Role of tensor force in light nuclei with tensor-optimized shell model

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Abstract. We investigate the structures of light nuclei focusing on the role of the tensor force. We describe the tensor and short-range correlations with the tensor optimized shell model (TOSM) and the unitary correlation operator method (UCOM), respectively. In the TOSM, the 2p2h states are fully optimized to describe the large tensor contribution that brings high momentum components explicitly in the wave function. We use a bare nucleon-nucleon interaction AV8$'$ and discuss the structures of the He and Li isotopes, such as the excitation energy spectra and the radii. We also investigate the structures of ^{8}Be from the ground-band states to the highly excited states. These states of ^{8}Be are indicated to have different structures of the 2α clustering states and the shell-model like states.

1. Introduction

One of the important issues in nuclear physics is to understand the nuclear structure in connection with the properties of the nucleon-nucleon (NN) interaction. The nucleon-nucleon (NN) interaction has strong tensor forces at long and intermediate distances caused by the pion exchange, and also strong central repulsions at short distance caused by the quark dynamics [1, 2]. It is important to investigate the nuclear structure by treating these characteristics of the NN interaction. The tensor force mainly comes from the pion exchange interaction, which brings the high momentum component in nuclei. The tensor force is of intermediate range and we are able to express the strong tensor correlation in a reasonable shell model space[3]. We name this method as tensor optimized shell model (TOSM), in which the wave function is constructed in terms of the shell-model basis states with full optimization of the two particle-two hole (2p2h) states. The spatial shrinkage of the particle states is essential to obtain convergence of the tensor contribution involving high momentum, and we do not put any truncation to describe the particle states in TOSM. The other is the unitary correlation operator method (UCOM) to treat the short-range correlation coming from the short-range NN repulsion [4]. We shall combine two methods, TOSM and UCOM, to describe nuclei using bare interaction. In this paper, we report our recent applications of TOSM to the He and Li isotopes and discuss their

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structures focusing on the roles of the tensor force on the energies and configurations. We also discuss the structure of ^{8}Be from the ground-band states to the highly excited states in the aspects of the $\alpha - \alpha$ clustering and the shell-model like states.

2. Tensor Optimized Shell Model (TOSM)

We employ the Hamiltonian having the bare nucleon-nucleon interaction, AV8′[1]. We explain the TOSM wave function Ψ as

$$\Psi \;=\; \sum_{k_0} A_{k_0} |0\text{p}0\text{h}\rangle_{k_0} + \sum_{k_1} A_{k_1} |1\text{p}1\text{h}\rangle_{k_1} + \sum_{k_2} A_{k_2} |2\text{p}2\text{h}\rangle_{k_2}. \tag{1}$$

The 1p1h and 2p2h states have the various radial components for particle states (p) which are distinguished by the labels k_0, k_1 and k_2, respectively. The variational coefficients are given as $\{A_k\}$ and are determined by the diagonalization of the Hamiltonian matrix elements. The hole states (h) are described by harmonic oscillator basis states in the TOSM. For the p-shell nuclei, the hole states are defined as the $0s$ and $0p$ shells and the other higher shells are tread as the particle states. The 0p0h states are regarded as the standard shell-model state with A nucleons. In the TOSM, we consider the $0s$+$0p$ space of the hole states and allow up to the two particle excitations from the $0s$ orbit to the $0p$ orbits owing to the presence of the two-body interactions. From each configurations in the 0p0h states, up to two nucleons can be excited to the particle states to make the 1p1h and 2p2h configurations in the TOSM. The orbits of particle states are taken as much as possible until we get the convergence of the solutions such as the energy and the configurations. Therefore, there is no truncation of the particle states included in the 1p1h and 2p2h states in the TOSM.

For the description of the particle states, we employ the Gaussian basis functions to express single-particle states in the higher shell. We prefer the Gaussian wave functions over the shell-model states in order to effectively include the necessary high-momentum components due to the tensor force [3, 5]. When we superpose a sufficient number of Gaussian wave functions with various length parameters, the radial components of the particle states can be fully expressed. Gaussian basis states should be orthogonalized to the hole states and among themselves. This condition is imposed by using the Gram-Schmidt orthonormalization. We construct the following orthonormalized single-particle basis function ψ^n_α using a linear combination of Gaussian bases $\{\phi_\alpha\}$ with length parameter $b_{\alpha,i}$:

$$\psi^n_\alpha(\boldsymbol{r}) \;=\; \sum_{i=1}^{N_\alpha} d^n_{\alpha,i}\; \phi_\alpha(\boldsymbol{r}, b_{\alpha,i}), \tag{2}$$

$$\langle \psi^n_\alpha | \psi^{n'}_{\alpha'} \rangle \;=\; \delta_{n,n'}\; \delta_{\alpha,\alpha'}, \qquad \text{for}\;\; n \;=\; 1, \cdots, N_\alpha, \tag{3}$$

where N_α is a number of basis functions for the orbit α, and i is an index to distinguish the bases with a Gaussian length of $b_{\alpha,i}$. The explicit form of the Gaussian basis function is written as

$$\phi_\alpha(\boldsymbol{r}, b_{\alpha,i}) \;=\; N_l(b_{\alpha,i}) r^l e^{-(r/b_{\alpha,i})^2/2} [Y_l(\hat{\boldsymbol{r}}), \chi^\sigma_{1/2}]_j, \tag{4}$$

where l and j are the orbital and total angular momenta of the basis states, respectively. $N_l(b_{\alpha,i})$ is a normalization factor. The weight coefficients $\{d^n_{\alpha,i}\}$ are determined to satisfy the overlap condition in Eq. (3). Following this procedure, we obtain the new single-particle basis states $\{\psi^n_\alpha\}$ in Eq. (2) used in the TOSM. The particle states in 1p1h and 2p2h states are prepared to specify the basis wave functions, whose amplitudes are determined by the variational principle. In the numerical calculation, we prepare at most 10 Gaussian basis functions with various range parameters to get a convergence of the energy and Hamiltonian components.

We use short-range UCOM[4] to include the short-range correlation caused by the NN interaction in the TOSM basis states. We employ the form of the correlator of UCOM in the same manner as proposed by Ref. [4] for four channels of spin-isospin pairs. The parametrization of the correlator is explained in our previous papers on He and Li isotopes [6, 7].

For energy variation, we minimize the total energies E by optimizing the amplitudes $\{A_k\}$ in Eq. (1) and the length parameters $\{b_{\alpha,i}\}$ of the Gaussian bases for all the hole states and the particle states in each J^π state of the particular nucleus. The variational equations are given by

$$\frac{\partial \langle \Psi | H - E | \Psi \rangle}{\partial b_{\alpha,i}} = 0 \, , \qquad \frac{\partial \langle \Psi | H - E | \Psi \rangle}{\partial A_{k_j}} = 0 \qquad \text{for } j = 0, 1, 2. \tag{5}$$

3. Results of TOSM+UCOM

3.1. He and Li isotopes

We explain the results of He and Li isotopes, where the partial wave of the particle states is taken as 10 to get a sufficient convergence. The excitation energies of He isotopes are shown in Fig. 1, and also shown in Fig. 2 for Li isotopes. For the excitation energies, we see good agreement with the experimental ones, so that we can discuss the structure differences between energy levels. We also predict some energy levels, which has not been observed experimentally yet or not settled to assign the spin of the states, in particular, in the neutron-rich side, ^{7,8}He and ^{8,9}Li. In most of the He and Li isotopes, the ground states have the largest contributions of the tensor force for a particular nucleus [6, 7]. The tensor contributions are dominantly produced by the 2p2h excitations into the particle states from the 0p0h hole states. The corresponding high momentum components induced by the tensor force are confirmed as the enhancement of the kinetic energies from the values of the naive shell model configurations.

It is also found that the resulting level spacing of the Li isotopes is slightly compact than the experimental spectra. For example, in ^{9}Li, the small energy difference between the lowest $3/2^-$ and $1/2^-$ states can be seen. These characteristics are commonly obtained in the GFMC calculation within the two-body interaction level [1]. The additional genuine three-body interaction can be one of the components to reproduce the experimental situation.

In the TOSM, the length parameters of the basis states are determined variationally, hence we can discuss the radial properties of nuclei. The matter radii of He and Li isotopes are shown in Fig. 3 and the systematic trend agrees with the experiments. For ^{6}He and ^{8}He, the results are good to explain the enhancement of radius from ^{4}He because of loosely binding extra neutrons in two nuclei, which are however, slightly smaller than the experiments. This difference can be recovered including the continuum effect of extra neutrons. The value of ^{6}Li is also smaller than the experiment, which can be recovered including the asymptotic α+d clustering component in addition to the shell model basis states in the TOSM.

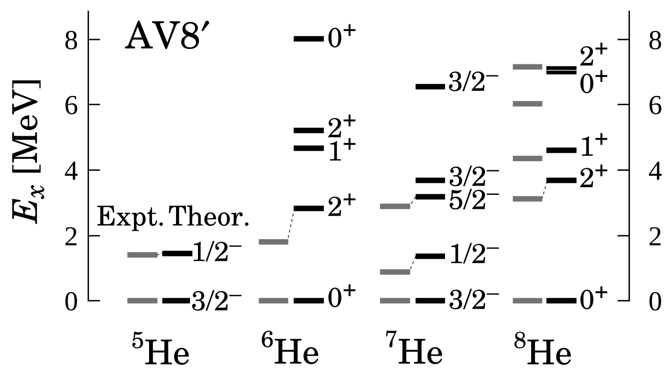

Figure 1. Excitation energies of He isotopes with TOSM+UCOM using AV8′.

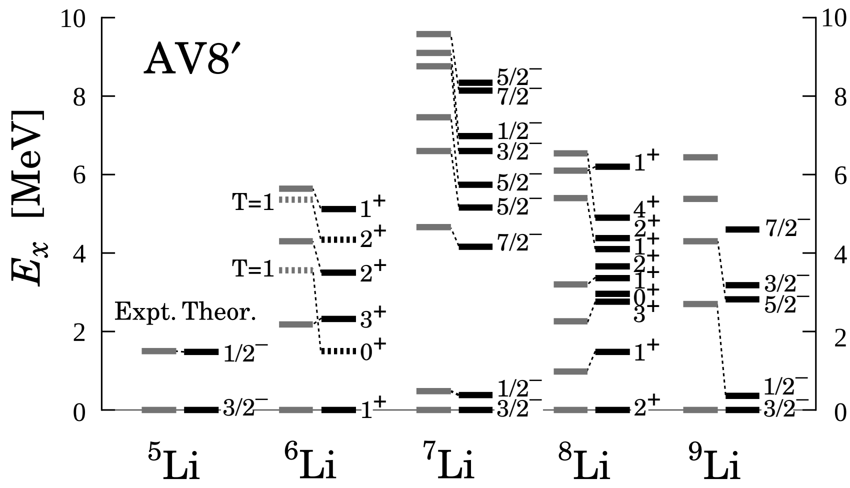

Figure 2. Excitation energies of Li isotopes with TOSM+UCOM using AV8′.

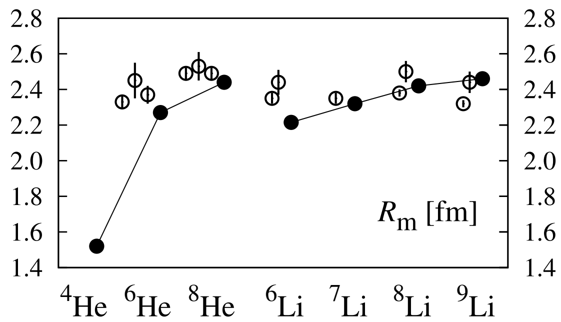

Figure 3. Matter radii of He and Li isotopes in fm (solid circles) with the experiments (open circles)[8, 9, 10].

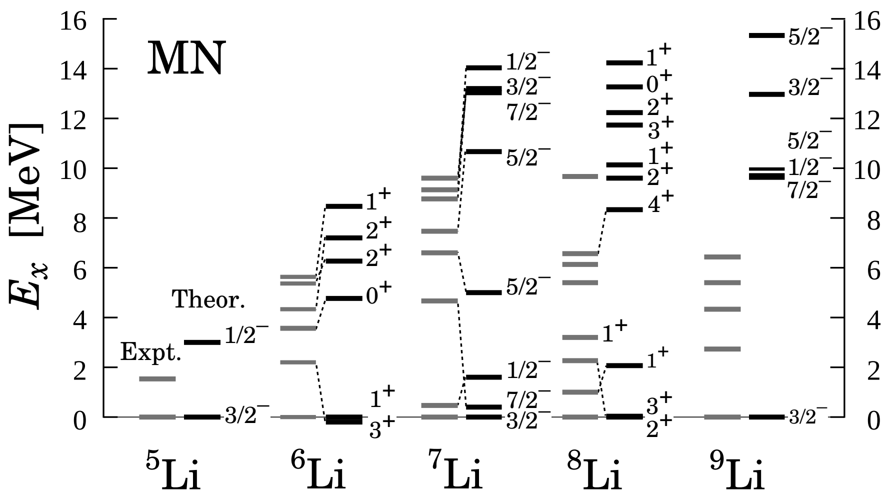

Figure 4. Excitation energies of Li isotopes using the Minnesota interaction (MN) with TOSM.

Table 1. Occupation numbers in each orbit of ^{4}He using AV8$'$ and Minnesota (MN) interactions.

^{4}He(J^π)	$0s_{1/2}$	$p_{1/2}$	$p_{3/2}$	$1s_{1/2}$	$d_{3/2}$	$d_{5/2}$
AV8$'$	3.72	0.07	0.05	0.05	0.04	0.02
MN	3.94	0.01	0.03	0.01	0.004	0.006

To see the effect of the tensor force in the TOSM clearly, we show the results with the effective Minnesota (MN) NN interaction, which does not have the tensor force. We choose the u parameter as 0.95 for the central force and use set III of the LS force [11, 12]. The energy spectra of Li isotopes using the MN interaction are shown in Fig. 4. In the MN case, the excitation energies are split in two regions in comparison with the experiments. One of the reasons is the too large splitting energy between $p_{3/2}$ and $p_{1/2}$ components mainly given by the LS interaction. When we adjust the MN interaction such as changing the strength of the LS interaction to fit the LS splitting energy of ^{5}Li, it is still difficult to reproduce the whole trend of the excitation energies of Li isotopes consistently. A large difference between the AV8$'$ and MN interactions is the tensor force. The comparison of the energy spectra using the two interactions indicates the effect of the tensor force, which has a large impact on the excitation energy spectra of the He and Li isotopes.

We compare the occupation numbers of nucleons in the α particle (^{4}He) between AV8$'$ and MN interactions in Table 1. In AV8$'$, it is shown that the $p_{1/2}$ orbit has the largest contribution among the particle states according to the large 2p2h mixing. This selectivity of the $p_{1/2}$ orbit can be explained considering the properties of the tensor operator S_{12} [13, 3, 7], which changes both the orbital angular momentum and spin by two with the opposite direction of their z-components. In the MN case, it is found that the component of the $0s$ orbit is larger than the AV8$'$ case and the enhancement of the $p_{1/2}$ orbit is not obtained. The $(0s)^4$ configuration in MN is quite dominant in ^{4}He by 96.6% using MN interaction. This comparison means that the tensor force introduces the specific excitations from the s-shell to the p- and sd-shells in ^{4}He. This tensor-driven excitation produces the Pauli-blocking effect on the excited states of the neutron-rich He and Li isotopes [6, 7].

3.2. ^{8}*Be*

The tensor force contributes to making the α particle strongly. It is known that the tensor contribution in the α particle is generally large [2]. In light nuclei, it is often observed that the α particles are strongly developed as a cluster, e.g., in ^{8}Be and the Hoyle state of ^{12}C [14]. For ^{8}Be, the ground, 2^+_1, and 4^+_1 states are regarded as states consisting of two weakly interacting α particles. On the other hand, the excited states above 4^+_1 can be considered to have shell-like structures, because the α decay is not always favored for the states. In the highly excited states, the T=1 states are degenerated with the T=0 states and are considered as isobaric analog states of ^{8}Li and ^{8}B. These facts of ^{8}Be indicate that the internal structures of the highly excited states are quite different from the ground-band states. In this sense, the ^{8}Be nucleus possesses different features in the ground band and highly excited states. It is theoretically known that a single α particle contains the strong tensor correlation and thus it would be interesting to see how the tensor force affects the variety of structures of ^{8}Be.

In this study, we investigate the different structures of ^{8}Be mentioned above from the viewpoint of the tensor force [16]. For the α clustering, we discuss how well the TOSM describes the two-α components of ^{8}Be. We show the excitation energy spectra of ^{8}Be in Fig. 5. We normalize the energy spectrum to the $2^+(T{=}1)$ state in the highly excited states, because this state is the isobaric analog state of ^{8}Li and the TOSM has nicely described the structures of Li

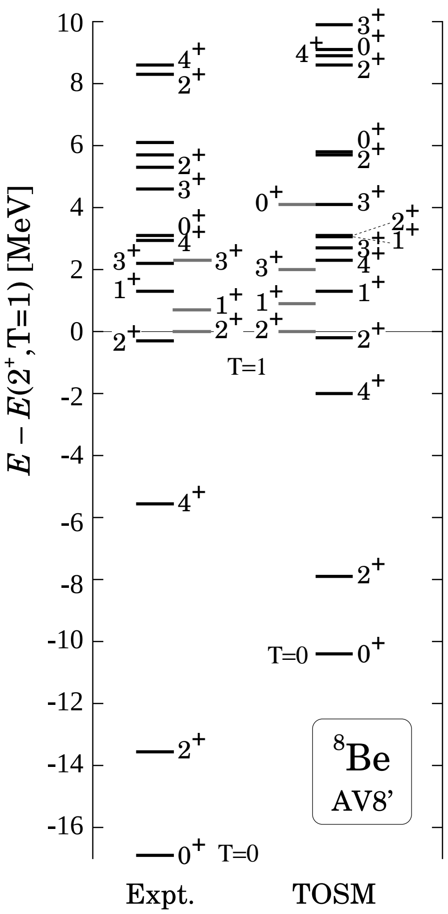

Figure 5. Energy spectrum of ^{8}Be, normalized to the $2^{+}(T{=}1)$ state.

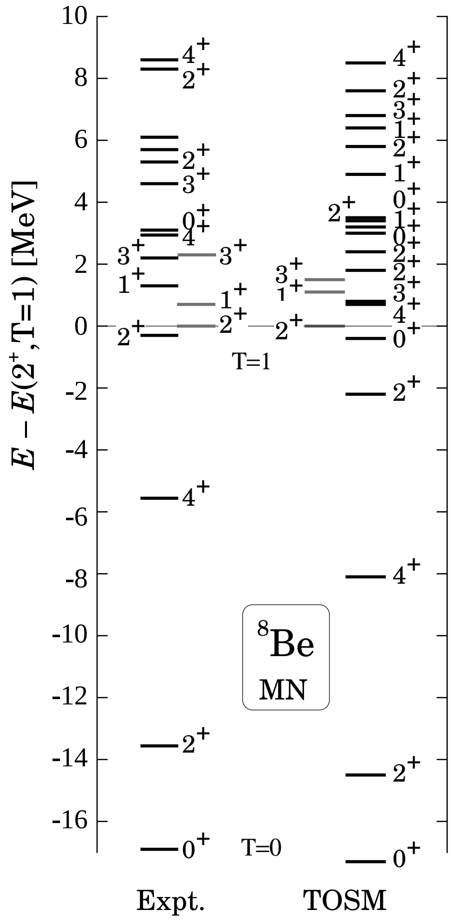

Figure 6. Excitation energy spectrum of ^{8}Be using the Minnesota interaction (MN).

isotopes as shown in Fig. 2 [7]. It is found that there are two groups of states in the spectrum; one is the three ground-band states of 0_1^+, 2_1^+, and 4_1^+, and the other is the highly excited states above 4_1^+. The lowest three states are considered as deformed (clustered) rotational states. The highly excited states include the $T = 1$ states starting from 2^+, the isobaric analog state of the ^{8}Li ground state. In the experiment, the $2^+(T = 1)$ state is almost degenerated with the $2^+(T = 0)$ state in energy and this situation is nicely reproduced in the TOSM.

It is found that the TOSM reproduces fairly well the excitation energy spectrum of ^{8}Be, except for the energy spacing between the ground-band states (0^+, 2^+ and 4^+) and the highly excited states. The resulting small energy spacing is related to the lack of the α clustering component in the grand-band states in the TOSM. In fact, the ^{8}Be ground state possesses almost twice of the values of the α particle for kinetic energies and the central contributions, which can be the signature of two-α clustering of ^{8}Be [16]. From the viewpoint of the shell model, it is generally difficult to express the asymptotic form of the spatially developed α clustering states that contain high shell quanta components. In the TOSM, it is found that specific 2p2h excitations involving high-momentum components are essential to incorporate the tensor correlation in the single α particle [3, 6]. This fact indicates that when two-α particles are established in ^{8}Be, each α particle independently needs the 2p2h components to express the tensor correlation. The other possibility is that many kinds of particle-hole excitations in the shell model bases are also important to describe the formation of the separated two-α clusters in space. In the present TOSM, we take up to 2p2h excitations and this approximation restricts the spatial cluster formation in ^{8}Be, which can require the 4p4h excitations for the α clustering. In addition, the tensor correlations in these α particles might require higher excitations.

In the highly excited states, the calculated spectrum in the TOSM reproduces the experimental level order and the relative energies of each level for both T=0 and T=1 states. The results indicate that the highly excited states of ^{8}Be are regarded as the shell-model like states and the TOSM can treat these states. The tensor contributions in the T=0 states are obtained as stronger than the T=1 states [16], which is consistent to the state dependence of the tensor force.

We compare the energy spectrum of ^{8}Be with those using the effective MN interaction to find the effect of the tensor force. In this case, we reduce the strength of the LS force by 30% to give the same LS splitting energy as ^{5}He obtained using AV8$'$ [6, 7]. We discuss the excitation energy spectrum using MN shown in Fig. 6, which reproduces the overall excitation energies. We obtain ground-band states and the rotational structure is nicely reproduced. Above 14 MeV excitation energy, many spin states are obtained. For T=0 states, the relative energy between the 4^+_1 and 2^+_2 states is fairly well reproduced, which is different from the AV8$'$ results shown in Fig. 5. In MN, the effect of the tensor force is renormalized into the model space, hence the 2p2h excitation induced by the tensor force is not necessary to describe ^{4}He. As a result, some amount of two-α clustering component in ^{8}Be can be described in the TOSM in terms of the 2p2h excitations, which could be the reason for the good energy spacing between 4^+_1 and 2^+_2. This situation is different from the case of AV8$'$ with the tensor force.

For highly excited states, the $T = 0$ states are located lower in energy than the $T = 1$ states by about 2 MeV. This comes from the strong $T = 0$ channel of the central force in MN. In T=0, it is found that the level order is often different from the experiment. From the results of AV8$'$ and MN, the level order of the higher excited states are rather better described in AV8$'$ in Fig. 5 than the MN case. This difference is related to the tensor force that is missing in MN explicitly. The tensor force can give the correct state dependence to reproduce the excitation energy spectrum of ^{8}Be.

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