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Multiple-scattering effects in proton- and alpha-nucleus reactions with Glauber theory

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Abstract. We study the total reaction and elastic differential cross sections for proton-nucleus and ^{4}He-nucleus reactions in the framework of the Glauber theory which describes multiple-scattering processes. The input wave functions are obtained using the Skyrme-Hartree-Fock method and prepared for a wide range of mass numbers, O, Ca, Ni, Sn, and Pb isotopes. The theory reproduces experimental data very well. An effect of the multiple scattering is discussed by comparing with a standard optical-limit approximation. We see that the multiple-scattering effects play a crucial role, especially in enhancing the elastic differential cross sections at large scattering angles.

1. Introduction

A study of new unstable nuclei has become possible in new radioactive beam facilities. In order to relate nuclear structure with observables, we need reaction theory which reflects the nuclear structure. The Glauber theory is widely used to evaluate cross sections in high-energy nuclear reactions [1]. In the Glauber theory, the scattering amplitude of two colliding nuclei with mass number A_P and A_T is obtained by evaluating a $[3 \times (A_P + A_T - 2)]-$fold multiple integral induced by an $(A_P + A_T - 2)-$body multiple-scattering operator. The optical-limit approximation (OLA) is a standard way to approximate the scattering amplitude. The complicated multiple-scattering operator is approximated to a one-body (two-body) operator in proton-nucleus (nucleus-nucleus) reactions, and thus the wave function reduced to a one-body density. However, the use of one-body densities may wash out some pieces of structural information. In fact, the multiple-scattering effects play an important role in the nuclear reaction involving unstable nuclei [2, 3, 4, 5, 6]. In this paper, we extend the application range of the Glauber theory to a wide range of mass numbers and discuss the effect of the multiple-scattering processes to the cross sections.

The paper is organized as follows: In Section 2, we explain how to calculate the scattering amplitude in the Glauber theory and its approximations. Section 3 presents our results of the total reaction and elastic differential cross sections. A role of multiple-scattering effects is discussed in this section. A summary is given in Section 4.

2. Formalism

The Glauber theory is formulated based on eikonal and adiabatic approximations [1]. The final state wave function ψ_f is expressed by a product of the phase-shift function and the initial wave

function, ψ_i:

$$\psi_f = e^{i\chi}\psi_i. \tag{1}$$

The total reaction cross sections and elastic differential cross sections with phase-shift function are calculated by

$$\sigma_R = \int \left(1 - |e^{i\chi(\boldsymbol{b})}|^2\right) d\boldsymbol{b}, \tag{2}$$

$$\frac{d\sigma}{d\Omega} = \left|\frac{k}{2\pi}\int e^{i\boldsymbol{q}\cdot\boldsymbol{b}}\left(e^{i\chi(\boldsymbol{b})} - 1\right) d\boldsymbol{b}\right|^2, \tag{3}$$

where k is the wave number of the relative motion between the two nuclei, $\boldsymbol{q}$ is a momentum transfer vector, and $\boldsymbol{b}$ is an impact parameter vector. In a proton-nucleus (pT, target: T) reaction, the phase-shift function of the pT reaction is

$$e^{i\chi_{pT}(\boldsymbol{b})} = \left\langle \psi_T(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_{A_T}) \middle| \prod_{j=1}^{A_T} e^{i\chi_{pN}(\boldsymbol{b}-\boldsymbol{s}_j)} \middle| \psi_T(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_{A_T}) \right\rangle, \tag{4}$$

where $e^{i\chi_{pN}}$ is a proton-nucleus (pN) phase-shift function, $\boldsymbol{s}$ is a two-dimensional vector perpendicular to the beam direction, and ψ_T is the wave function of the target nucleus. The NN phase-shift function employed here is determined so as to reproduce the NN scattering properties [7]. It should be noted that the above equation includes the multiple integral.

The optical-limit approximation (OLA) offers the most simple expression, which is obtained by taking only the first-order term of the cumulant expansion of Eq. (4) [1], that is

$$e^{i\chi_{pT}^{\mathrm{OLA}}(\boldsymbol{b})} = \exp\left\{-\int d\boldsymbol{r}\rho_T(\boldsymbol{r})(1 - e^{i\chi_{NN}(\boldsymbol{b}-\boldsymbol{s})})\right\}, \tag{5}$$

where the ρ_T is a one-body density of the target nucleus.

In a nucleus-nucleus (PT, projectile: P) reaction, we employ the nucleon target formalism in the Glauber theory (NTG) proposed in Ref. [8]. In the NTG, the pN phase-shift function in Eq. (4) is simply replaced with a PN one,

$$e^{i\chi_{PT}(\boldsymbol{b})} = \left\langle \psi_T(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_{A_T}) \middle| \prod_{j=1}^{A_T} e^{i\chi_{PN}(\boldsymbol{b}-\boldsymbol{s}_j)} \middle| \psi_T(\boldsymbol{r}_1, \cdots, \boldsymbol{r}_{A_T}) \right\rangle. \tag{6}$$

The phase-shift function of PN in the NTG is obtained in the spirit of the OLA by

$$e^{i\chi_{PN}(\boldsymbol{b})} = 1 - \exp\left\{-\int d\boldsymbol{r}'\rho_P(\boldsymbol{r}')(1 - e^{i\chi_{NN}(\boldsymbol{b}+\boldsymbol{s}')})\right\}, \tag{7}$$

where ρ_P is the one-body density in the projectile nucleus. The phase-shift function in the OLA is given by

$$e^{i\chi_{PT}^{\mathrm{OLA}}(\boldsymbol{b})} = \exp\left\{-\iint d\boldsymbol{r}'d\boldsymbol{r}\rho_P(\boldsymbol{r}')\rho_T(\boldsymbol{r})(1 - e^{i\chi_{NN}(\boldsymbol{b}+\boldsymbol{s}'-\boldsymbol{s})})\right\}. \tag{8}$$

The wave function of those nuclei are generated by the Skyrme-Hartree-Fock (HF) method on three-dimensional coordinate space [9]. The SkM* interaction [10] is employed. The wave function is expressed in a Slater determinant that greatly reduces the computational cost of evaluating the phase-shift functions defined in Eqs. (4) and (6) [11, 12]. Since the multiple-scattering operator is written by a product of one-body operators, the multiple integration is reduced to three-dimensional. We now systematically investigate the cross sections for a wide range of mass number without *ad hoc* parameters. In this study, the wave functions for $^{16-24}$O, $^{40-70}$Ca, ^{56,58}Ni, $^{100-140}$Sn, and $^{190-214}$Pb are analyzed.

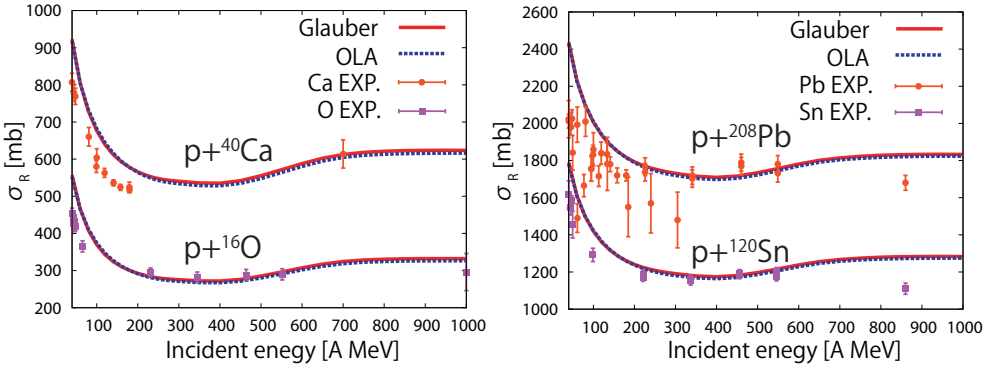

Figure 1. Total reaction cross sections of proton-nucleus reactions incident at 40-1000A MeV calculated with the Glauber theory and the OLA on (left) ^{16}O and ^{40}Ca, and (right) ^{120}Sn and ^{208}Pb targets. The experimental data are taken from [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

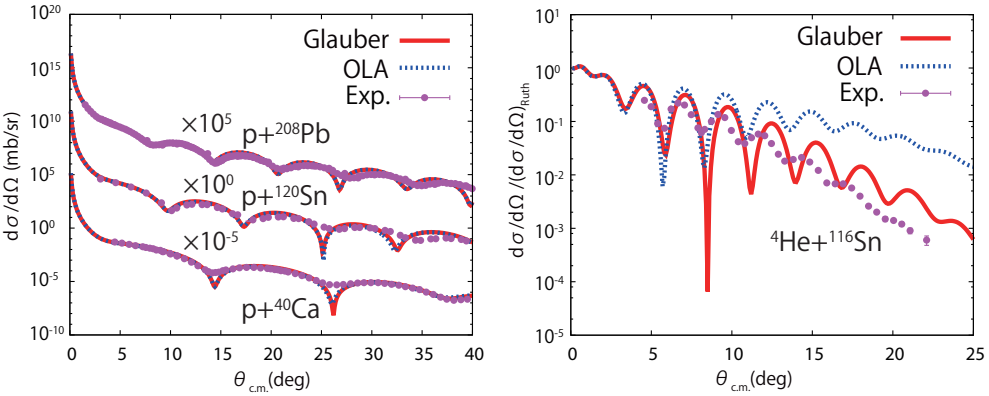

Figure 2. (Left) Elastic differential cross sections of proton-nucleus reactions on ^{40}Ca, ^{120}Sn, and ^{208}Pb targets at 300A MeV calculated with the Glauber theory and the OLA. (right) Same as the left panel but for a ^{4}He-^{116}Sn reaction at 120A MeV. The experimental data are taken from [30, 31, 32, 33, 34, 35].

3. Results

We show our results of the total reaction cross sections, σ_R, and elastic differential cross sections for proton-nucleus and ^{4}He-nucleus reactions. Fig. 1 displays σ_R of proton-nucleus reactions on ^{16}O, ^{40}Ca, ^{120}Sn, and ^{208}Pb targets. The calculated cross sections at the energy higher than 200A MeV are in reasonable agreement with the experimental data. At the energy lower than 200A MeV, the theory overestimate the data. At such a low energy, the validity of the Glauber model is questioned and the Fermi motion and the Pauli blocking effects may not be negligible [28, 29]. In most energies the Glauber theory gives larger cross sections than those of the OLA due to the multiple-scattering effects.

Fig. 2 shows the elastic differential cross sections of proton-nucleus reactions on ^{40}Ca, ^{120}Sn, and ^{208}Pb targets, and for a ^{4}He-^{116}Sn reaction. The calculations include Coulomb effect. For the proton-nucleus reactions, as displayed in the left panel of Fig. 2 both the Glauber theory and the OLA reproduce the experimental data very well and give virtually the same results. For the ^{4}He-^{116}Sn reaction, the two calculated cross sections give the same results at small scattering

angles up to 5° but deviate significantly at larger angles as shown in the right panel of Fig. 2. The Glauber theory gives a much better description at larger scattering angles than the OLA.

4. Summary

We have applied the Glauber theory to high-energy proton-nucleus and nucleus-nucleus reactions and calculate the total reaction and elastic differential cross sections. The wave functions are generated by the Skyrme-Hartree-Fock method for a wide range of mass number. The theory agrees with the experimental data very well in the energy higher than $200A$ MeV. Our systematic analysis shows the effects of multiple scattering to the cross sections by comparing with the standard optical-limit approximation. The effects are small when the proton-nucleus reactions are used. In the ^{4}He-^{116}Sn reaction, the effects are large that the multiple scattering processes significantly enhance the elastic differential cross sections at large scattering angles. A detailed analysis is in progress and will be reported elsewhere soon.

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References

[1] Glauber R J, 1959 *in Lectures on Theoretical Physics* vol 1, ed W E Brittin and L C Dunham (New York: Interscience) p 315
[2] Bertsch G F, Esbensen H and Sustich A 1990 *Phys. Rev.* C **42** 758
[3] Ogawa Y, Yabana K and Suzuki Y 1992 *Nucl. Phys.* A **543** 722
[4] Al-Khalili J S *et al* 1996 *Phys. Rev. Lett.* **76** 3903 ; Al-Khalili J S *et al* 1996 *Phys. Rev.* C **54** 1843
[5] Yabana K, Ogawa Y and Suzuki Y 1992 *Phys. Rev.* C **45** 2909
[6] Al-Khalili J S, Tostevin J A and Brooke J M 1997 *Phys. Rev.* C **55** R1018
[7] Abu-Ibrahim B, Horiuchi W, Kohama A and Suzuki Y 2008 *Phys. Rev.* C **77** 034607
[8] Abu-Ibrahim B and Suzuki Y 2000 *Phys. Rev.* C **62** 034608
[9] Vautherin D and Brink D M 1972 *Phys. ReV.* C **5** 626
[10] Bartel J, Quentin P, Brack M, Guet C and Håkansson H B 1982 *Nucl. Phys.* A **386** 79
[11] Abu-Ibrahim B, Iwasaki S and Horiuchi W 2009 *J. Phys. Soc. Jpn.* **78** 044201
[12] Bassel R H and Wilkin C 1968 *Phys. Rev.* **174** 1179
[13] Carlson R F, Cox A J, Nimmo J R and Davison N E *et al* 1975 *Phys. Rev.* C **12** 1167
[14] Renberg P U *et al* 1972 *Nucl. Phys.* A **183** 81
[15] Ingemarsson A, Nyberg J and Renberg P U *et al* 1999 *Nucl. Phys.* A **653** 341
[16] Slaus I, Margaziotis D J, Carlson R F, Van Oers W T H and Richardson J R 1975 *Phys. Rev.* C **12** 1093
[17] Auce A, Igemarsson A and Johansson R *et al* 2005 *Phys. Rev.* C **71** 064606
[18] Johansson A, Svanberg U and Sundberg O 1961 *Arkiv Fysik* **19** 527
[19] Kirkby P and Link W T 1966 *Can. J. Phys.* **44** 1847
[20] Anderson B D, Bevington P R and Cverna F H *et al* 1979 *Phys. Rev.* C **19** 905
[21] Menet J J H, Gross E E, Malanify J J and Zucker A 1971 *Phys. Rev.* C **4** 1114
[22] Carlson R F, Cox A J, Eliyakut-Roshko T and van Oers W T H 1995 *Can. J. Phys.* **73** 512
[23] Chen F F, Leavitt C P and Shapiro A M 1955 *Phys. Rev.* **99** 857
[24] Meyer V, Eisberg R M and Carlson R F 1960 *Phys. Rev.* **117** 1334
[25] Goloskie R and Strauch K 1962 Nucl. Phys. **29** 474
[26] Cassels J M and Lawson J D 1954 *Proc. Phys. Soc.* A **67** 125
[27] Millburn G P, Birnbaum W, Crandall W E and Schecter L 1954 *Phys. Rev.* **95** 1268
[28] DiGiacomo N J, DeVries R M and Peng J C 1980 *Phys. Rev. Lett.* **45** 527 ; 1981 *Phys. Lett.* B **101** 383
[29] Takechi M, Fukuda M and Mihara M *et al* 2009 *Phys. Rev.* C **79** 061601(R)
[30] Kelly J J, Boberg P and Feldman A E *et al* 1991 *Phys. Rev.* C **44** 2602
[31] Terashima S, Sakaguchi H and Takeda H *et al* 2008 *Phys. Rev.* C **77** 024317
[32] Takeda H 2003 *Memoirs of the Faculty of Science, Kyoto University - Series of physics, astrophysics, geophysics and chemistry* **44-1** 1
[33] Hutcheon D A, Olsen W C and Sherif H S *et al* 1988 *Nucl. Phys.* A **483** 429
[34] Zenihiro J, Sakaguchi H and Murakami T *et al* 2010 *Phys. Rev.* C **82** 044611
[35] Bonin B, Alamanos N and Berthier B *et al* 1985 *Nucl. Phys.* A **445** 381