

# Rotational character of the $^8\text{Be}$ and $^{12}\text{C}$ spectra investigated through inelastic cross sections via photon emission

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**Abstract.** The electric quadrupole transitions between  $0^+$ ,  $2^+$ , and  $4^+$  states in  $^8\text{Be}$  and  $^{12}\text{C}$  are investigated by discretization of the continuum with a box boundary condition. The  $\gamma$ -emission cross sections and the corresponding transition strengths are computed. The consistency of these transition strengths with the expected behavior for transitions between states in a rotational band is investigated.

## 1. Introduction

In this work we investigate the spectra of  $^8\text{Be}$  and  $^{12}\text{C}$ , understood as a two-alpha and a three-alpha system, respectively. In particular, we focus on the electric quadrupole  $\gamma$ -emission processes between the states in these two spectra. Experimentally it is well known that in both nuclei,  $^8\text{Be}$  and  $^{12}\text{C}$ , it is possible to identify several sequences of states with angular momentum and parity  $0^+$ ,  $2^+$ , and  $4^+$ . This fact strongly suggests that these sequences of states could correspond to states in a rotational band. The goal of this work is to investigate the rotational character of the spectra, and, at the same time, extract as much information as possible about the continuum properties of  $^8\text{Be}$  and  $^{12}\text{C}$ .

We first calculate cross sections defined by specified initial and final state energies. From the cross sections we can obtain the  $\mathcal{B}^{(E2)}$  transition strengths. These strengths are used to investigate the rotational character of the spectrum. A rotational band is defined as the sequence of states arising from quantization of the rotational motion of an (almost) frozen deformed structure. This leads to the well-known sequence of excited states with energies following the  $J(J+1)$ -rule where  $J$  is the angular momentum quantum number. However, this behavior of the energy spectrum is not enough. The wave functions for different angular momenta should simultaneously describe the same rotating structure. This condition leads to electromagnetic transition probabilities from one of these states to another given by simple geometric factors depending only on the angular momentum quantum numbers. In this work we investigate to what extent the spectra of  $^8\text{Be}$  and  $^{12}\text{C}$  fulfill this condition for the transition strengths.

We also investigate the dependence on the initial and final state energies and on the chosen  $\alpha - \alpha$  interaction. In fact, even if two different two-body potentials reproduce equally well the  $\alpha - \alpha$  phase shifts, they can still give rise to two-body wave functions with different short-distance



behavior. This different behavior at short distances could produce different cross sections, since the radial matrix element of the electromagnetic operator is a crucial ingredient.

## 2. Theoretical procedure: Cross sections

At the two-body level, given an initial continuum state with energy  $E$  and angular momentum  $J$ , the cross section for a  $\gamma$ -transition into some other continuum state with energy  $E'$  and angular momentum  $J'$  is given by [1]:

$$\frac{d\sigma^{(\lambda)}}{dE'} = \nu \frac{2(2J+1)}{(2J_a+1)(2J_b+1)} \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{1}{k^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} \frac{d\mathcal{B}^{(\lambda)}}{dEdE'} \Bigg|_{J \rightarrow J'}, \quad (1)$$

where  $J_a$  and  $J_b$  are the angular momenta of the two particles,  $E_\gamma = E - E'$  is the photon energy,  $\lambda$  is the multipolarity,  $\nu$  is the number of identical particles, and  $k^2 = 2\mu_{ab}E/\hbar^2$ , where  $\mu_{ab}$  is the reduced mass of the two-body system made of particles  $a$  and  $b$ .

For three-body systems the expression analogous to the one above takes the form [2]:

$$\frac{d\sigma^{(\lambda)}}{dE'} = \nu! \frac{2(2J+1)}{(2J_a+1)(2J_b+1)(2J_c+1)} \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{32\pi}{\kappa^5} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} \frac{d\mathcal{B}^{(\lambda)}}{dEdE'} \Bigg|_{J \rightarrow J'}, \quad (2)$$

where again,  $J_a$ ,  $J_b$ , and  $J_c$  are the angular momenta of the three particles, and  $\kappa^2 = 2mE/\hbar^2$ , and  $m$  is the normalization mass used to define the Jacobi coordinates.

The double differential transition strength contained in Eqs.(1) and (2) is given by:

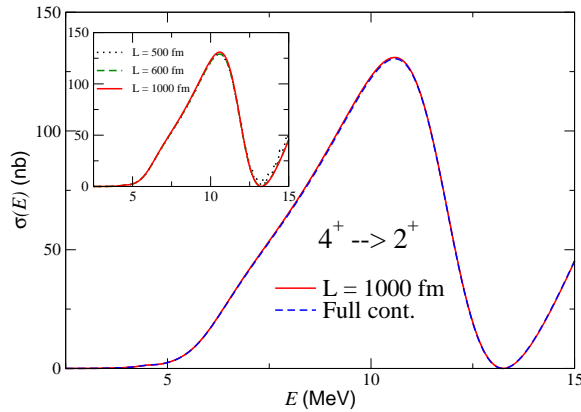
$$\frac{d\mathcal{B}^{(\lambda)}}{dEdE'} \Bigg|_{J \rightarrow J'} = \frac{1}{2J+1} \left| \langle \Phi_J(E) | \hat{O}_\lambda | \Phi_{J'}(E') \rangle \right|^2 \quad (3)$$

where  $\Phi_J(E)$  and  $\Phi_{J'}(E')$  are the initial and final continuum two- or three-body wave functions, and  $\hat{O}_\lambda$  is the electromagnetic operator with multipolarity  $\lambda$ .

The total cross section for a scattering process with incident energy  $E$  is obtained after integration of Eq.(1) or (2) over  $E'$ . However, to integrate over all possible final energy values does not make much sense. In fact, experimentally the final energy is restricted to some energy window,  $\Delta E'$ , which is taken around the energies of interest, typically the resonance energies. In particular, since resonances do not have a well defined energy we have that given an initial energy  $E$  we have to consider the transitions into all the states within some energy range around the resonance energy. In this work we choose this energy window as  $\Delta E' = E'_R \pm \Gamma'_R$ , where  $E'_R$  is the energy of the resonance and  $\Gamma'_R$  is the resonance width. Obviously, this choice is a matter of taste, and therefore there is always an ambiguity in the definition of the cross section.

An important point is that the calculations are made on the real energy axis. The wave functions in Eq.(3) are continuum wave functions at a certain energy, and there is no particular treatment of the resonances. Therefore, in order to know the position and the width of the final-state energy windows it is necessary to obtain by some other procedure the energy and width of the resonances. In this work this is made by use of the complex scaling method.

As a final technical remark, let us mention that since we are dealing with continuum states, they should be normalized in the continuum sense, i.e., in terms of a Dirac delta. This can be easily implemented at the two-body level, since in this case the asymptotic behavior of the wave function for two charged particles in the continuum is well known. However, for three charged particles this is not true anymore, and the correct normalization of the continuum three-body wave functions becomes a very delicate task. For this reason, for  $^{12}\text{C}$  it is much more convenient to discretize the continuum spectrum, for instance by imposing a box boundary condition. When done, the “discrete” continuum wave functions are normalized in terms of a Kronecker delta, just like bound states. The details about the equivalence of these two methods can be found in Ref.[3], and it is illustrated in Fig.1 for the case of the  $4^+ \rightarrow 2^+$  transition in  $^8\text{Be}$ .



**Figure 1.** Cross section for the  $4^+ \rightarrow 2^+$  transition in  $\alpha + \alpha$  scattering as a function of the incident energy  $E$ . The curves are obtained after integration of Eq.(1) over the final energy  $E'$  within the energy window  $\Delta E' = 3 \pm 1$  MeV. The big panel shows the results with and without discretization of the continuum (solid and dashed curves, respectively). The discretization has been made in a box with a size of 1000 fm. The inset shows the results obtained for different sizes of the discretization box.

### 3. Transition strengths

In this section we discuss the  $\mathcal{B}^{(E2)}$  transition strengths obtained for  $^8\text{Be}$  and  $^{12}\text{C}$ . The results reported here have been obtained using the Ali-Bodmer  $\alpha - \alpha$  potential specified in Ref.[4]. No relevant differences have been found when the Buck potential is used. For both nuclei a previous complex scaling calculation has been performed in order to determine the corresponding resonance energies and widths, which are obtained as poles of the  $S$ -matrix. For  $^8\text{Be}$  a sequence of  $0^+$ ,  $2^+$ ,  $4^+$ ,  $6^+$ , and  $8^+$  resonances are found [4]. The computed energies and widths of the first three resonances agree with the known experimental values. For  $^{12}\text{C}$  three  $0^+$  states (the first one bound), three  $2^+$  states (the first one bound), and three  $4^+$  resonances have been found [5]. These states form three  $\{0^+, 2^+, 4^+\}$  bands. The possible rotational character of the first two bands will be investigated. The third band is excluded due to the fact that the energy of the third  $0^+$  state is experimentally known to be above the third  $2^+$  state.

**Table 1.**  $\mathcal{B}^{(E2)}$  strengths (in  $e^2\text{fm}^4$ ) for the transitions in  $^8\text{Be}$  indicated in the first column. The second column shows the results obtained in this work. The third column gives the strengths computed in Ref.[6]. The last two columns are the estimates given by rotational model (see text). The numbers within parenthesis are the ratios in Eq.(6) taking the first reaction as reference.

	This work	Ref.[6]	Rotational model	
			$Z_0 = 3 \text{ fm}$	$Z_0 = \sqrt{\langle r^2 \rangle}$
$2^+ \rightarrow 0^+$	79.1 (1)	71.3 (1)	6.4 (1)	84.0 (1)
$4^+ \rightarrow 2^+$	22.1 (0.28)	18.0 (0.25)	9.2 (1.43)	18.1 (0.22)
$6^+ \rightarrow 4^+$	10.1 (0.13)	–	10.1 (1.57)	9.1 (0.11)
$8^+ \rightarrow 6^+$	13.0 (0.16)	–	10.6 (1.65)	7.6 (0.09)

The differential  $E2$ -transition strengths,  $d\mathcal{B}^{(E2)}/dE$ , have been obtained by integrating the cross section in Eq.(1) for  $^8\text{Be}$ , and in Eq.(2) for  $^{12}\text{C}$ , over  $E'$ , with  $E'$  restricted to the final energy window  $\Delta E' = E'_R \pm \Gamma'_R$ , where  $E'_R$  and  $\Gamma'_R$  are the energy and width of the final state resonance. Subsequently, the integrated cross section is divided by all the factors that in Eqs.(1) and (2) multiply the differential transition strength. Finally, the total strength  $\mathcal{B}^{(E2)}$  for a transition between some specific initial state into the selected final state is obtained after integration over the initial energy  $E$ , where, as in the case of  $E'$ , the energy  $E$  is limited to the corresponding energy window around the initial resonance energy.

For  $^8\text{Be}$ , the computed transition strengths between the different states are given in the second column of Table 1. The computed values for the  $2^+ \rightarrow 0^+$  and the  $4^+ \rightarrow 2^+$  reactions agree with the results given in Ref.[6], where a cluster model was also used. Also, the  $22.1 e^2\text{fm}^4$  obtained for the  $4^+ \rightarrow 2^+$  transition agrees with the experimental value of  $25 \pm 8 e^2\text{fm}^4$  reported in Ref.[7].

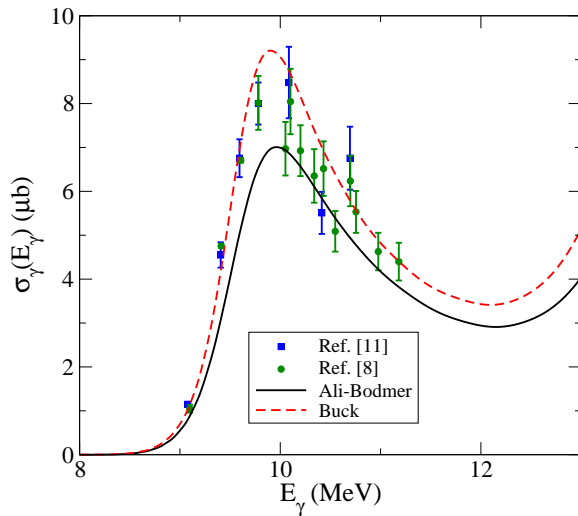
For  $^{12}\text{C}$ , the computed transition strengths are given in the third column of Table 2. For those reactions involving the second  $2^+$  state (the lowest  $2^+$  resonance), two different results are given, which correspond, respectively, to placing the resonance energy at 1.8 MeV and 2.5 MeV. The first energy corresponds to the three-body calculation such that a phenomenological three-body force is used to fit the experimental separation energy of the bound  $2^+$  state, and in the second case the three-body force is weakened in order to obtain a resonance energy in better agreement with the recent experimental value given in Ref.[8]. In the second column of the table the available experimental data are given.

**Table 2.**  $\mathcal{B}^{(E2)}$  strengths (in  $e^2\text{fm}^4$ ) for the transitions in  $^{12}\text{C}$  indicated in the first column. The available experimental data are given in the second column. The third column are the results in this work. For the cases where the second  $2^+$  state enters the results for two energies of the  $2^+$  resonance are given. The fourth and fifth columns are the strengths given in Refs.[9] and [10], respectively. The last column is the prediction by the rotational model.

Transition	Exp. [9, 11, 8]	This work		Ref.[9]	Ref.[10]	Rot. model
$2_1^+ \rightarrow 0_1^+$	$7.6 \pm 0.4$	10.6		9.16	8.4	18
$4_1^+ \rightarrow 2_1^+$		14.5			15.8	25
		$E_{2^+}=1.8 \text{ MeV}$	$E_{2^+}=2.5 \text{ MeV}$			
$2_2^+ \rightarrow 0_2^+$		153	185		102	155
$4_2^+ \rightarrow 2_2^+$		0.6	1.7		595	185
$2_1^+ \rightarrow 0_2^+$	$2.6 \pm 0.4$	1.1		0.84	5.1	
$4_1^+ \rightarrow 2_2^+$		3.6	5.2		7.5	
$2_2^+ \rightarrow 0_1^+$	$0.73 \pm 0.13$	2.6	1.8	1.99	0.4	
	$1.57 \pm 0.14$					

Our results agree reasonably well with the cluster-model calculations described in Ref.[9] (fourth column in the table). When compared to the AMD calculations in Ref.[10] significant differences are observed for some of the reactions, specially for the  $4_2^+ \rightarrow 2_2^+$  transition. Unfortunately, in Ref.[9] this transition was not considered.

For the  $2_2^+ \rightarrow 0_1^+$  transition the experimental values given in Table 2 are taken from Refs.[11, 8], which are obtained after photo-dissociation of the  $^{12}\text{C}$  ground state. The experimental data, whose analysis is obviously still to be clarified, are shown by the square and the circles in Fig.2, respectively. The solid and dashed curves in the figure are the cross sections obtained with the Ali-Bodmer and Buck potentials. As seen in the figure, the shape of the cross section is nicely reproduced. However, in order to get this agreement the computed curves have been divided by a factor of three. This point reflects the fact that a description of the  $^{12}\text{C}$  ground state as a three-alpha system is not appropriate, and only a small component of the ground state wave function corresponds to such structure.



**Figure 2.** Photo-dissociation cross section for the  $^{12}\text{C} + \gamma \rightarrow \alpha + \alpha + \alpha$  reaction between the  $0^+$  ground state and the continuum  $2^+$  states as a function of the photon energy. The computed curves have been divided by a factor of 3. The experimental data are from Refs.[8, 11]. Only the results with the  $2^+$  resonance at 2.5 MeV are shown.

#### 4. Rotational model

For transitions between states within a rotational band the quadrupole strength is given by

$$\mathcal{B}^{(E2)}(J \rightarrow J') = \frac{5}{16\pi} e^2 Q_0^2 \langle J0; 20 | J'0 \rangle^2, \quad (4)$$

where the rotational band has been assumed to zero projection of the angular momentum over the intrinsic symmetry axis, and where the intrinsic quadrupole moment  $Q_0$  is given by:

$$Q_0 = \frac{4}{3} \langle \sum_i Z_i r_i^2 \rangle \delta, \quad (5)$$

where  $i$  runs over all the charged particles (with charge  $Z_i$  and center of mass coordinate  $r_i$ ) and  $\delta$  is the deformation parameter. From the definition above we have that  $Q_0 = 4\delta\langle r^2 \rangle/3$  and  $Q_0 = 2\delta\langle \rho^2 \rangle/3$  for a two- $\alpha$  and a three- $\alpha$  system, respectively, where  $r$  is the relative distance between the two particles, and  $\rho$  is the hyperradius defined in the three-body system.

The intrinsic quadrupole moment  $Q_0$  is the same for all the states in the band, which is related to the frozen structure of a rotating rigid rotor. Therefore, it is clear that for transitions between states in a rotational band the following ratio between transition strengths holds:

$$\frac{\mathcal{B}^{(E2)}(J \rightarrow J')}{\mathcal{B}^{(E2)}(\tilde{J} \rightarrow \tilde{J}')} = \frac{\langle J0; 20 | J'0 \rangle^2}{\langle \tilde{J}0; 20 | \tilde{J}'0 \rangle^2}. \quad (6)$$

In Ref.[3] it is shown how the average distance between the two alpha-particles in  $^8\text{Be}$  is far from being constant for the different resonances. In fact, if a constant distance of 3 fm is assumed, the rotational estimate given by Eq.(4), which is given in the fourth column in Table 1, clearly disagrees with our results (second column). In the table we show within parenthesis the results of the ratios in Eq.(6) when the first transition is taken as a reference. Nevertheless, if we compute the intrinsic quadrupole moment in Eq.(5) using the different computed  $\alpha$ - $\alpha$  distance for each resonance, and this value is used in Eq.(4), we then get the transition strengths and ratios given in the last column of Table 2. These results agree much better with our calculations. Therefore, the  $^8\text{Be}$  spectrum has a rotational character provided that the  $\alpha$ - $\alpha$  distance is angular momentum dependent, and the principal  $\alpha$ -cluster structure is maintained.

About  $^{12}\text{C}$ , it is found that the computed value of  $\langle \rho^2 \rangle$  is reasonably constant for the  $0^+$ ,  $2^+$ , and  $4^+$  states belonging to each of the two bands under investigation. This is consistent with

the expected constant intrinsic quadrupole moment for all the states within a rotational band. Using the computed values of  $\langle \rho^2 \rangle$ , see Ref.[5], we obtain by means of Eq.(4) the transition strengths shown in the last column of Table 2.

For the two transitions within the first band,  $2_1^+ \rightarrow 0_1^+$  and  $4_1^+ \rightarrow 2_1^+$ , the values obtained with the rotational picture agree reasonably well with our calculation. The agreement would be even better if we use the known relation between the static and intrinsic quadrupole moments, which permit us to estimate the deformation parameter, which is found to be  $\delta^2 \approx 0.55$ . Using this value we get that the rotational picture predicts transition strength values of about  $10 e^2\text{fm}^4$  and  $14 e^2\text{fm}^4$  for these two transitions.

For the transitions within the second band,  $2_2^+ \rightarrow 0_2^+$  and  $4_2^+ \rightarrow 2_2^+$ , the situation is different. For the first of them we can consider that our calculations and the rotational estimate agree. However, our results for the  $4_2^+ \rightarrow 2_2^+$  transition and the rotational estimate are very far from each other. One could think that a very small deformation parameter could correct the problem, but that would imply to break the reasonable agreement found in the  $2_2^+ \rightarrow 0_2^+$  case. Simultaneous agreement between our calculations and the rotational picture for the two reactions is not possible.

For completeness in the lower part of Table 2 we show the strengths corresponding to several transitions between states located in different bands.

## 5. Conclusions

In this work continuum to continuum transitions are used to investigate the properties of the  $^8\text{Be}$  and  $^{12}\text{C}$  spectra. The rotational character of the  $\{0^+, 2^+, 4^+, \dots\}$  bands is investigated.

We have found that the idea of  $^8\text{Be}$  as a rigid rotor of two  $\alpha$ -particles at a fixed distance is questionable. This picture does not agree with the computed transition strengths. However, if the distance between the two particles is allowed to depend on the angular momentum, the agreement is then good.

For  $^{12}\text{C}$  the conclusion is that the  $E2$ -transition strengths between the  $0_1^+$ ,  $2_1^+$ , and  $4_1^+$  states are consistent with the behavior expected for states that belong to a rotational band. For the  $0_2^+$ ,  $2_2^+$ , and  $4_2^+$  states only the strength corresponding to the  $2_2^+ \rightarrow 0_2^+$  transition can be considered to agree with a rotational behavior, but the  $4_2^+$  state does not seem to belong to such rotational band, since the strength for the transition into the  $2_2^+$  state is much smaller than the expected rotational value.

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