Journal of Physics: Conference Series **569** (2014) 012070 doi:10.1088/1742-6596/569/1/012070

A solution to the old puzzle of 0^+ resonances above the 0_2^+ Hoyle state in ^{12}C
— *A new analysis with complex-scaled 3α OCM* —

S Ohtsubo[1], Y Fukushima[1], M Kamimura[2] and E Hiyama[2]

[1] Department of Applied Physics, Fukuoka University, Fukuoka 814-0180, Japan
[2] RIKEN Nishina Center, RIKEN, Wako 351-0198, Japan

E-mail: mkamimura@riken.jp

Abstract. We have developed the complex-scaling method (CSM) by using the complex-range (or oscillating) Gaussian basis functions that are suited for describing highly oscillating few-body wave functions. The eigenvalue distribution of the complex scaled Hamiltonian becomes much more precise and the maximum scaling angle becomes drastically larger than those given by the use of real-range Gaussians. Owing to this advantage, we were able to isolate the S-matrix pole of the new broad 0_3^+ resonance from the 3α continuum. This confirms the Kurokawa-Kato's prediction (2005) of the new 0_3^+ resonance, which is considered to correspond to the newly observed 0_3^+ resonance ($E_x = 9.04$ MeV,$\Gamma = 1.45$ MeV) by Itoh *et al.* (2013). As a result the long-standing puzzle for the 0^+ and 2^+ resonances above the 0_2^+ Hoyle state in ^{12}C was solved. In this paper, the negative parity resonances with $J = 1^-, 2^-, 3^-, 4^-$ and 5^- are newly calculated.

1. Introduction

There was a long-standing puzzle, as shown in Fig. 1, for the 0^+ and 2^+ resonances above the 0_2^+ Hoyle state in ^{12}C; observation [1] and the typical 3α-cluster-model calculations are inconsistent. Recently, this puzzle has been almost solved by the new cluster-model calculation by Kurokawa and Kato [2] and the new experiment by Itoh *et al.* [3] as shown in Fig. 2.

The authors of Ref. [2] predicted a new broad 0_3^+ resonances at $E_{\rm x} = 8.95$ MeV with $\Gamma = 1.48$ MeV using the complex-scaling method (CSM) (see Ref. [4] for a review) within the 3α orthogonality-condition model (OCM). However, the prediction is not very satisfactory from the following point: Their CSM calculation was performed at the scaling angle of 16° that is the largest angle available in their calculation using the real-range Gaussian basis. Since the angle is not enough for separating the low-lying broad 0_3^+ resonance from the continuum eigenvalues, they made an extrapolation by the method of analytic continuation of the coupling constant (ACCC) [5] to derive the complex energy of the 0_3^+ resonance (cf. Fig. 6 below).

In order to overcome this difficulty and confirm the prediction in Ref. [2], we proposed [6] to use the complex-range Gaussian basis functions [7, 8],

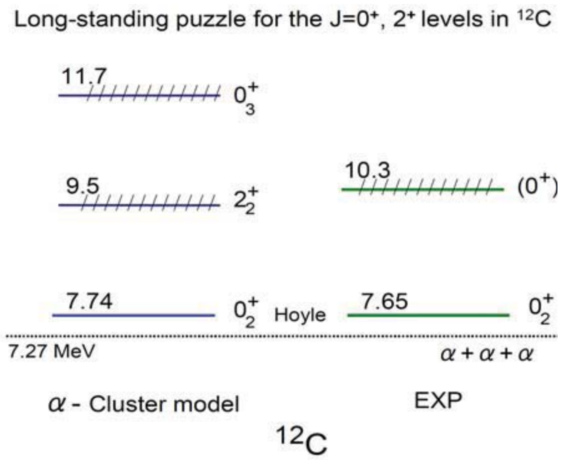

Figure 1. The 0^+ and 2^+ resonances in ^{12}C above the 0_2^+ Hoyle state that were inconsistent between the experiments [1] and the typical cluster-model calculations.

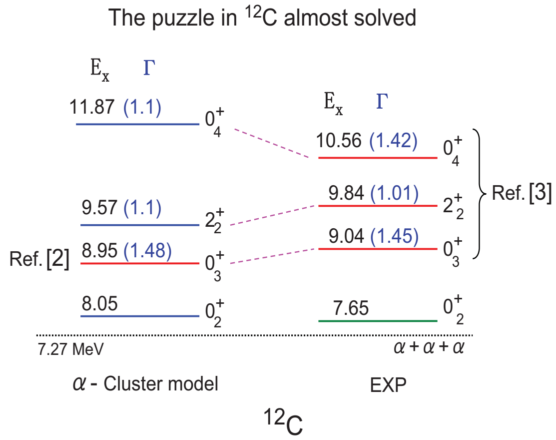

Figure 2. The new cluster-model calculation for the 0_3^+ resonance [2] and the new experiment for the 0_3^+, 2_2^+ and 0_4^+ resonances [3]. All the calculated results are cited from Ref. [2]. The energy $E_{\rm x}$ and the width Γ are in MeV.

$$r^l\, e^{-(1\pm i\,\omega)(r/r_n)^2} Y_{lm}(\widehat{\mathbf{r}}), \qquad \text{or equivalently} \qquad r^l\, e^{-(r/r_n)^2} \Big\{ {\cos\omega(r/r_n)^2 \atop \sin\omega(r/r_n)^2} \Big\} Y_{lm}(\widehat{\mathbf{r}}) \tag{1}$$

with r_n in a geometric progression, in the calculation of three-body resonances with CSM in which use is often made of the real-range Gaussian basis functions [9, 7, 4],

$$r^l\, e^{-(r/r_n)^2} Y_{lm}(\widehat{\mathbf{r}}). \tag{2}$$

Though the latter set is suitable for describing the short-distance structure and the asymptotic decaying behavior of few-body systems [7], the former basis set is more powerful when describing the resonant and non-resonant continuum states in CSM with highly oscillating amplitude at large scaling angles θ.

We applied the new basis functions (both for the two Jacobi coordinates, $\mathbf{r}$ and $\mathbf{R}$) to the 3α OCM-CSM calculation [6] of the 3α resonances with $J = 0^+, 2^+$ and 4^+ in ^{12}C (see Ref. [6] for details). In this paper, we newly calculate the negative parity resonances with $J = 1^-, 2^-, 3^-, 4^-$ and 5^-. We took the same model and the Hamiltonian as used in the 3α OCM-CSM calculation in Ref. [2].

2. The broad new 0_3^+ resonance

Figure 3, taken from Ref. [2], shows the 0^+ eigenvalue distribution of the complex scaled Hamiltonian calculated with the real-range Gaussian basis functions. The scaling angle $\theta = 16°$ was the maximum angle available in the calculation. The new broad 0_3^+ state was predicted at $E_{\rm res} = 1.66 - i\,0.74$ MeV ($E_{\rm x} = 8.95 - i\,0.74$ MeV) but this complex energy is not isolated from the continuum states in Fig. 3 at $\theta = 16°$. The energy was derived by an extrapolation based on the method of analytic continuation of the coupling constant [5] (cf. Fig. 6 below).

Figure 4 illustrates the result of our CSM calculation [6] for the $J = 0^+$ states at $\theta = 16°$ and $26°$. The distribution of the S-matrix becomes very much improved and precise. However, even at $\theta = 26°$, the 0_3^+ resonance is not isolated from the continumm states.

Journal of Physics: Conference Series **569** (2014) 012070 doi:10.1088/1742-6596/569/1/012070

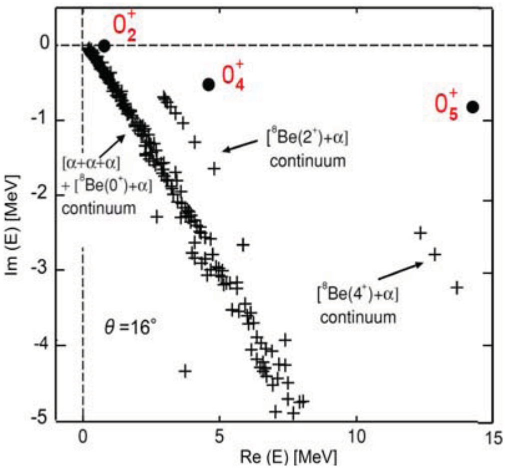

Figure 3. The 0^+ eigenvalue distribution of the complex scaled Hamiltonian for the 3α system obtained by Kurokawa and Katō [2] using the real-range Gaussian basis functions. The scaling angle is $\theta = 16°$. The 0_3^+ state was predicted at $E = 1.66 - i\,0.74$ MeV (see the text).

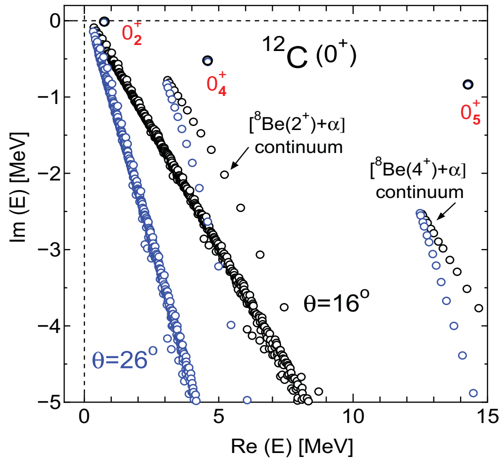

Figure 4. The 0^+ eigenvalue distribution of the complex scaled Hamiltonian for the 3α system with the use of the complex-range Gaussian basis functions [6]. The scaling angles are $\theta = 16°$ (black) and $26°$ (blue). This figure is to be compared with Fig. 3.

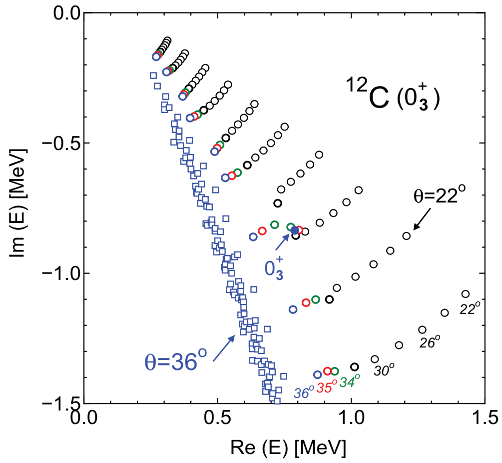

Figure 5. The 0^+ eigenvalue distribution of the complex scaled Hamiltonian, where the angle θ is varied from $22°$ to $36°$ [6]. The 0_3^+ resonance appears, as the closed blue circle ($36°$), at $E = 0.79 - i\,0.84$ MeV. Only for $\theta = 36°$ both the continua of $[\alpha + \alpha + \alpha]$ (open blue boxes) and $[^8\text{Be}(0^+) + \alpha]$ (open blue circles) are illustrated, but the former is omitted for $\theta < 36°$ for clarity of the figure.

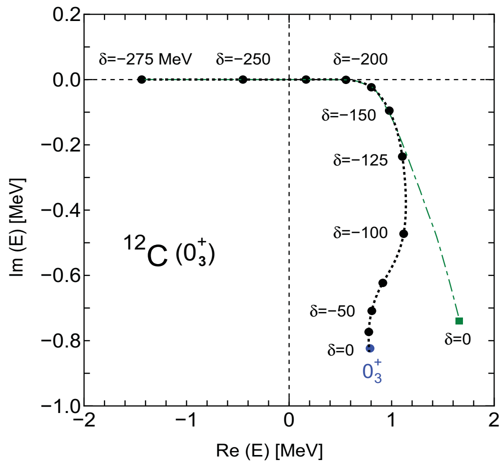

Figure 6. Trajectory of the 0_3^+ state obtained by changing the strength parameter δ of the auxiliary three-body potential [6]. The blue closed circle at $\delta = 0$ corresponds to the 0_3^+ resonance in Fig. 5. The green box denotes the 0_3^+ state predicted by Kurokawa and Katō [2] on the basis of the extrapolation (the dash-dotted green curve) using the ACCC+CSM.

Journal of Physics: Conference Series **569** (2014) 012070 doi:10.1088/1742-6596/569/1/012070

We illustrate, in Fig. 5, the 0^+ distribution of complex eigenvalues. Only for $\theta = 36°$ both the $[\alpha+\alpha+\alpha]$ continuum (open blue boxes) and the $[^8\mathrm{Be}(0^+)+\alpha]$ continuum (open blue circles) are given, but the former is omitted for $\theta < 36°$ to avoid complexity of the figure. We observe a converged resonance pole at $E_{\mathrm{res}} = 0.79 - i\,0.84$ MeV as shown by the closed blue circle. We identify it as the third 0^+ state that was predicted in Ref. [2]. The position and width of the resonance, however, differ slightly from the result of Ref. [2], $E_{\mathrm{res}} = 1.66 - i\,0.74$ MeV.

We explain the reason of this difference with Fig. 6 which is to be compared with Fig. 2 of Ref. [2]. Fig. 6 illustrates the trajectory of the 0_3^+ state on the complex energy plane, which was obtained by changing the strength parameter δ of the auxiliary three-body potential $V_{\mathrm{aux.}} = \delta \exp[-\mu(r_1^2 + r_2^2 + r_3^2)]$ with $\mu = 0.15\ \mathrm{fm}^{-2}$ (Eq. (4) of Ref. [2]) that is added to the 3α OCM Hamiltonian. The closed blue circle for $\delta = 0$ in Fig. 6 is the same as that for the 0_3^+ resonance in Fig. 5. On the other hand, in Ref. [2], the direct CSM calculation of the 0_3^+ resonance was not possible when the auxiliary 3α potential is less attractive than $\delta = -120$ MeV. The green box that indicates the 0_3^+ state of Ref. [2] was therefore estimated by the extrapolation (the dash-dotted green curve) using the ACCC+CSM. We thus understand that the difference in the resonance-pole position between the two calculations comes from the error of the extrapolation by the ACCC.

We concluded [6] that we confirmed the prediction by Kurokawa-Katō [2] about the appearance of a new 0_3^+ broad resonance which is located slightly above the Hoyle state (0_2^+).

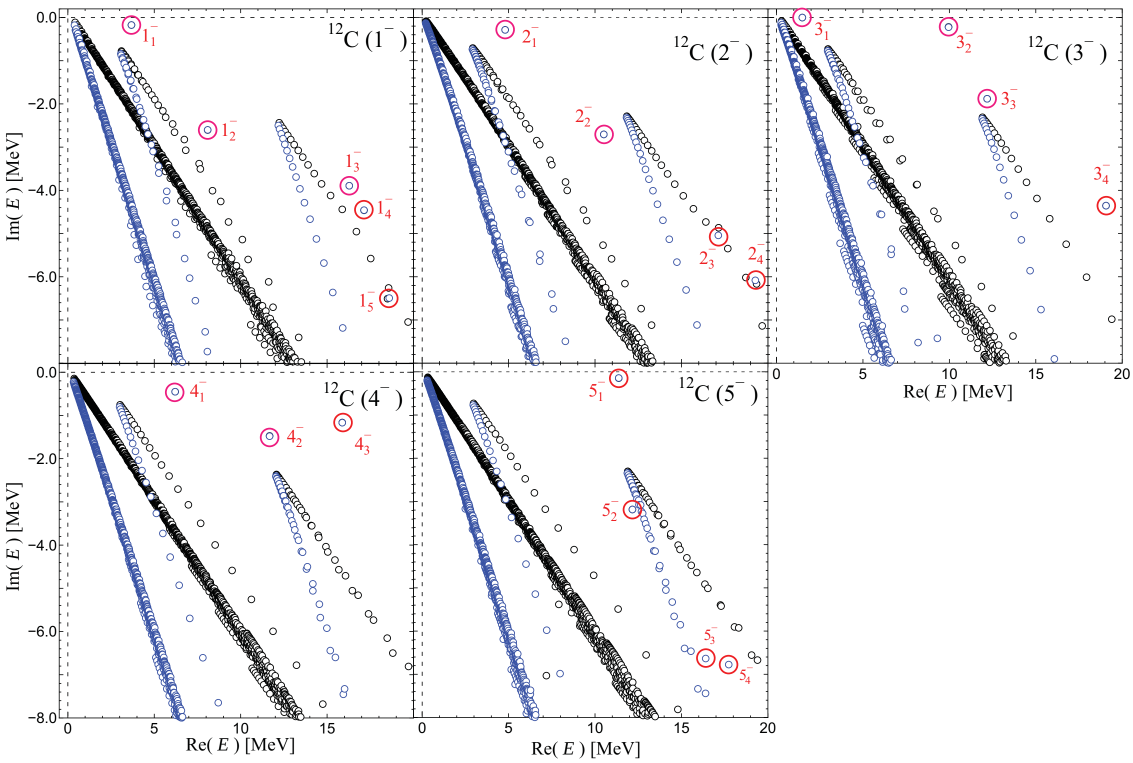

Figure 7. The eigenvalue distribution of the complex scaled Hamiltonian for $J = 1^-, 2^-, 3^-, 4^-$ and 5^- states of the 3α system with the use of the complex-range Gaussian basis functions [6]. The scaling angles are $\theta = 16°$ (black) and $26°$ (blue). The resonance poles are stable against the change of the scaling angle, and their energies and widths are summarized in Table 1.

Journal of Physics: Conference Series **569** (2014) 012070 doi:10.1088/1742-6596/569/1/012070

3. Negative-parity states

In this paper, we newly calculated the negative parity resonances with $J = 1^-, 2^-, 3^-, 4^-$ and 5^- using the same model and interactions as those in Refs. [2, 6]. Figure 7 shows the eigenvalue distribution of the complex scaled Hamiltonian for the negative-parity states at the scaling angles $\theta = 16°$ (black) and 26° (blue).

The resonance poles are stable against the change of the scaling angle, and their energies and widths are summarized in Table 1 together with the calculated result by Kurokawa and Kato [2] and the experimental data [1]. Owing to the advantage of using the complex-range Gaussian basis functions, the very broad resonances that were not given in Ref. [2] are obtained. The experimental data for $J = 1^-_1, 2^-_1, 3^-_1$ and 4^-_1 are reproduced well. Further observation of the negative-parity states are expected to examine our prediction in Table 1.

Table 1. Summary of the negative parity resonaces in ^{12}C by the present work together with the result by Ref. [2] and the experimental data [1]. All quantities are given in MeV.

^{12}C	present work			Ref. [2]			Experimental data [1]		
J^π	E_x	E_r	Γ	E_x	E_r	Γ	E_x	E_r	Γ
3^-_1	8.79	1.49	2.1×10^{-3}	8.80	1.51	2.0×10^{-3}	9.641(5)	2.366	3.4×10^{-2}
1^-_1	10.98	3.68	0.35	10.94	3.65	0.30	10.844(16)	3.569	0.315(25)
2^-_1	12.10	4.80	0.57	11.97	4.68	0.42	11.828(16)	4.553	0.260(25)
4^-_1	13.51	6.21	0.45	12.45	5.16	0.12	13.352(17)	6.077	0.375(40)
1^-_2	15.40	8.10	5.20						
3^-_2	17.25	9.95	0.45	18.29	11.0	0.5			
2^-_2	17.80	10.50	5.40	16.60	9.31	4.65			
5^-_1	18.66	11.36	0.29	18.69	11.4	0.3			
4^-_2	18.97	11.67	2.93						
5^-_2	19.47	12.17	6.37						
3^-_3	19.49	12.19	3.67						
4^-_3	23.15	15.85	2.34						
1^-_3	23.58	16.28	7.79						
5^-_3	23.70	16.40	13.27						

References

[1] Ajzenberg-Selobe F 1990 *Nucl. Phys.* A **506** 1
[2] Kurokawa C and Katō K 2005 *Phys. Rev.* C **71** 021301(R); 2007 *Nucl. Phys.* A **792** 87
[3] Itoh M *et al.* 2011 *Phys. Rev.* C **84** 054308
[4] Aoyama S, Myo T, Katō K and Ikeda K 2006 *Prog. Theor. Phys.* **116** 1
[5] Kukulin V I and Krasnopol'sky V M 1977 *J. Phys.* A **10** L33
[6] Ohtsubo S Fukushima Y Kamimura M and Hiyama E 2013 *Prog. Theor. Exp. Phys.* **2013** 073D02
[7] Hiyama E Kino Y and Kamimura K 2003 *Prog. Part. Nucl. Phys.* **51** 223
[8] Hiyama E 2012 *Prog. Theor. Exp. Phys.* **2012** 01A204
[9] Kamimura M 1988 *Phys. Rev.* A **38** 621