

Fragmentation in nuclear reaction and its relation to EOS

Akira Ono

Department of Physics, Tohoku University, Sendai 980-8578, Japan

E-mail: ono@nucl.phys.tohoku.ac.jp

Abstract. Heavy-ion collisions in the incident energy region from several ten to several hundred MeV/nucleon are closely related to the properties of nuclear matter under various conditions of the density, the temperature and the neutron-proton asymmetry. The emission of a large number of clusters in collisions implies strong cluster correlations in nuclear matter that is expanding and breaking up into many fragments. The antisymmetrized molecular dynamics approach has been extended in order to describe the cluster emissions properly. Calculations show that the formation of α clusters affects strongly the observables, such as the ${}^3\text{H}/{}^3\text{He}$ spectrum ratio, that are usually considered as probes of the density dependence of nuclear symmetry energy.

1. Heavy-ion collisions, nuclear matter EOS, and cluster correlations

Heavy-ion collisions go through various conditions of nuclear matter depending on the incident energy and the impact parameter. In central collisions at several ten to several hundred MeV/nucleon, the initially compressed system expands toward low densities where many intermediate mass fragments are formed, which is closely related to the instability of uniform matter and/or the liquid-gas phase transition. It has been expected that one can explore the nuclear matter equation of state (EOS) in a wide density region from low ($\sim \frac{1}{10}\rho_0$) to high ($\sim 2\rho_0$) by carefully studying the heavy-ion collision observables, where ρ_0 is the nuclear saturation density. The knowledge of the EOS is also important for astrophysical problems such as the properties of neutron stars and the explosion mechanism of supernovae. The temperature of the order of 1-10 MeV achieved in heavy-ion collisions is similar to the situation of supernovae, while the information from heavy-ion collisions is still useful for neutron star problems as far as one can take care of the temperature dependence.

Recently much effort of experimental and theoretical studies has been made in order to get some information of the EOS of asymmetric nuclear matter by varying the neutron to proton ratio of the reaction system. The energy of nuclear matter per nucleon can be written as $E/A = E_0(\rho) + S(\rho)\delta^2 + \dots$, where $\delta = (\rho_n - \rho_p)/\rho$ represents the neutron-proton asymmetry. If the EOS of symmetric nuclear matter is known, the symmetry energy $S(\rho)$, which is a function of the density ρ , is the key quantity to characterize the EOS of asymmetric nuclear matter. Constraints on $S(\rho)$ obtained so far from nuclear physics information are summarized in Refs. [1, 2]. The masses of neutron-rich nuclei place a stringent constraint on the value of $S(\rho_1)$ at $\rho_1 \approx \frac{2}{3}\rho_0$, while the E1 strength distribution (such as the electric dipole polarizability) is sensitive to the parameter $L = 3\rho_0(dS/d\rho)_{\rho_0}$ which characterizes the density dependence of the symmetry



energy. Analyses of heavy-ion observables have also placed some constraints in the plane of L and $S(\rho_0)$. The representative is that from the isospin diffusion analysis of the exchange of neutrons and protons between the projectilelike and targetlike parts in semiperipheral collisions of nuclei with different neutron-to-proton ratios [3]. It seems that these constraints are consistent with the ones from nuclear structure information. However, these relatively reliable observables are constraining $S(\rho)$ only at subsaturation densities $\frac{1}{3}\rho_0 \lesssim \rho \lesssim \rho_0$. It is important to obtain some information at higher densities $\rho \sim 2\rho_0$ in near future from experiments at RIBF/RIKEN and future facilities.

Nuclear matter information at much lower densities $\rho \sim \frac{1}{10}\rho_0$ have been extracted by a Texas A&M group by analyzing the yields of clusters emitted from the intermediate velocity source in heavy-ion collisions [4, 5, 6]. Their essential assumption is that the clusters with the same velocity (at the surface of the source) were emitted simultaneously from an equilibrated source which is expanding. They have extracted the evolution of the temperature as a function of the density, the symmetry energy at low densities, the binding energies of clusters in medium, and so on. Some results are compared with the calculation that explicitly takes into account the clusters [7, 8]. These studies clearly show that cluster correlations play essential roles in low-density nuclear matter EOS. It should be noted that, due to the existence of cluster correlations, the low-density EOS exhibits nontrivial dependence on ρ , δ and T that cannot be simply represented by $S(\rho)$.

Isospin diffusion in collisions at around 50 MeV/nucleon is known to be sensitive to the symmetry energy at higher subsaturation densities $\frac{1}{3}\rho_0 \lesssim \rho \lesssim \rho_0$ [3]. An interesting and important question is whether cluster correlations are important in the isospin diffusion reactions. Coupland et al. [9] compared two BUU calculations with and without cluster correlations, and found that the fragmentation dynamics and the degree of isospin diffusion depend on the existence of cluster correlations. In this approach with clusters [10], collision terms couple the BUU equations for the phase-space distribution functions of different species of clusters with $A \leq 3$. This example shows that the study of nuclear matter properties should also treat clusters carefully to obtain a robust constraint on EOS.

Collisions at higher incident energies of several hundred MeV/nucleon are generally suitable to get information of high density EOS since the system is compressed up to a high density ($\sim 2\rho_0$) in the early stage of a reaction. Calculations have shown that the neutron-to-proton ratio in the high density part of the compressed system is sensitive to the symmetry energy $S(\rho)$ at high densities in neutron-rich systems. The yield ratio of charged pions π^-/π^+ has been proposed as a probe for this high density effect, since many of the pions originate from this high density part where Δ resonances are produced by two-nucleon collisions [11, 12]. Currently, different transport model calculations resulted in contradicting conclusions on the high density symmetry energy [13, 14], based on the same existing FOPI data [15]. Theoretical work is required to resolve the origin of this contradiction, while experimental data for collisions with various neutron-to-proton ratios will be useful to clarify the situation. The study of pions should be done together with the study of other observables, such as the neutron and proton flows, to obtain a consistent picture. It should also be remarked that, even in these high energy collisions, a major part of nucleons are bound in clusters at the end of the collisions [16], and therefore cluster formation may influence the global expansion dynamics and the emissions of particles.

Thus the formation and/or existence of clusters seem to be essential not only for themselves but also for the study of nuclear matter EOS in many situations. It may be required that transport models should be able to describe clusters in order to obtain reliable conclusions on EOS from heavy-ion collisions. In the following, we will review the basic idea to treat cluster correlations in the antisymmetrized molecular dynamics (AMD) approach. The composition of clusters is investigated in the calculation of heavy-ion collisions, which suggests that the α cluster formation should be carefully studied in order to get the symmetry energy information

from typical observables such as the $^3\text{H}/^3\text{He}$ ratio.

2. Cluster correlations in AMD

2.1. Basic version of AMD

AMD [17] solves the time evolution of many-nucleon system starting with two boosted nuclei with a given impact parameter and a suitable distance between them. To describe the state at each time, AMD employs a single Slater determinant of Gaussian wave packets for A nucleons

$$\langle \mathbf{r}_1 \dots \mathbf{r}_A | \Phi(Z) \rangle \propto \det_{ij} \left[\exp \left\{ -\nu (\mathbf{r}_i - \mathbf{Z}_j / \sqrt{\nu})^2 \right\} \chi_{\alpha_j}(i) \right], \quad (1)$$

where χ_{α_i} are the spin-isospin states with $\alpha_i = p \uparrow, p \downarrow, n \uparrow$, or $n \downarrow$. Thus the many-body state $|\Phi(Z)\rangle$ is parametrized by a set of complex variables $Z = \{\mathbf{Z}_i\}_{i=1,\dots,A}$. The width parameter $\nu = (2.5 \text{ fm})^{-2}$ is treated as a constant parameter common to all the wave packets. The time evolution of the wave packet parameters Z is determined, up to the extensions described later, by applying the time-dependent variational principle from which the equation of motion for Z is obtained, $\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$ with suitably defined Poisson brackets. The Hamiltonian $\mathcal{H}$ is the expectation value of the Hamiltonian operator with an additional correction for the spurious zero-point energies of the center-of-mass motions of fragments [17]. The effective interactions for the mean-field calculations, such as the Gogny force and the Skyrme force, have been usually employed. The calculated results presented in this paper were obtained with the Skyrme SLy4 force [18] unless otherwise mentioned. The equation of motion can be interpreted intuitively as representing the motion of individual wave packets in the mean-field potential.

In addition to the mean-field effect, the two-nucleon collisions play important roles. This effect is treated as a stochastic transition from an AMD state to one of the possible other AMD states,

$$W_{i \rightarrow f} = \frac{2\pi}{\hbar} |\langle \Psi_f | V | \Psi_i \rangle|^2 \delta(E_f - E_i). \quad (2)$$

When the nucleons 1 and 2 collide in the initial state $|\Psi_i\rangle$ that has corresponding physical coordinates $W_i = \{\mathbf{W}_1, \dots, \mathbf{W}_A\}$ [17], the set of final states $\{|\Psi_f\rangle\}$ is usually constructed by changing only the two physical coordinates $W_f = \{\mathbf{W}'_1, \mathbf{W}'_2, \mathbf{W}_3, \dots, \mathbf{W}_A\}$, under the constraints of the momentum and energy conservation and the Pauli principle. The transition probability is given based on the differential cross section $(d\sigma/d\Omega)_{\text{NN}}$ of the two-nucleon scattering, which depends on the scattering energy and may be modified in nuclear medium. In the calculations presented here, we employ the in-medium density-dependent total cross sections by Li and Machleidt [19] for the scattering energies $E > 35 \text{ MeV}$ in the nucleon-nucleon center-of-mass system, while the cross sections at $E = 35 \text{ MeV}$ are employed at lower energies. Angular distributions are assumed to be similar to the experimental data in the free space.

This basic version of AMD, however, cannot well describe fragmentation reactions in most cases except for very light systems. For example [20], in central Xe + Sn collisions at 50 MeV/nucleon, the calculation often produces two heavy fragments which are remnants of initial nuclei but do not exist in experimental data. Too many protons and too few α particles are emitted compared with the data.

2.2. Consideration on cluster correlations

In low density nuclear matter, clusters such as deuterons and α particles can exist as if they have a bound intrinsic state as shown by some calculations [10, 8, 7]. The existence of a bound cluster depends on the density and the temperature of the medium, and the momentum of the cluster relative to the medium.

Figure 1 shows the internal binding energy of an α cluster that is placed in a ^{124}Sn nucleus. The center-of-mass of the AMD wave function for the ^{124}Sn bound state is placed at the origin

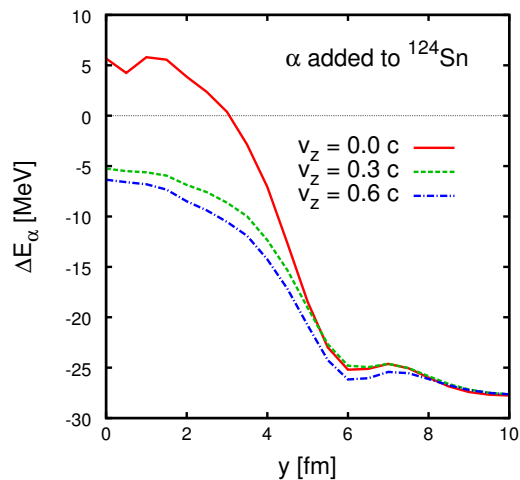


Figure 1. Internal binding energy of an α cluster placed in the ground state ^{124}Sn nucleus as a function of the position $(0, y, 0)$ and the velocity $(0, 0, v_z)$ of the cluster relative to the center of the nucleus. See the text for the definition of the binding energy.

in the phase space, and an α cluster, that is represented by four Gaussian wave packets with different spin and isospin states, is placed at $(0, y, 0)$ and $(0, 0, v_z)$ in the coordinate and velocity spaces, respectively. The total wave function is always antisymmetrized. The energy of this state E_α is evaluated relative to the energy of the ^{124}Sn ground state. Similarly single-nucleon energies ($E_{p\uparrow}$, $E_{p\downarrow}$, $E_{n\uparrow}$ and $E_{n\downarrow}$) are evaluated by adding a nucleon at $(0, y, 0)$ and $(0, 0, v_z)$ to ^{124}Sn . The α binding energy B_α is defined by $-B_\alpha = \Delta E_\alpha = E_\alpha - (E_{p\uparrow} + E_{p\downarrow} + E_{n\uparrow} + E_{n\downarrow})$ as a function of y and v_z . The calculated result shows that the α cluster at rest ($v_z = 0$) has a positive binding energy ($\Delta E_\alpha < 0$) at the surface of the nucleus and outside it, while the α binding energy is always positive ($\Delta E_\alpha < 0$) if a sufficient momentum is given as in the cases of $v_z = 0.3c$ and $0.6c$ in the figure.

The above result suggests that the AMD wave function is suitable for the description of cluster correlations. However, this is not sufficient for the proper description of emergence of cluster correlations in nuclear reactions. The probability of formation of bound clusters is dominantly influenced by the phase space or the density of states. When a bound state exists, the correct density of states for the internal state of the subsystem contains the bound state contribution as $D(E) = \delta(E - \Delta E_\alpha) + D_{\text{continuum}}(E)$. On the other hand, when the AMD equation of motion $\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$ is solved (with the usual two-nucleon collisions included), the system tends to follow the classical phase space $\tilde{D}(E)$. If the cluster binding energy is correctly described by AMD, then $\tilde{D}(E) = 0$ for $E \leq \Delta E_\alpha$ and it starts at $E = \Delta E_\alpha$ continuously. The probability of finding a cluster is related to the bound phase-space volume $\int_{\Delta E_\alpha}^0 \tilde{D}(E) dE$ which is likely much smaller than the correct quantum-mechanical value $\int_{\Delta E_\alpha}^0 D(E) dE = 1$. This is probably the reason why the usual AMD calculations underestimate the yield of clusters.

2.3. AMD with cluster correlations

Guided by the above consideration, we try to recover the correct quantum-mechanical probability of cluster formation by choosing the set of the final states $\{|\Psi_f\rangle\}$ of each two-nucleon collision in such a way that the configuration of bound cluster formation is explicitly included in the set [20]. We assume here that clusters with $A = 2, 3$ and 4 should be treated explicitly.

To obtain the transition probabilities to clusterized states, we employ an approximation similar to Ref. [10]. As one of possible final states $\{|\Psi_f\rangle\}$ for a collision of two nucleons N_1 and N_2 with the initial relative velocity v_{NN} , let us consider a case that N_1 (N_2) forms a cluster C_1 (C_2) with another nucleon B_1 (B_2) in the final state. The partial differential cross section to

this final channel is given by

$$v_{\text{NN}} d\sigma(N_1 B_1 N_2 B_2 \rightarrow C_1 C_2) = \frac{2\pi}{\hbar} |\langle \varphi'_1 | \varphi_1^{+\mathbf{q}} \rangle|^2 |\langle \varphi'_2 | \varphi_2^{-\mathbf{q}} \rangle|^2 |M|^2 \delta(\mathcal{H} - E) \frac{p_{\text{rel}}^2 dp_{\text{rel}} d\Omega}{(2\pi\hbar)^3}, \quad (3)$$

where M is the matrix element for the two-nucleon scattering to the final state with the relative momentum p_{rel} and the scattering angle Ω in the two-nucleon center-of-mass system. The overlap matrix $\langle \varphi'_1 | \varphi_1^{+\mathbf{q}} \rangle$ is taken between $|\varphi_1^{+\mathbf{q}}\rangle = e^{i\mathbf{q}\cdot\mathbf{r}_1} |\varphi_1\rangle$ and $|\varphi'_1\rangle$, where $|\varphi_1\rangle$ and $|\varphi'_1\rangle$ are the initial and final states of the $N_1 + B_1$ system, respectively, and the operator $e^{i\mathbf{q}\cdot\mathbf{r}_1}$ gives the momentum transfer to the nucleon N_1 . The clusterized states $|\varphi'_1\rangle$ and $|\varphi'_2\rangle$ are approximated by the simple harmonic oscillator $(0s)^n$ configuration with the oscillator constant associated with the wave packet width ν in AMD, so that any final state of the collision is represented by an AMD wave function. By identifying the two-nucleon matrix element M with that of the usual two-nucleon scattering, the cluster-forming cross section can be expressed by using the two-nucleon collision cross section that is an input to the calculation.

The actual situation of a two-nucleon collision requires more considerations because there are many possible ways of forming a cluster with different nucleons (B's) in the system for each of the scattered nucleons N_1 and N_2 . It is important to take care of the non-orthogonality of the final states corresponding different ways of cluster formation.

The procedure is repeated for the cluster formation with nucleons (B's) with other spin and isospin directions until formation of an α cluster is considered. The particle N should be regarded as a cluster, instead of a nucleon, if a (sub)cluster has been already formed in previous steps of the repetition.

Many of light nuclei (Li, Be *etc.*) have only one or a few bound states which may be regarded as bound states of internal clusters. Then the same issue exists about the mismatch of classical and quantum-mechanical density of states. Therefore, for a better description, inter-cluster correlation is introduced as a stochastic process of bonding clusters. The relative momentum between clusters is replaced by zero if moderately separated clusters ($R_{\text{rel}} < 5$ fm) are moving away from each other with a small relative momentum ($\mathbf{R}_{\text{rel}} \cdot \mathbf{P}_{\text{rel}} < 0$ and $P_{\text{rel}}^2/2\mu < 8$ MeV).

3. Cluster emission in fragmentation reactions

This version of the AMD approach with cluster correlations has been shown to be able to describe the important features in multifragmentation reactions at $E \gtrsim 50$ MeV/nucleon reasonably well, including the yields of protons, clusters and heavier nuclei [20]. In fact, the effect of cluster correlations is so strong that it changes not only the yields of emitted clusters but also the whole dynamics of expansion and fragmentation of the system.

We choose here a typical central $^{124}\text{Sn} + ^{124}\text{Sn}$ collisions at 50 MeV/nucleon. Such a neutron-rich system is suitable to see the effect of the density dependence of the symmetry energy or the parameter $L = 3\rho_0(dS/d\rho)_{\rho_0}$. In the early stage of collisions, the difference of the calculated neutron and proton densities in the high density region shows a very clear effect of the density dependence of the symmetry energy. In the late stage of the reaction, the calculation clearly shows that the symmetry energy influences the degree of fractionation, i.e., how much the gas part is neutron rich compared with the liquid part of the system. Fragment nuclei with $Z \gtrsim 3$ are usually regarded as belonging to the liquid part, while the gas part consists of emitted nucleons and light clusters. The effects in some final observables are shown in Ref. [21]. Direct information of the liquid part may be obtained from the isotope distribution of fragment nuclei. The width of the isotope distribution has been found to be sensitive to the symmetry energy parameter L , as well as the mean value of the distribution. However, in order to obtain a robust constraint on the L parameter from the comparison with experimental data, it is necessary to better understand the decay of primary fragments. On the other hand, direct information of the gas part should be observed in the yields and spectra of nucleons and light clusters. In fact,

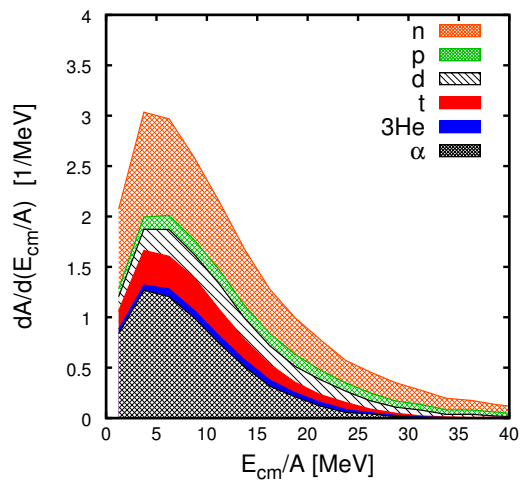


Figure 2. Calculated distributions of the kinetic energy per nucleon E_{cm}/A for the nucleons and clusters emitted in transverse directions $70^\circ < \theta_{\text{cm}} < 110^\circ$ in central $^{124}\text{Sn} + ^{124}\text{Sn}$ collisions at 50 MeV/nucleon. Each colored area shows the number of the specific cluster multiplied by the cluster mass number.

the dependence of the calculated ^3H and ^3He spectra on the L parameter can be understood as a consequence of fractionation that is sensitive to the symmetry energy at low densities [21].

In order to better understand the composition of the gas part, Fig. 2 shows the distributions of the kinetic energy per nucleon E_{cm}/A for the nucleons and clusters emitted in transverse directions $70^\circ < \theta_{\text{cm}} < 110^\circ$. Each colored area shows the number of the specific cluster multiplied by the cluster mass number A , so that the figure shows the decomposition of nucleons into clusters for each emission velocity. Except for the high velocity part, the α cluster formation is quite important since about half of gas nucleons are bound in α particles. The gas part is generally much more neutron-rich than the total system as a consequence of fractionation. For the lowest velocity part $0 < E_{\text{cm}} < 5$ MeV, the neutron-to-proton ratio is $N_{\text{gas}}/Z_{\text{gas}} = 2.07$ for this calculated case. As the α particles contain equal numbers of protons and neutrons, the gas part becomes more neutron-rich if the α particles are excluded. In fact, the neutron-to-proton ratio for $A \leq 3$ particles is $(N_{\text{gas}} - 2M_\alpha)/(Z_{\text{gas}} - 2M_\alpha) = 3.92$ in this case. This is the reason of the large n/p and $^3\text{H}/^3\text{He}$ ratios. In the calculated result with the SLy4 interaction ($L = 46$ MeV), the $^3\text{H}/^3\text{He}$ is about 2 for large velocity ($E_{\text{cm}}/A \sim 20$ MeV) while it becomes much larger (about 5-6) for low energies $0 < E_{\text{cm}}/A < 5$ MeV. This behavior is qualitatively similar to the experimental data from MSU [22] but the ratio observed in experiment is larger than the calculated ratio at low energies. However, it should be kept in mind that the $^3\text{H}/^3\text{He}$ ratio is influenced not only by the density dependence of the symmetry energy but also by the formation of α particles as mentioned above. In fact, a 10% change of α multiplicity for the low energy part will result in a 15% change of the $^3\text{H}/^3\text{He}$ ratio, if the total numbers of gas protons and neutrons do not change. Therefore, a careful comparison of α particle spectra between theory and experiment is quite important.

4. Summary

Clusters are important for the study of heavy-ion collisions to explore EOS, because formation and existence of light clusters influence very much the global reaction dynamics and the bulk nuclear matter properties. AMD has been extended to include cluster correlations in the final states of two-nucleon collisions. Some cluster observables, such as $^3\text{H}/^3\text{He}$ ratio, are sensitive to the density dependence of symmetry energy. It is important to describe precisely the α -particle formation for the study of nuclear symmetry energy.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Numbers 21540253 and 24105008. The numerical calculations were partly carried out on SR16000 at YITP in Kyoto University.

References

- [1] Tsang M B *et al* 2012 *Phys. Rev. C* **86** 015803
- [2] Horowitz C J, Brown E F, Kim Y, Lynch W G, Michaels R, Ono A, Piekarewicz J, Tsang M B and Wolter H H 2014 *J. Phys. G: Nucl. Part. Phys.* **41** 093001
- [3] Tsang M B, Zhang Y, Danielewicz P, Famiano M, Li Z, Lynch W G and Steiner A W 2009 *Phys. Rev. Lett.* **102** 122701
- [4] Qin L *et al* 2012 *Phys. Rev. Lett.* **108** 172701
- [5] Wada R *et al* 2012 *Phys. Rev. C* **85** 064618
- [6] Hagel K *et al* 2012 *Phys. Rev. Lett.* **108** 062702
- [7] Typel S, Röpke G, Klähn T, Blaschke D and Wolter H H 2010 *Phys. Rev. C* **81** 015803
- [8] Röpke G 2011 *Nucl. Phys. A* **867** 66
- [9] Coupland D D S, Lynch W G, Tsang M B, Danielewicz P and Zhang Y 2011 *Phys. Rev. C* **84** 054603
- [10] Danielewicz P and Beardsch G F 1991 *Nucl. Phys. A* **533** 712
- [11] Li B A, Yong G C and Zuo W 2005 *Phys. Rev. C* **71** 014608
- [12] Ferini G, Gaitanos T, Colonna M, Di Toro M and Wolter H H 2006 *Phys. Rev. Lett.* **97** 202301
- [13] Xiao Z, Li B A, Chen L W, Yong G C and Zhang M 2007 *Phys. Rev. Lett.* **102** 062502
- [14] Feng Z Q and Jin G M 2010 *Phys. Lett. B* **683** 140
- [15] Reisdorf W *et al* 2007 *Nucl. Phys. A* **781** 459
- [16] Reisdorf W *et al* 1997 *Nucl. Phys. A* **612** 493
- [17] Ono A, Horiuchi H, Maruyama T and Ohnishi A 1992 *Prog. Theor. Phys.* **87** 1185
- [18] Chabanat E, Bonche P, Haensel P, Meyer J and Schaeffer R 1998 *Nucl. Phys. A* **635** 231
Chabanat E, Bonche P, Haensel P, Meyer J and Schaeffer R 1998 *Nucl. Phys. A* **643** 441(E)
- [19] Li G Q and Machleidt R 1993 *Phys. Rev. C* **48** 1702
Li G Q and Machleidt R 1994 *Phys. Rev. C* **49** 566
- [20] Ono A 2013 *J. Phys.: Conf. Ser.* **420** 012103
- [21] Ono A 2013 *J. Phys.: Conf. Ser.* **436** 012068
- [22] Liu T X *et al.* 2012 *Phys. Rev. C* **86** 024605