Study of cluster structure in ^{13}C with AMD+HON-constraint method

Yohei Chiba and Masaaki Kimura

Department of Physics, Hokkaido University, 060-0810 Sapporo, Japan

E-mail: chiba@nucl.sci.hokudai.ac.jp

Abstract. The $3\alpha + n$ cluster states of ^{13}C are discussed on the basis of antisymmetrized molecular dynamics with the constraint on the harmonic oscillator quanta. We predict two different kinds of the cluster states, the hoyle analogue state and the linear-chain state. The former is understood as the 0_2^+ state (Hoyle state) of ^{12}C accompanied by a valence neutron occupying the s-wave. The latter constitute the parity doublet bands of $K^\pi = 1/2^\pm$ owing to its parity asymmetric intrinsic structure.

1. Introduction

It is known and expected that two different types of 3α cluster states, gas-like [1, 2] and linear-chain states [3, 4, 5], appear in ^{12}C. Naturally, their analog states accompanied by a valence neutron are expected in ^{13}C, and how the valence neutron will modify those 3α cluster structure is very interesting issue to understand the interplay between the clustering of the core nucleus and the single particle motion of the valence neutron. For example, it was pointed out that a p-wave valence neutron attached to the 0_2^+ state will feel very weak spin-orbit interaction due to the very low matter density. As a result, the neutron p-waves coupled to the ^{12}C(0_2^+) will show very reduced spin-orbit splitting [6]. However, if such cluster states really exist or not is unknown. Another example is a stabilization of the linear-chain configuration. The valence neutrons may stabilize the linear-chain configuration [7, 8, 9, 10] as they were in the case of Be isotopes. ^{13}C is a very suitable system to investigate the stabilization mechanism of the linear chain.

For those purpose, we investigated the excited states of ^{13}C on the basis of antisymmetrized molecular dynamics (AMD) [11, 12]. Our aims in this study are (1) the search for the hoyle analogue states with s or p-wave valence neutron, and (2) the search for the linear-chain states.

2. Theoretical framework

We start from the microscopic Hamiltonian with an effective NN interaction of Gogny D1S [13], and employ the parity-projected wave function as the variational wave function,

$$\Phi^\pi = \frac{1 + \pi \hat{P}_x}{2} \Phi_{int}, \quad \Phi_{int} = \mathcal{A}\{\varphi_1, \varphi_2, ..., \varphi_A\}, \tag{1}$$

$$\varphi_i = \exp\left\{ - \sum_{\sigma=x,y,z} \nu_\sigma \left(r_\sigma - \frac{Z_{i\sigma}}{\sqrt{\nu_\sigma}} \right)^2 \right\} \otimes (a_i \chi_\uparrow + b_i \chi_\downarrow) \otimes (\text{proton or neutron}). \tag{2}$$

Here φ_i is the single particle wave packet whose spatial part is represented by the deformed Gaussian [14], and the variational parameters $\boldsymbol{Z}_i$, a_i, b_i and ν_σ are optimized to minimize the total energy under the constraint. In many AMD studies, the constraint on the quadrupole deformation parameters which is very effective to describe the low-lying yrast states are often used. However, it is not sufficient to describe the non-yrast states, in particular, cluster states. Therefore, we have extended the constraint on the harmonic oscillator quanta used in Ref. [15]. We introduce the following quantities,

$$N = N_x + N_y + N_z, \quad \lambda = N_z - N_y, \quad \mu = N_y - N_x, \quad (N_z \geq N_y \geq N_x), \qquad (3)$$

where N_x, N_y, N_z represents the expectation values of the harmonic oscillator quanta in each direction. The constraint on the values of N, λ and μ which call HON constraint generates the wave functions with various excited nucleon configurations. As we will show below, HON constraint is found very effective to generate various cluster structures without the assumption on nuclear structure.

After the variational calculation, we perform the angular momentum projection and generator coordinate method (GCM) by superposing the wave functions with different values of N, λ, μ,

$$\Psi_{M\alpha}^{J\pi} = \frac{2J+1}{8\pi^2} \sum_{Ki} g_{Ki\alpha}^{J\pi} \int d\Omega D_{MK}^{J*}(\Omega) \hat{R}(\Omega) \Phi^\pi(N_i, \lambda_i, \mu_i), \qquad (4)$$

and the coefficients $g_{Ki\alpha}^{J\pi}$ and eigenenergies $E_\alpha^{J\pi}$ are determined by the Hill-Wheeler equation. To investigate the internal structure of the excited states, we calculate the spectroscopic factor (S-factor) that is defined as the volume integral of the multipole component $\varphi_{jl}^{\pi_1\pi_2}(r)$ of the overlap function $\varphi(\boldsymbol{r})$ between the ^{12}C and ^{13}C wave functions,

$$\varphi(\boldsymbol{r}) = \sqrt{A+1}\langle \Phi_{M_2}^{J_2\pi_2}(^{12}\text{C})|\Psi_{M_1}^{J_1\pi_1}(^{13}\text{C})\rangle = \sum_{jl}\langle J_1 M_1|J_2 M_2 j M_1 - M_2\rangle \varphi_{jl}^{\pi_1\pi_2}(r)[Y_l \otimes \chi]_{jM_1-M_2},$$

$$S_{jl}^{\pi_1\pi_2} = \int_0^\infty dr \; r^2 |\varphi_{jl}^{\pi_1\pi_2}(r)|^2. \qquad (5)$$

3. Results

By applying the HON constraint, various $3\alpha + n$ cluster structures which constitute the non-yrast states are generated as shown in Fig. 1. The fact that these cluster structures are obtained without any assumption clearly demonstrates that the clustering is an essential degree-of-freedom of nuclear excitation together with the single-particle and collective motions. The panels (a)-(d) show the triangular configurations of $3\alpha + n$ which constitute the $1/2_2^+$ state analogous to the 0_2^+ state of ^{12}C, and the panel (e) shows the linear-chain configuration of 3α particles with a valence neutrons which constitutes the doublet band of $K^\pi = 1/2^\pm$ as discussed below.

By performing the GCM calculation, we obtained many yrast and non-yrast states shown in Fig. 2 (a). It should be noted that many non-yrast states corresponding to the observations are described by the HON constraint, although their excitation energies are overestimated by 5 MeV in average. Among those states we focus on the $1/2_2^+$ state at 15.7 MeV and the $K^\pi = 1/2^\pm$ bands built on the $1/2^\pm$ states at 18.0 and 14.9 MeV, respectively.

3.1. The hoyle analogue $1/2_2^+$ state
To search for the hoyle analogue states with a s or p-wave valence neutron, we have investigated the ^{12}C(0_2^+) $\otimes n(s_{1/2}, p_{1/2}, p_{3/2})$ S-factors for all of the obtained $1/2^+$, $1/2^-$ and $3/2^-$ states. As a result, it was found that the ^{12}C(0_2^+) $\otimes n(p_{1/2}, p_{3/2})$ S-factors are strongly fragmented into many states (it is 0.2 at most for a single state) and does not exist as a single-particle states.
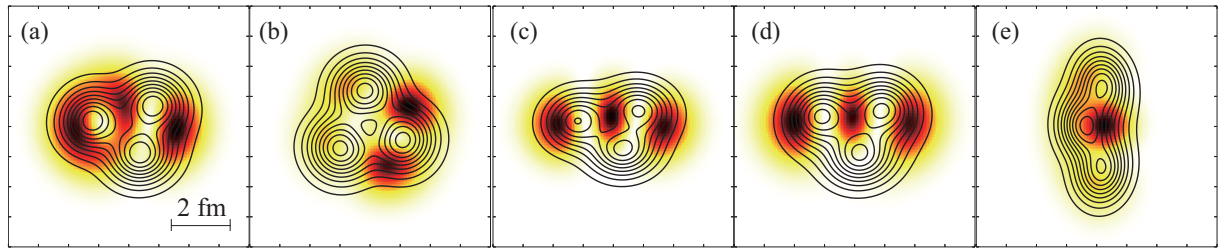
Figure 1. The density distributions of the $3\alpha + n$ cluster states obtained by applying the HON constraint. The contour lines show the proton density distributions that are almost same as those of the 3α cluster core, and the color plots show the valence neutron orbits. (a)-(d) are the dominant component of the $1/2_2^+$ state, while (e) constitutes the doublet bands of the $K^\pi = 1/2^\pm$.
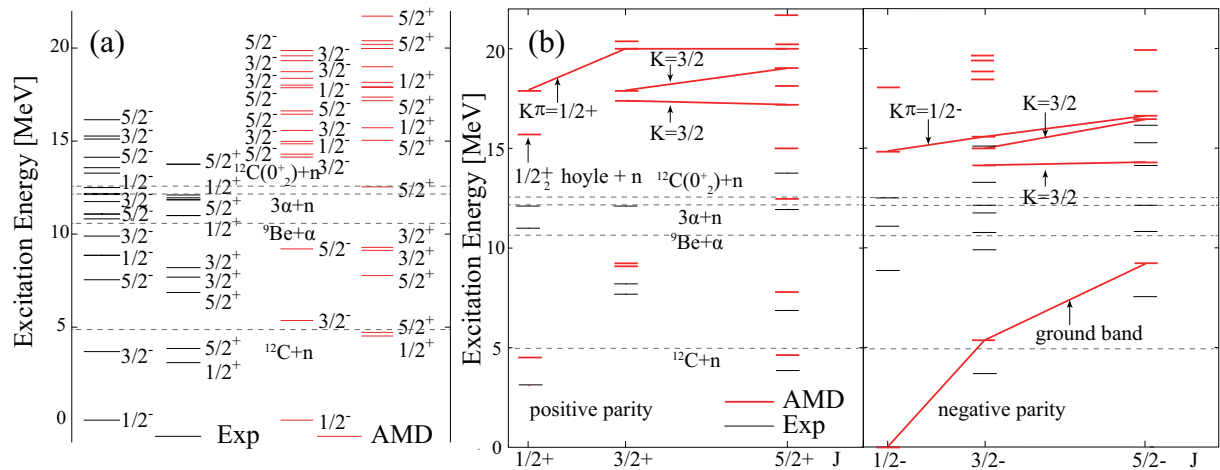
Figure 2. The panel (a) compares the calculated and observed spectra up to $5/2^\pm$ states. The panel (b) shows the spectra as function of the angular momentum for each parity. The lines in panel (b) are the guide of eyes to show the ground band and linear-chain parity doublet.

Therefore, we conclude that the p-wave states coupled to the hoyle state discussed in Ref. [6] may not exist. On the other hand, the $1/2_2^+$ state carries 0.5 of the $^{12}\text{C}(0_2^+) \otimes n(s_{1/2})$ S-factor, and hence, it is understood as a single-particle s-wave state coupled to the Hoyle state. Indeed, similar to the Hoyle state, this state cannot described by a single Slater determinant but a linear combination of many Slater determinants with different cluster configurations as illustrated in Fig. 1 (a)-(d). Furthermore, this state has a larger radius (2.78 fm) compared to the ground state (2.52 fm) indicating analogous nature to the Hoyle state. However, it also should be noted that the gas-like nature is distorted considerably due to the interaction between the core and valence neutrons. For example, the radius of the $1/2_2^+$ is much smaller than the Hoyle state (2.90 fm), and it also couples to the other channels such as $^{12}\text{C}(0_1^+) \otimes n(s_{1/2})$.

3.2. $3\alpha + n$ linear-chain doublet bands
Different from the $1/2_2^+$ state, the $1/2_3^+$ state is well described by a single Slater determinant shown in Fig. 1 (e) indicating the strong-coupling between 3α and valence neutron, and rigid rotor nature. Indeed a rotational band with $K^\pi = 1/2^+$ is built on this state that has very enhanced intra-band $E2$ transition probabilities. Therefore, we conclude that this band is a

$3\alpha + n$ linear-chain band with very strong deformation. Furthermore, this linear-chain wave function has parity asymmetry, and hence, constitute the parity-doublet band. As shown in Fig. 2 (b), $K = 1/2$ rotational bands with very large moment-of-inertia appear in both parity, and both of them are well described by a single Slater determinant shown in Fig. 1 (e).

4. Summary

In summary, we have investigated the $3\alpha + n$ cluster states on the basis of AMD calculation. By introducing HON constraint, $3\alpha + n$ cluster states are described well. We predict two different kinds of the cluster states, the hoyle analogue state and the linear-chain state. By the analysis of the S-factor, we found that the $1/2_2^+$ state is understood as the 0_2^+ state (Hoyle state) of ^{12}C accompanied by a valence neutron occupying the s-wave. We also conclude that the p-wave states coupled to the hoyle state may not exist as a single state. The linear-chain state appears as a $K = 1/2$ rotational bands in both parity owing to its parity asymmetric structure.

Acknowledgments

References

[1] Tohsaki A, Horiuchi H, Schuck P and Röpke G 2001 *Phys. Rev. Lett.* **87** 192501
[2] Funaki Y, Tohsaki A, Horiuchi H, Schuck P and Röpke 2003 *Phys. Rev. C* **67** 051306
[3] Morinaga H 1956 *Phys. Rev.* **101** 254
[4] Kanada-En'yo Y 1998 *Phys. Rev. Lett.* **81** 5291
[5] Chernykh M, Feldmeier H, Neff T, von Neumann-Cosel P and Richter A 2007 *Phys. Rev. Lett.* **98** 032501
[6] Yamada T, Horiuchi H and Schuck P 2006 *Mod. Phys. Lett.* A **21** 2373
[7] Itagaki N, Okabe S, Ikeda K and Tanihata I 2001 *Phys. Rev. C* **64** 014301
[8] von Oertzen W, *et al. Eur. Phys. J.* A **21** 193
[9] Suhara T and Kanada-En'yo K 2010 *Phys. Rev. C* **82** 044301
[10] Furutachi N and Kimura M 2011 *Phys. Rev. C* **83** 021303(R)
[11] Kanada-En'yo Y, Kimura M and Horiuchi H 2003 *C. R. Physique* **4**, 497
[12] Kanada-En'yo Y, Kimura M and Ono A 2012 *PTEP* **2012** 01A202
[13] Berger J F, Girod M and Gogny D 1991 *Comput. Phys. Comm.* **63** 365
[14] KimuraM 2004 *Phys. Rev. C* **69** 044319.
[15] Kanada-En'yo Y and Kimura M 2005 *Phys.Rev. C* **72** 064322