [°C] is the ambient air temperature, $T^{\mathrm{s}}(t)$ [°C] refers to the supply air temperature, $\Delta(t) \in \mathcal{U}^{\mathrm{N}} = \{0, 0.1, \ldots, 0.9\}$ is the damper position and $P^{w}(t)$ represents the disturbance power load generated for example by occupancy of the lecture hall or its equipment. Note that the damper is fully opened to the ambient air when $\Delta(t) = 0$ and it cannot be fully closed due to fresh air circulation, therefore $\max\left(\Delta(t)\right) = 0.9$. The thermal capacity of the lecture hall is $C^{\mathrm{i}} = 7.8\cdot 10^{6}$ $[\mathrm{J}\cdot{}^{\circ}\mathrm{C}^{-1}]$, the specific thermal capacity of supply air is $c^{\mathrm{s}} = 1012$ $[\mathrm{J}\cdot\mathrm{kg}^{-1}\cdot{}^{\circ}\mathrm{C}^{-1}]$, mass flow rate of the supply air is $\dot{m}^{\mathrm{s}} = 1$ $[\mathrm{kg}\cdot\mathrm{s}^{-1}]$ and the external wall thermal resistance is $R^{\mathrm{ow}} = 7\cdot 10^{-4}$ $[{}^{\circ}\mathrm{C}\cdot\mathrm{W}^{-1}]$. The increase or decrease in temperature of the supply air caused by heating or cooling $T^{\mathrm{hc}}(t)$ [°C] is defined by the coil switching $s^{\mathrm{hc}}(t) \in \mathcal{U}^{\mathrm{L}} = \{-1, 0, 1\}$, the heating power $P^{\mathrm{hc}} = 4\cdot 10^{4}$ [W] of the AHU, and the coil efficiency $\eta^{\mathrm{hc}} = 0.6$. The system input is $s^{\mathrm{hc}}(t)$ together with $\Delta(t)$ and the set of admissible inputs is defined as $\mathcal{U} = \mathcal{U}^{\mathrm{L}} \times \mathcal{U}^{\mathrm{N}}$. In the paper, it is assumed that the ambient air temperature remains constant, i.e. $\dot{T}^{\mathrm{a}}(t) = 0$, and the considered fault is represented by the damper stuck at the fully open position. A possible fault of the system is represented by damper stuck at the position fully opened to the ambient air.

The nonlinear model (16) discretized by the forward Euler method with the sampling period $T_{\mathrm{s}} = 300$ [s] has the following form

$$f_i\left(\mathbf{x}_k, \mathbf{u}_k\right) = \mathbf{A}\mathbf{x}_k + \mathbf{B}^{\mathrm{L}}\mathbf{u}_{k,1} + \mathbf{B}_i^{\mathrm{N}}\left(\mathbf{x}_{k,1}-\mathbf{x}_{k,2}\right)\mathbf{u}_{k,2} + \mathbf{w}_k, \tag{17}$$

where $i \in \mathcal{M} = \{1, 2\}$ and the following definitions hold

$$\begin{aligned} &\mathbf{A} = \begin{bmatrix} 1-K_1T_{\mathrm{s}}-K_2T_{\mathrm{s}} & K_1T_{\mathrm{s}}+K_2T_{\mathrm{s}} \\ 0 & 1 \end{bmatrix}, \mathbf{B}^{\mathrm{L}} = \begin{bmatrix} K_1K_3T_{\mathrm{s}} \\ 0 \end{bmatrix}, \mathbf{B}_1^{\mathrm{N}} = \begin{bmatrix} K_1T_{\mathrm{s}} \\ 0 \end{bmatrix}, \mathbf{B}_2^{\mathrm{N}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\\ &\mathbf{x}_k = \begin{bmatrix} \mathbf{x}_{k,1} \\ \mathbf{x}_{k,2} \end{bmatrix} = \begin{bmatrix} T_k^{\mathrm{i}} \\ T_k^{\mathrm{a}} \end{bmatrix}, \mathbf{u}_k = \begin{bmatrix} \mathbf{u}_{k,1} \\ \mathbf{u}_{k,2} \end{bmatrix} = \begin{bmatrix} s_k^{\mathrm{hc}} \\ \Delta_k \end{bmatrix}, \mathbf{w}_k = \begin{bmatrix} \mathbf{w}_{k,1} \\ \mathbf{w}_{k,2} \end{bmatrix},\\ &K_1 = \frac{\dot{m}^{\mathrm{s}}c^{\mathrm{s}}}{C^{\mathrm{i}}}, K_2 = \frac{1}{C^{\mathrm{i}}R^{\mathrm{ow}}}, K_3 = \frac{P^{\mathrm{hc}}\eta^{\mathrm{hc}}}{\dot{m}^{\mathrm{s}}c^{\mathrm{s}}}. \end{aligned} \tag{18}$$

The state noise $\mathbf{w}_k$ is independent of the system state. The noise $\mathbf{w}_{k,1}$ represents disturbance power load and has the Laplace distribution with the location parameter $\varpi = 0$ and the scale parameter $\beta = 0.08$. The noise $\mathbf{w}_{k,2}$ is described by Dirac delta function $\delta\left(\mathbf{w}_{k,2}\right)$. The system initially starts as fault-free, i.e. $P\left(\mu_0 = 1\right) = 1$. The switching between fault-free and faulty models is defined by the transition probabilities $P\left(\mu_{k+1} = i|\mu_k = j\right) = 0.02$ for $i, j \in \mathcal{M}, i \neq j$.

The detection cost function $L^{\mathrm{d}}(\mu_k, d_k)$ and control cost function $L^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k)$ are chosen as

$$L^{\mathrm{d}}(\mu_k, d_k) = \begin{cases} 0 & \text{if } d_k = \mu_k, \\ 1 & \text{otherwise}, \end{cases}$$
$$L^{\mathrm{c}}(\mathbf{x}_k, \mathbf{u}_k) = \sum_{i=1}^{n_u} |\mathbf{p}_i^{\mathrm{hc}} \mathbf{u}_{k,i}| + \mathbf{q}_1 \left(1 - e^{-\mathbf{q}_2 \left(\mathbf{x}_{k,1} - x^{\mathrm{ref}}\right)^2}\right), \tag{19}$$

where $\mathbf{p}^{\mathrm{hc}} = \left[\mathbf{p}_1^{\mathrm{hc}}, \mathbf{p}_2^{\mathrm{hc}}\right]^{\mathrm{T}} = [1, 0]^{\mathrm{T}}$ and $\mathbf{q} = [\mathbf{q}_1, \mathbf{q}_2]^{\mathrm{T}} = [60, 1]^{\mathrm{T}}$ are parameters, and $x^{\mathrm{ref}} = 23$ [°C] is the reference indoor temperature. From (19) it follows that the detection aim is to minimize the probability of making a wrong decision about an actual system model. The control aim is designed to minimize costs of a compromise between a weighted price of control actions and following the reference temperature. It can be derived that the detector is given by the following decision rule

$$d_k = \begin{cases} 1 & \text{if } P(\mu_k = 1 | \mathbf{x}_0^k, \mathbf{u}_0^{k-1}) \geq 0.5, \\ 2 & \text{otherwise}. \end{cases} \tag{20}$$

The design parameter of the UT is set to $\kappa = 3$, the discount factor $\lambda = 0.98$, and the weighting factor is $\alpha = 0.99$. For the constant ambient air temperature $T^{\mathrm{a}} = 21$ [°C], the uniform grid is defined by $\mathcal{S}_1^{\mathrm{g}} = \{5, 5.1, \ldots, 30\}$, $\mathcal{S}_2^{\mathrm{g}} = \{21\}$, and $\mathcal{S}_3^{\mathrm{g}} = \{0, 0.01, \ldots, 1\}$.

The VI algorithm was terminated after performing $n_{\mathrm{iter}} = 30$ iterations. The corresponding approximate Bellman function $\bar{V}$ for fixed $\mathbf{s}_{k,2} = 21$ [°C] is shown in Figure 1. The global minimum of $\bar{V}$ is around the value of the indoor reference temperature which is caused by the control cost function L^{c}. It can also be seen that approximate Bellman function $\bar{V}$ attains a local maximum when the uncertainty of the model is high, i.e. $\mathbf{s}_{k,3}$ is close to 0.5. When the indoor air temperature is close to the ambient air temperature, $\bar{V}$ slightly increases as well. This is caused by the definition (17) of the fault-free and faulty models which differ just in $\mathbf{B}^{\mathrm{N}}$.

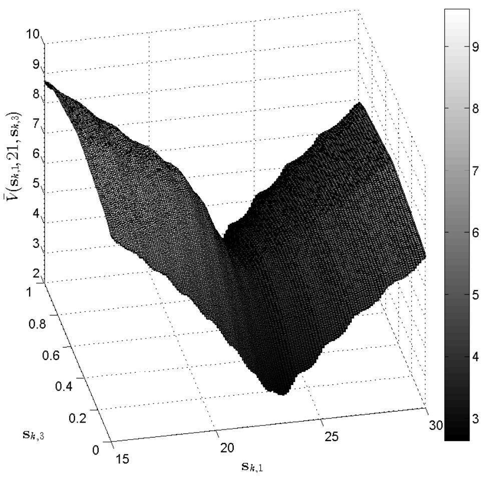

Figure 1. The approximate Bellman function for fixed $\mathbf{s}_{k,2} = 21$ [°C].

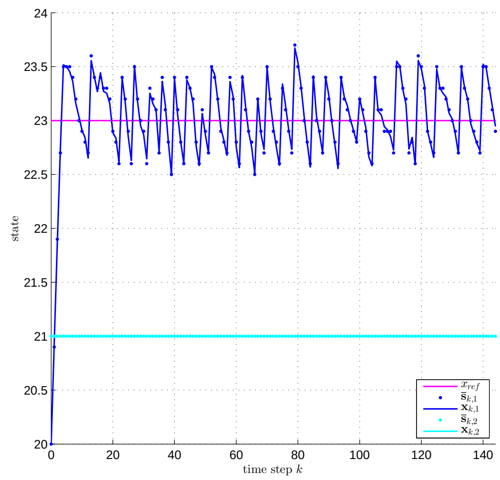

Figure 2. Typical state trajectories of the system with the AFDC for a time horizon of 12 hours.

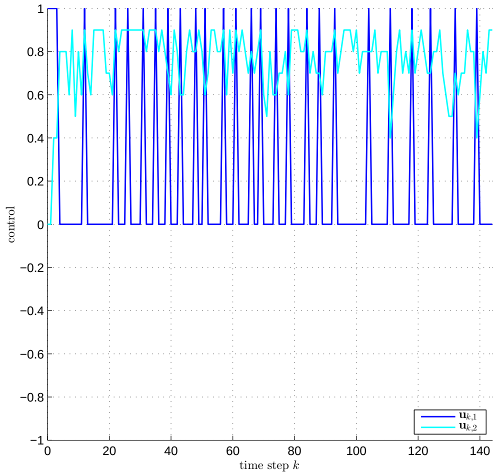

Figure 3. Typical control actions of the AFDC for a time horizon of 12 hours.

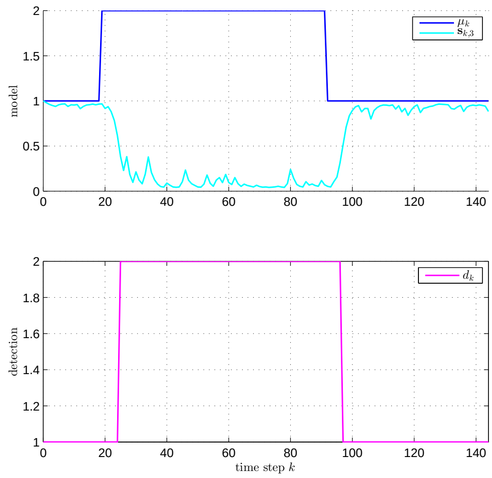

Figure 4. Typical true model, probability of the fault-free model, and a decision of the AFDC for a time horizon of 12 hours.

Next, a simulation of system trajectories was performed for a time horizon of 12 hours. Typical state trajectories of the system with the AFDC are depicted in Figure 2. A development of the state trajectories indicates that the indoor air temperature follows the reference value with oscillations caused by a discrete amount of power delivered during the sampling period. The trajectories of $\mathbf{u}_{k,1}$ and $\mathbf{u}_{k,2}$ are depicted in Figure 3. Since the ambient air temperature is lower than the reference air temperature, the AFDC keeps the damper closed to the ambient air. Figure 4 shows that the true model is correctly detected with a delay of approximately 5 steps. It was shown that the AFDC fulfilled the detection aim as well as the control aim. The designed AFDC is able to stabilize the system both for fault-free and faulty model as depicted in the Figures 2, 3, and 4. However, in general a stability is not guaranteed.

Each action of the approximate control policy $\hat{\gamma}(\bar{\mathbf{s}}_k) = [\mathbf{u}_{k,1}, \mathbf{u}_{k,2}]^{\mathrm{T}}$ of the AFDC is shown in Figures 5 and 6. These two figures represent the control policy functions for coil switching and damper position. A value of control is defined by color hue. Neither heating nor cooling is needed when the actual indoor temperature is close to the reference indoor temperature as shown in Figure 5. Heating or cooling is demanded when the actual indoor temperature is lower or higher than the reference indoor temperature, respectively. Figure 6 shows that when the indoor and ambient air temperatures attain approximately the same values it may be difficult to decide between fault-free and faulty model. In such a case the damper fully opens to ambient air to differentiate the temperatures. The damper is closed $\mathbf{u}_{k,2} = 0.9$ for majority of states to prevent unwanted cooling of the indoor air.

In the numerical example of the AHU, the weighting factor α that compromises detection and control aims was set to $\alpha = 0.99$. To illustrate an influence of the parameter α on detection and control aims, the following set of α values $\mathcal{A} = \{0, 0.01, 0.5, 0.99, 0.999, 1\}$ was chosen. For each of the $\mathcal{A}$ element an estimation of detection J^{d} and control J^{c} part of the design criterion (4)

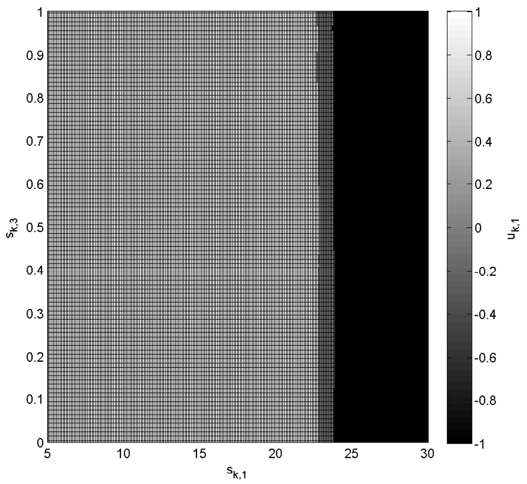

Figure 5. The component $\mathbf{u}_{k,1}$ of the approximate control policy $\hat{\boldsymbol{\gamma}}$ for the AHU.

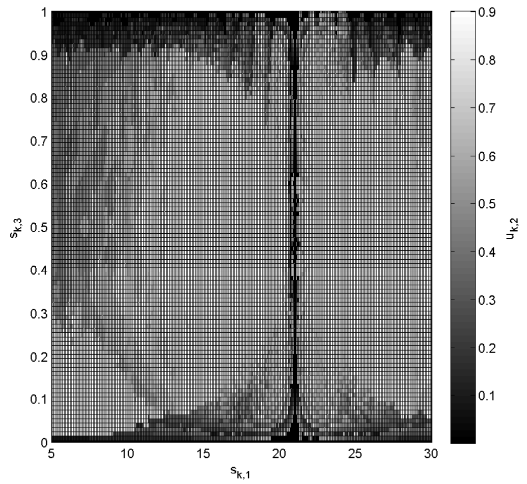

Figure 6. The component $\mathbf{u}_{k,2}$ of the approximate control policy $\hat{\boldsymbol{\gamma}}$ for the AHU.

was performed. The detection and the control parts are expressed as

$$J^{\mathrm{d}} = \lim_{F\to+\infty} \mathbb{E}\left\{\sum_{k=0}^{F} \lambda^k L^{\mathrm{d}}\left(\mu_k, d_k\right)\right\},$$
$$J^{\mathrm{c}} = \lim_{F\to+\infty} \mathbb{E}\left\{\sum_{k=0}^{F} \lambda^k L^{\mathrm{c}}\left(\mathbf{x}_k, \mathbf{u}_k\right)\right\}. \tag{21}$$

The values J^{d} and J^{c} were computed approximately using 10 000 Monte Carlo simulations of state trajectories for a finite time horizon $F = 500$. The Figure 7 shows that the estimates $\hat{J}^{\mathrm{d}}$ and $\hat{J}^{\mathrm{c}}$ attain their lowest values for $\alpha = 1$ and $\alpha = 0$, respectively. Note that the solutions on the Pareto front are not uniformly distributed with respect to the weighting factor α. The value $\hat{J}^{\mathrm{c}}$ remains approximately the same for $\alpha \in [0, 0.99]$ and only quality of detection changes. Thus one might consider only $\alpha \geq 0.99$ when a compromise between active fault detection and control should be considered in this case.

5. Conclusion

This paper is one of the first to present a design of an approximate active fault detector and controller (AFDC) for a class of nonlinear stochastic systems over an infinite time horizon. The problem formulation allows a compromise to be made between detection and control aims. The approximate AFDC was designed by solving the Bellman equation over a discrete grid of hyper-states and successfully demonstrated in a numerical example of an air handling unit.

Acknowledgments

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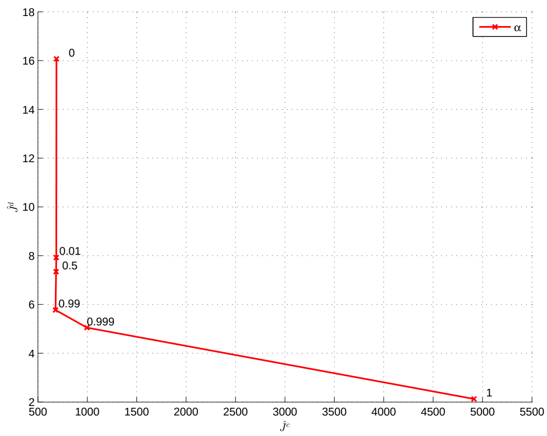

Figure 7. Pareto front for the AHU optimization problem indicating an influence of parameter α on estimates of detection and control parts of criterion (4).

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