A resonance problem on the low-lying resonant state in the ^{9}Be system

Myagmarjav Odsuren[1], **Kiyoshi Katō**[2], **Yuma Kikuchi**[3], **Masayuki Aikawa**[2] **and Takayuki Myo**[4,5]

[1]Nuclear Research Center, National University of Mongolia, Ulaanbaatar 210646, Mongolia
[2]Nuclear Reaction Data Centre, Faculty of Science, Hokkaido University, Sapporo 060-0810, Japan
[3]RIKEN Nishina Center, Wako, 351-0198, Japan
[4]General Education, Faculty of Engineering, Osaka Institute of Technology, Osaka 535-8585, Japan
[5]Research Centre for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan

E-mail: odsurenn@gmail.com

Abstract. The photo-disintegration cross section of ^{9}Be is investigated in a framework of the $\alpha + \alpha + n$ three-body cluster model. Much interest is concentrated on the nature of the $1/2^+$ state of ^{9}Be located just above the three-body threshold. The existence of this $1/2^+$ resonance is a long-standing problem and has been a topic of quite interest in relation to the breakup mechanism of ^{9}Be into the ^{8}Be(0^+)+n and $(\alpha + \alpha + n)$ channels. The newly measured breakup cross sections of the $1/2^+$ state of Ref. [1] is inconsistent with the old experimental values [2, 3]. The purpose of this study is to clarify the properties of the $1/2^+$ state of ^{9}Be with the possibility of a genuine three-body $(\alpha + \alpha + n)$ resonance or the two-body (^{8}Be(0^+)+n) virtual state.

1. Introduction

The neutron capture (n,γ) and its inverse (γ,n) reactions are the highly interesting topics in current nuclear physics, because these reactions play important roles in nuclear reactor physics and astronuclear physics. So far, the resonant structure [4, 5] and a virtual characteristic state [6] of ^{9}Be($1/2^+$) have been studied based on the various theoretical approaches and photo-dissociation experiments. However, a complete understanding on the nature of ^{9}Be($1/2^+$) state has not yet been obtained. It is desired to perform a more comprehensive study based on an $\alpha+\alpha+n$ three-body calculations which can treat the bound ground state and also the unbound resonant states of ^{9}Be in an unified framework. In this work, we apply the complex scaling method (CSM) [7] to an $\alpha+\alpha+n$ three-cluster model for understanding the nuclear structure and (γ,n) reactions for low-lying states in ^{9}Be.

For the purpose of this work we treat the unbound nature of the $1/2^+$ state of ^{9}Be and investigate the E1 transition between $1/2^+$ and $3/2^-$ states, which contributes to the (γ,n) reaction dominantly. We examine the resonance formation in the (γ,n) reaction to see whether a direct three-body breakup or a two-step process through the ^{8}Be+n intermediate resonant state is dominant.

2. Three-body model and complex scaling method

The Hamiltonian for the relative motion of the $\alpha+\alpha+n$ three-body system for ^{9}Be is given as

$$\hat{H} = \sum_{i=1}^{3} t_i - T_{c.m} + \sum_{i=1}^{2} V_{\alpha n}(\xi_i) + V_{\alpha\alpha} + V_3 + V_{PF}. \tag{1}$$

The Hamiltonian consists of the kinetic energy operators of the first and second terms, the two-body potential terms of subsystems for $\alpha+\alpha$ and $\alpha+n$, the three-body $\alpha+\alpha+n$ potential term and the last term that projects out the Pauli forbidden (PF) state from the relative motion. The details are given in Ref. [8]. The KKNN potential [9] and the folding potential of an effective nuclear and the Coulomb forces are applied to $V_{\alpha n}$ and $V_{\alpha\alpha}$, respectively. Here ξ_i is the relative coordinate between two clusters (α and n). The three-body (three-cluster) potential V_3 is explicitly given by the following one-range Gaussian form with the strength v_{3b}:

$$V_3(r_1,\ r_2) = \mathrm{v}_{3b} \exp{(-\mu\rho^2)}, \tag{2}$$

where ρ is the hyper-radius of the $\alpha+\alpha+n$ system.

The lowest threshold of ^{9}Be is a three-body breakup into $\alpha+\alpha+n$. Only the ground state of ^{9}Be is bound, so ^{9}Be is a Borromean system. We investigate the properties of the unbound states of ^{9}Be using the CSM. The CSM has been applied to many kinds of nuclear systems, particularly as a useful method to describe many-body resonance states together with bound states [7, 8, 10]. The complex-scaled Schrödinger equation is expressed using the complex-scaled Hamiltonian $H(\theta)$ as

$$H(\theta)\Psi(\theta) = E\Psi(\theta). \tag{3}$$

The relative coordinates $\vec{r}_j$ of the three-body system $\alpha+\alpha+n$ are transformed as $\vec{r}_j \to \vec{r}_j e^{i\theta}$ ($j = 1, 2$) with a real parameter θ. We can approximate the eigen function $\Psi^{\nu}_J(\theta)$ for the state ν with the total spin J in Eq. (3) using the following basis expansion:

$$\Psi^{\nu}_J(\theta) = \sum_{\beta=1}^{N} c^{\nu}_{\beta}(\theta)\psi^J_{\beta}, \tag{4}$$

where ψ^J_β is the basis function for three-body states of ^{9}Be and given as

$$\psi^J_\beta = \left[\left[\phi^{i_\beta}_{l_\beta}(\vec{r}^{\,c}_1) \otimes \phi^{j_\beta}_{\lambda_\beta}(\vec{r}^{\,c}_2)\right]_{L_\beta} \otimes \chi^{\sigma}_{1/2}\right]_J. \tag{5}$$

The index β represents a set of $\{c, i, l, j, \lambda, L\}$ where c =1 and 2 specifies a channel of $(\alpha+\alpha)+n$ and $(\alpha+n)+\alpha$, respectively. The orbital angular momenta l and λ are corresponding to the relative motion for $\vec{r}_1$ and $\vec{r}_2$, respectively, in the channel c. Furthermore, L is the total orbital angular momentum. The indices i and j are to distinguish the radial basis functions. We employ the Gaussian basis functions to describe the radial component for each partial wave:

$$\phi^k_\ell(\vec{r}) = N^k_\ell r^\ell e^{-\frac{a_k}{2}r^2} Y_\ell(\hat{r}), \qquad a_k = a_0\eta^{k-1}, \tag{6}$$

where the details are explained in Ref. [11].

The cross section $\sigma_{\gamma n}$ of the photo-disintegration

$$^9\mathrm{Be}(3/2^-) + \gamma \to n + \alpha + \alpha. \tag{7}$$

is calculated in terms of the multi-pole response can be expressed as the following form

$$\sigma^{\gamma}_{E\lambda}(E_\gamma) = \frac{(2\pi)^2(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(E\lambda, E_\gamma)}{dE_\gamma}. \tag{8}$$

Journal of Physics: Conference Series **569** (2014) 012072 doi:10.1088/1742-6596/569/1/012072

In this work, we consider the $E1$ transitions from the $3/2^-$ ground state to the unbound $1/2^+$ state in ^{9}Be using the complex-scaled Green's function [7, 8], which contribute to the ^{9}Be(γ,n) ^{8}Be cross section as

$$\sigma_{E1}^{\gamma}(E_\gamma) = \frac{16\pi^3}{9\hbar c} E_\gamma \frac{dB(E1, E_\gamma)}{dE_\gamma}. \tag{9}$$

3. Results and Discussion

The $3/2^-$ ground state of ^{9}Be is calculated at -2.16 MeV from the $\alpha+\alpha+n$ threshold with no three-body potential (v_{3b}=0) [12]. The two-cluster potentials $V_{\alpha n}$ and $V_{\alpha\alpha}$ are fixed so as to reproduce the experimental data of the corresponding sub-systems. Here, we introduce a repulsive three-body potential (v_{3b}=6.57 MeV, μ = 0.1 fm^{-2}) to fit the observed binding energy (-1.574 MeV) of ^{9}Be. The Hamiltonian Eq. (1) reproduces all the threshold energies of $\alpha+\alpha+n$, ^{8}Be+n and ^{5}He+α in the ^{9}Be nucleus. Using this Hamiltonian, we do not obtain the $1/2^+$ resonance of ^{9}Be. This result is the same as that obtained in Refs. [6, 12]. To clarify the resonance nature of the $1/2^+$ state, we carry out three-body calculations of the $1/2^+$ state by adding the three-body potential. With a strong attractive three-body potential with a negative value of v_{3b}, we can obtain the $1/2^+$ state as a bound state as shown in Fig. 1. From calculations for various values of v_{3b}, we find that the $1/2^+$ state becomes bound with the value of v_{3b} below -26 MeV.

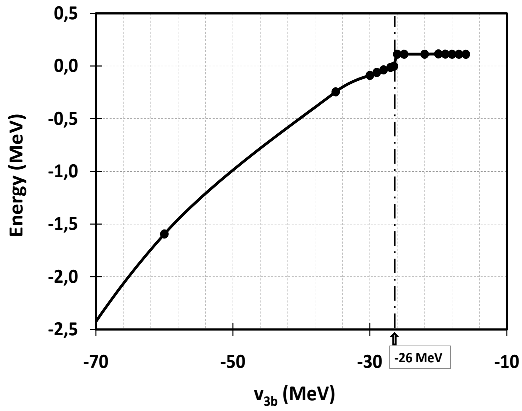

Figure 1. Energies of the $1/2^+$ state measured from the α+α+n threshold, as a function of the strength three-body potential v_{3b}.

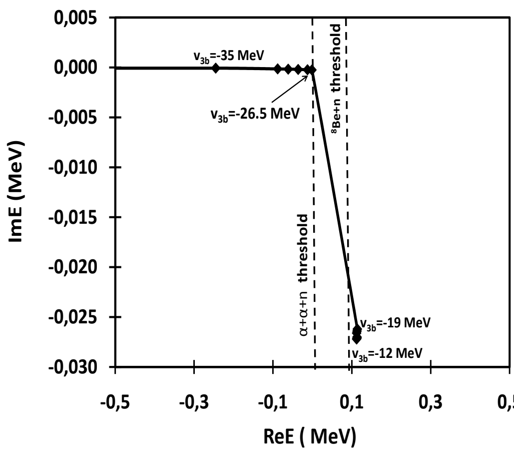

Figure 2. The eigenvalue trajectory of the $1/2^+$ state in the complex-energy plane.

Increasing the value of v_{3b} from -26 MeV gradually, we search for the $1/2^+$ resonance solutions in the CSM, whose eigenvalue trajectory is shown in Fig. 2. We see a jump from the $\alpha+\alpha+n$ threshold energy to a complex value, the real part of which corresponds to a energy just above the ^{8}Be+n threshold. This discontinuity of the trajectory may be understood by considering the properties of solutions in the CSM. In the CSM, resonance poles existing below the 2θ-line of continuum states starting from the lowest threshold of $\alpha+\alpha+n$ cannot be obtained as isolated eigenvalues. Because of the analyticity condition for the present complex scaled Hamiltonian, the θ value must be smaller than 45° [7]. For virtual states (whose energy eigenvalue is located at the negative real axis of the second Riemann surface), the angle θ has to be larger than 90°. When a virtual state exists in the ^{8}Be+n channel, a jump is expected in the resonance trajectory as discussed in Ref. [13]. The present case is not a two-body system

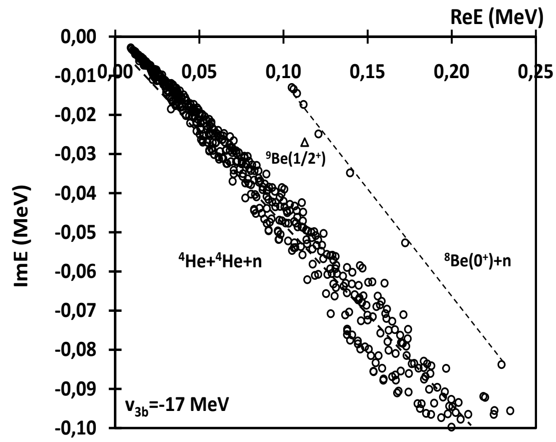

Figure 3. The $1/2^+$ energy eigenvalue distribution in the complex-scaled $\alpha + \alpha + n$ model.

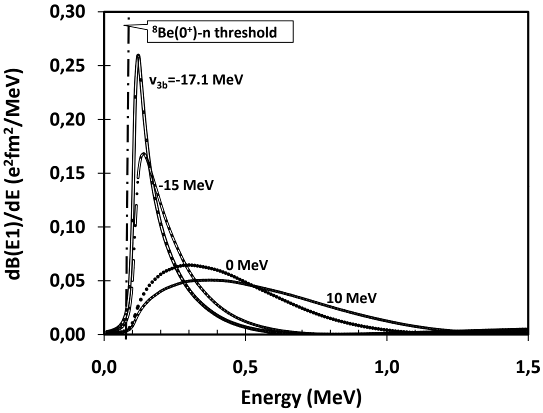

Figure 4. The $E1$ transition strength for ^{9}Be$(3/2^- \to 1/2^+)$ as a function of energy E, changing the strength v_{3b} of the three-body potential.

but the $\alpha+\alpha+n$ three-body one. It is difficult to trace the jump behavior of the pole solutions in the CSM.

Figure 3 displays an example of the eigenvalue distribution of ^{9}Be$(1/2^+)$ for v_{3b}=-17 MeV with $\theta = 15°$. Besides the triangle corresponding to the resonance, all solutions for continuum states described by open circles lie on two straight lines starting from positions of the $\alpha+\alpha+n$ three- and ^{8}Be$(0^+)+n$ two-body thresholds.

Using the solutions of $1/2^+$ and ground states, we calculate the $E1$ transition strength to investigate the effect of the $1/2^+$ resonance of ^{9}Be. In Fig. 4, the results are shown as a function of the energy measured from the $\alpha + \alpha + n$ threshold for several values of the three-body strength v_{3b}. We applied attractive ($\mathrm{v}_{3b} = -15$ and -17.1 MeV) or repulsive ($\mathrm{v}_{3b} = 0$ and 10 MeV) three-body potentials. The $E1$ transition strength for $\mathrm{v}_{3b} = -17.1$ MeV rises sharply from the ^{8}Be$(0^+)+n$ two-body threshold. The strength reaches a maximum of 0.26 e^2fm^2/MeV

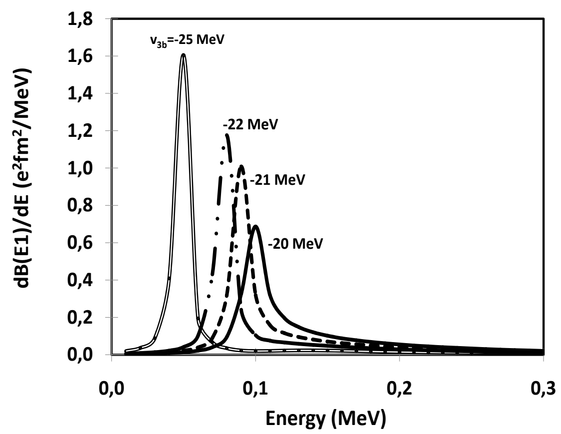

Figure 5. The $E1$ transition strength of ^{9}Be$(3/2^- \to 1/2^+)$ calculated for $\mathrm{v}_{3b} = -20,\ -21,\ -22,\ -25$ MeV.

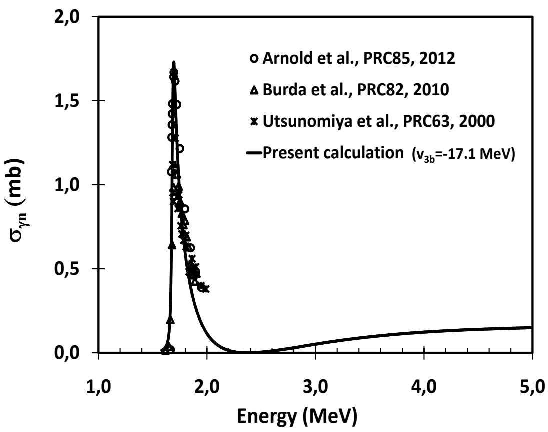

Figure 6. The ^{9}Be$(\gamma, n)^8$Be cross section $\sigma_{\gamma n}$ as a function of excitation energy E. The experimental data are taken from Refs. [1, 2, 3].

at $E = 0.11$ MeV, and decreases to 0.01 $e^2\text{fm}^2$/MeV at $E = 0.5$ MeV. As v_{3b} increases, the peaks of the $E1$ transition strength decrease gradually and move into higher energies. In the cases of v_{3b}=0 and 10 MeV, the peaks of the $E1$ strength are broad and their energy positions are distant from the $^8\text{Be}(0^+) + n$ threshold. It is noted that the $E1$ transition strength is very small in the energy region below the $^8\text{Be}(0^+) + n$ two-body threshold.

The $E1$ transition strengths calculated for $\text{v}_{3b} = -25 \sim -20$ MeV are shown in Fig. 5. They have strengths in the energy region between $\alpha + \alpha + n$ and the $^8\text{Be}(0^+) + n$ thresholds. Each $E1$ transition strength seems to be expressed by the Breit-Wigner form. In the case of $\text{v}_{3b} < -25$ MeV, where the $1/2^+$ state becomes a bound state, the $E1$ transition strengths have not a characteristic structure around $\alpha + \alpha + n$ and $^8\text{Be}(0^+) + n$ threshold energies.

Comparing the calculated photo-disintegration cross section of the $1/2^+$ state with the recent new experiment [1], we find that $\text{v}_{3b} = -17.1$ MeV gives a good agreement as shown in Fig. 6. Using this v_{3b}, we obtain $1/2^+$ as a three-body resonance. This result indicates that the experimental cross section can be explained in terms of the $1/2^+$ resonance of ^{9}Be.

4. Summary

In this work, we investigate the properties of the $1/2^+$ state of ^{9}Be varying values of three-body potential. Without the three-body potential, we cannot obtain the $1/2^+$ resonance of ^{9}Be, which is consistent to the results in Refs. [6, 12]. Using the attractive three-body potential, we can obtain the $1/2^+$ resonance, which is responsible to explain the $E1$ transition strength of ^{9}Be with an enhancement near the $\alpha + \alpha + n$ threshold energy. The calculated photo-disintegration cross section is in good agreement with new experimental data [1] using the resonance solution of $^9\text{Be}(1/2^+)$ with an appropriate strength of the three-body potential.

Acknowledgements

The numerical calculations were done with use of computing system of the Nuclear Research Centre, National University of Mongolia and Theoretical Nuclear Physics Laboratory, Hokkaido University. Two of the authors, M.O. and K. K., are thankful for the support from a Grant-in Aid Scientific Research (No. 25400241) of the Ministry of Education, Science and Culture, Japan.

References

[1] Arnold C W, Clegg T B, Iliadis C, Karwowski H J, Rich G C, Tompkins J R and Howell C. R 2012 *Phys. Rev.* C **85** 044605
[2] Burda O, von Neumsnn-Cosel P, Richter A, Forssén C and van Overstraeten Brown B A 2010 *Phys. Rev.* C **82** 015808
[3] Utsunomiya H, Yonezawa Y, Akimune H, Yamagata T, Ohta M, Fujishiro M, Toyokawa H and Ohgaki H 2000 *Phys. Rev.* C **63** 018801
[4] Efros V D, von Neumann-Cosel P and Richter A 2014 *Phys. Rev.* C **89** 027301
[5] Gariddo E, Fedorov D V and Jensen A S 2010 *Phys. Lett.* B **684** 132
[6] Arai K, Descouvemont P, Baye D and Catford W N 2003 *Phys. Rev.* C **68** 014310
[7] Aoyama S, Myo T, Katō K and Ikeda K 2006 *Prog. Theor. Phys.* **116** 1
[8] Myo T, Katō K, Aoyama S and Ikeda K 2001 *Phys. Rev.* C **63** 054313
[9] Kanada H, Kaneko T, Nagata S and Nomoto M 1979 *Prog. Theor. Phys.* **61** 1327
[10] Odsuren M, Katō K, Aikawa M and Myo T 2014 *Phys. Rev.* C **89** 034322
[11] Hiyama E, Kino Y and Kamimura M 2003 *Prog. Part. Nucl. Phys.* **51** 223
[12] Kato M, Master Thesis in Hokkaido University (March, 2012) and private communication.
[13] Masui H, Aoyama S, Myo T Katō K, and Ikeda K 2000 *Nucl. Phys.* A **673** 207