

Systematics of fine structure in the α decay of deformed odd-mass nuclei

Zhongzhou Ren^{1,2,3,4} and Dongdong Ni^{1,2}

¹Department of Physics and Key Laboratory of Modern Acoustics, Institute of Acoustics, Nanjing University, Nanjing 210093, China

²Joint Center of Nuclear Science and Technology, Nanjing University, Nanjing 210093, China

³Kavli Institute for Theoretical Physics China, Beijing 100190, China

⁴Center of Theoretical Nuclear Physics, National Laboratory of Heavy-Ion Accelerator, Lanzhou 730000, China

E-mail: dongdongnick@gmail.com and zren@nju.edu.cn

Abstract.

We present a detailed analysis of the α -decay fine structure in 32 deformed odd-mass nuclei from $Z = 93$ to $Z = 102$. The α -decay half-lives are systematically calculated within the multichannel cluster model (MCCM), which turns out to well reproduce the experimental data and show the neutron deformed shell structure. The branching ratios for various daughter states are investigated in the MCCM and in the WKB barrier penetration approach, respectively. It is found that the MCCM results agree well with the experimental data, while the WKB results have relatively large deviations from the experimental data for the α transitions to the high-lying members of the rotational band.

1. Introduction

Nearly a century ago, Geiger and Nuttall [1] found that the logarithm of the half-life in α decay is inverse proportional to the energy of the outgoing α particle, i.e., the decay energy Q_α . To understand the Geiger-Nuttall rule, the quantum mechanical explanation of α -decay as a quantum tunneling effect was reported independently by Gamow [2] and by Condon and Gurney [3]. Later on, Viola and Seaborg [4] proposed a simple formula for α -decay half-lives of heavy nuclei. The Viola-Seaborg formula is very successful in systematizing the data of α decay and widely used to predict the half-lives of unknown nuclei. Recently, the new Geiger-Nuttall law has been proposed [5] where the effects of the quantum numbers of α -core relative motion are naturally embedded. Since the pioneering work of Gamow [2], α -decay half-lives have been interpreted with improved accuracy for both spherical and deformed α -emitters based on various models and methods [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16]. Nowadays the new challenge in α -decay studies is the understanding of the α -decay fine structure observed in heavy deformed nuclei. It has been investigated from both semiclassical and coupled-channel standpoints [17, 18, 19, 20, 21, 22, 23, 24]. The semiclassical methods [17, 18, 19] treat each decay channel as a separate event for one α decay, ignoring the coupling effect of various decay channels. In contrast, the coupled-channel methods [20, 21, 22, 23, 24], based on the three-dimensional Schrödinger equation, employ various channel wave functions with outgoing wave boundary conditions and consider the coupling among the channels.



Recently, we have presented the multichannel cluster model (MCCM) to investigate the α -decay fine structure including half-lives and branching ratios (BRs) [22, 23, 24]. As most α -decay studies, we carried out our investigation following the path from even-even nuclei, to odd-mass nuclei, to odd-odd nuclei. In the coupled-channel study of even-even rotational nuclei, the diagonalization technique [25] and the multipole expansion [26, 27] are separately used to deal with the interaction matrix. Both of them give precise descriptions of the fine structure especially for the BR for excited 4^+ states [22]. In the coupled-channel study of odd- A and odd-odd nuclei, enough decay channels are considered and the calculated results show good agreement with the experimental data [23, 24]. To our knowledge, it was the first coupled-channel description of the fine structure observed in odd- A and odd-odd nuclei. The purpose of this contribution is to present an additional analysis of the α -decay fine structure observed in odd-mass nuclei as a supplement to the previous publication. By comparison with the semiclassical methods, one can discern the reliability of the MCCM based on the coupled-channel approach for the α -decay fine structure.

2. Theoretical models for fine structure observed in α decay

The coupled channel approach has been found to be successful in describing the fine structure observed in α decay. In particular, the MCCM has good applicability for the α decays of well-deformed even-even, odd-mass, and odd-odd nuclei [22, 23, 24]. Within the MCCM, the total wave function of the decaying system is expanded into a sum of partial waves and the cluster radial wave functions satisfy the following coupled equations [20, 21, 22, 23, 24]:

$$\left[-\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right) - (Q_0 - E_I) \right] u_\alpha(r) + \sum_{\alpha'} V_{\alpha,\alpha'}(r) u_{\alpha'}(r) = 0, \quad (1)$$

where $\alpha \equiv (n\ell I)$ labels the channel quantum numbers, ℓ is the angular momentum carried by the emitted α particle, Q_0 is the Q_α value for the decay to ground states, E_I is the excitation energy of the daughter states involved in α transitions, and $V_{\alpha,\alpha'}(r)$ is the matrix element of the interaction V taken between channels α and α' . As usual, the deformed potential V consists of the nuclear part of a simple axially deformed Woods-Saxon (WS) form and the Coulomb part in the first-order $\sum_\lambda \beta_\lambda Y_{\lambda 0}(\theta)$ [22]. Also, the multipole expansion of the potential V is used to deal with the interaction matrix elements where the dynamics of rotational core nuclei are considered. The details of the evaluation of the interaction matrix elements can be found in [22, 26, 27].

The coupled equations (1) are solved with outgoing Coulomb-Hankel wave boundary conditions. Moreover, the experimental decay energies ($Q_0 - E_I$) of each channel are exactly reproduced, and the quantum numbers follow the Wildermuth rule $G = 2n + \ell$ [28] which is an approximation treatment of the Pauli exclusion principle. All these requirements can be simultaneously achieved by adjusting the depth of the nuclear potential. After one obtains the radial wave functions, the partial width of the channel $I\ell$ is given by [22, 23, 24, 26]

$$\Gamma_{I\ell} = \frac{\hbar^2 k_I}{\mu} \frac{|u_{n\ell I}(R)|^2}{G_\ell(k_I R)^2 + F_\ell(k_I R)^2}, \quad (2)$$

where R denotes large distances beyond the range of the nuclear potential and beyond the distance where the Coulomb potential can be regarded as spherically symmetric.

In view of the fact that the internal structure of nuclear states has some influences on α transitions as well, the hypothesis of the Boltzmann distribution (BD) for daughter states is proposed $\rho(E_I) = \exp(-cE_I)$ [22, 23, 24], where E_I (in MeV) is the excitation energy of the daughter state I . As early as 1917, Einstein [29] had proposed such a similar hypothesis on

the quantum theory of radiation for molecules with a set of discrete states, that is, canonical distribution of states. This hypothesis implies that α -preformation factors have an exponential dependence on the excitation energy of daughter states and show a gradual decline with increasing daughter spin. This is consistent with the other theoretical studies [30, 31]. In order to achieve absolute α -decay half-lives, one also needs to multiply the partial width by an α -preformation factor P_α . Base on the available experimental facts and theoretical analysis [15, 32, 33], the α -preformation factor is kept the same for one certain kind of α emitters (even-even, odd- A , or odd-odd nuclei), making the number of free parameters to a minimum. In this way, one can obtain: $P_\alpha = 0.36$ for even-even nuclei, $P_\alpha = 0.18$ for odd- A nuclei, and $P_\alpha \approx 0.12$ for odd-odd nuclei [22, 23, 24]. Ultimately, the total width is given by $\Gamma = \sum_{I\ell} P_\alpha \rho(E_I) \Gamma_{I\ell}$ and the total α -decay half-life is $T_{1/2} = \hbar \ln 2 / \Gamma$. The BR for a daughter state I is expressed as

$$\text{BR}_I^{\text{MCCM}} = P_\alpha \rho(E_I) \sum_{\ell} \Gamma_{I\ell} / \Gamma \times 100\%. \quad (3)$$

Moreover, one can also describe the α -decay fine structure using the one-dimensional Wentzel-Kramers-Brillouin (WKB) approximation. In this case, each decay channel is treated as a separate event, and the partial widths for various channels are separately calculated with slightly different decay energies and various centrifugal barriers. The WKB barrier penetration probability for the channel $I\ell$ is written as $P_{I\ell} = \exp(-G)$, with

$$G = \frac{2}{\hbar} \int_R^B \sqrt{2\mu [V(r) - (Q_0 - E_I)]} dr, \quad (4)$$

where $V(r)$ is the sum of the Coulomb and centrifugal potentials, $V(r) = Z_c Z_d e^2 / r + \ell(\ell + 1) \hbar^2 / (2\mu r^2)$, B is the classical turning point satisfying the expression $V(B) = Q_0 - E_I$, and R is the touching radius $R = 1.20 A^{1/3}$ fm. It should be particularly noted that the height of the centrifugal barrier at $r = R$ is significantly small relative to the height of the Coulomb barrier at $r = R$,

$$\xi = \frac{\ell(\ell + 1) \hbar^2}{2\mu R^2} / \frac{Z_c Z_d e^2}{R} \simeq 0.002 \ell(\ell + 1). \quad (5)$$

Using elementary integration techniques and some approximations, one can obtain [8, 24]

$$G = \frac{Z_c Z_d e^2 \pi \sqrt{2\mu}}{\hbar \sqrt{Q_0 - E_I}} - \frac{4 \sqrt{2\mu Z_c Z_d e^2 R}}{\hbar} \left(1 - \frac{\xi}{2}\right). \quad (6)$$

It is seen that the effect of the centrifugal barrier is small but non-negligible. Combining with the BD hypothesis, the BR for a daughter state I is written as

$$\text{BR}_I^{\text{WKB}} = \frac{\rho(E_I) \sum_{\ell} P_{I\ell}}{\sum_{I\ell} \rho(E_I) P_{I\ell}} \times 100\%. \quad (7)$$

3. Numerical results and discussion

For odd-mass α emitters, the structure of the decaying state is generally different from that of the ground-state rotational band in the daughter nuclei [34]. As a consequence, there exists competition between α transitions to the ground-state rotational band and to the favored rotational band whose structure is similar to that of the decaying state. In general, the transitions to the favored band play a major role in the α decay of odd- A nuclei [34]. In the following, we will focus on such α transitions. Note that enough channels are considered in numerically integrating the coupled equations (1) for proper convergence. This means that all

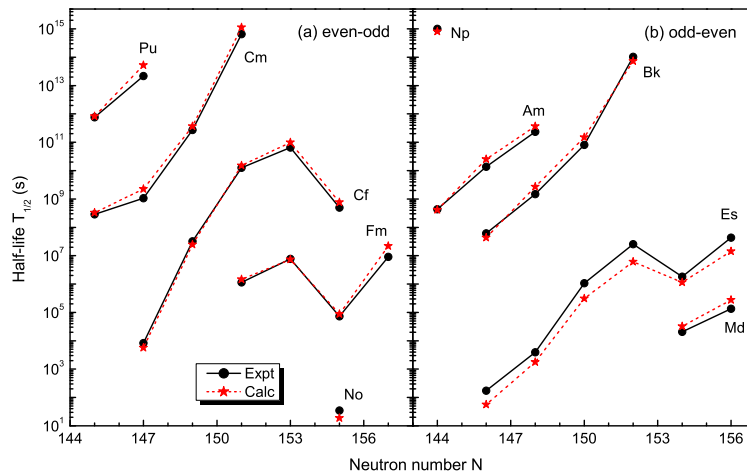


Figure 1. (Color online) Comparison of the calculated α -decay half-lives with the experimental data (a) for deformed even-odd nuclei with $Z = 94 - 102$ and (b) for deformed odd-even nuclei with $Z = 93 - 101$. The deformed neutron shell effect is shown at the neutron number $N = 152$.

partial waves with $\ell \leq 8$ are included in the present calculations [22, 23]. According to this, the number of partial waves required here becomes much larger as compared with the case of even-even nuclei. This is because an α transition $J^\pi \rightarrow I^{\pi'}$ in odd- A nuclei allows several decay channels that are characterized by the even ($\pi\pi' = 1$) or odd ($\pi\pi' = -1$) ℓ values in the range of $|J - I| \leq \ell \leq (J + I)$. For example, the α decay of odd- A nuclei from ground $5/2^+$ states to $K = 5/2^+$ rotational bands exhibits 21 decay channels, while the α decay of even-even nuclei from ground 0^+ states to ground-state 0^+ rotational bands exhibits only 5 decay channels.

First, we examine the evaluation of total α -decay half-lives. The comparison of the calculated α -decay half-lives with the experimental data is illustrated in figure 1 for the $Z = 93 - 102$ isotopes. Circles denote the experimental data and stars stand for the theoretical results. One can see that the theoretical results follow the experimental data well, although the half-lives span many orders of magnitude from 10^1 to 10^{15} s. In general, the decay energies of an isotopic chain decrease with increasing neutron number, leading to an increase in the half-life. But there is an abnormal decrease in the half-life across the neutron number $N = 152$, as shown in the Cf, Es, and Fm isotopic chains. This is attributed to the deformed neutron shell closure $N = 152$.

Next, we pay attention to the BRs for various daughter states. Figure 2 displays the comparison of the measured and calculated BRs for the α transitions to seven low-lying members of the favored rotational bands. Here, our calculations are separately performed within the MCCM and in the WKB approximation. In figure 2, the upper panel (a) shows the results calculated within the MCCM and the lower panel (b) shows the results calculated in the WKB approximation. For the MCCM calculations, the calculated results show good agreement with the experimental data including the BRs for the high-lying members of the favored band. Specifically, the values of $\log_{10}[\text{BR}(\text{calc})/\text{BR}(\text{expt})]$ are generally within the range of about ± 0.5 , which corresponds to the values of the ratio $\text{BR}(\text{calc})/\text{BR}(\text{expt})$ within the range of about 0.32–3.2. The largest deviation occurs at the α transition from ^{249}Cf to the fifth member of the favored band. For the WKB calculations, the calculated results agree well with the experimental data for the transitions to the low-lying three members of the favored band. But the deviations from the experimental data become quite large as we proceed to the other four members. For the transition to the seventh member of the favored band, the WKB calculations underestimate

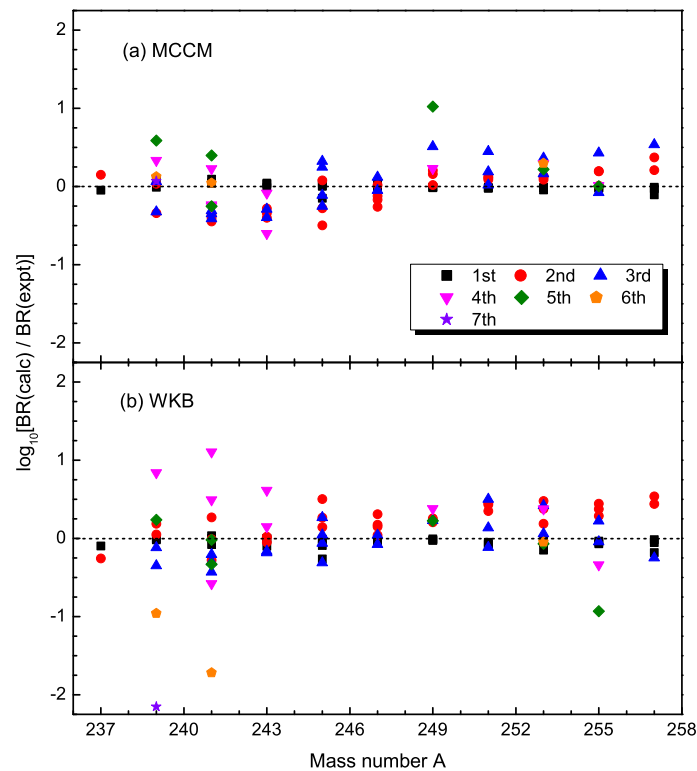


Figure 2. (Color online) Comparison of the calculated and experimental branching ratios as a function of the parent mass number A . Calculations within the MCCM and in the WKB approximation are, respectively, shown in the upper and lower panels. Squares, circles, uptriangles, downtriangles, diamonds, pentagons, and stars represent the transitions to the low-lying seven members of the favored rotational band, respectively.

the BR by about two orders of magnitude. On the whole, the points of all kind gather toward the line $\log_{10}[\text{BR}(\text{calc})/\text{BR}(\text{expt})] = 0$ more evidently within the MCCM than in the WKB approximation. This is particularly considerable for the transitions to the high-lying members of the favored band. In the present study, the standard deviation of the MCCM calculations for 103 BRs is $\sigma = \{\sum_{i=1}^{103} [\log_{10}(\text{BR}_{\text{calc}}^i/\text{BR}_{\text{expt}}^i)]^2/102\}^{1/2} = 0.254$ corresponding to a factor of roughly 1.80, while the standard deviation of the WKB calculations is 0.41 corresponding to a factor of about 2.57. This demonstrates that the MCCM is appropriate and successful in describing the α -decay fine structure. Along with the enhancement of experimental sensitivity and the development of experimental techniques, measurements of the BRs would be most welcome to further examine the validity and reliability of the MCCM.

4. Summary

In summary, the α -decay half-lives of 32 odd-mass nuclei with $Z = 93-102$ are calculated within the MCCM and compared with the experimental data. It is found that the theoretical results are in good agreement with the experimental data and the deformed neutron shell effect is reflected at $N = 152$. Moreover, the branching ratios for the transitions to the favored rotational band are

evaluated within the MCCM. For comparison, they are calculated in the WKB approximation as well. It turns out that the results obtained from the WKB approximation deviate from the experimental data evidently for the transitions to the high-lying members of the favored band, while the results obtained from the MCCM agree well with the experimental data. This confirms the good reliability of the coupled-channel approach so that coupled-channel predictions are allowed for the α -decay fine structure in the heavier mass region.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant Nos. 10735010, 10975072, 11035001, 11120101005, and 11375086), by 973 National Major State Basic Research and Development of China (Grants Nos. 2013CB834400 and 2010CB327803), by a Project Funded by the Priority Academic Programme Development of Jiangsu Higher Education Institutions (PAPD), by the Research Fund of Doctoral Point (RFDP; Grant No. 20100091110028), by the Science and Technology Development Fund of Macau (Grant No. 068/2011/A), and by a China Postdoctoral Science Foundation funded project.

References

- [1] Geiger H and Nuttall J M 1911 *Philos. Mag.* **22** 613
- [2] Gamow G 1928 *Z. Phys.* **51** 204
- [3] Condon E U and Gurney R W 1928 *Nature* **122** 439
- [4] Viola V E and Seaborg G T 1966 *J. Inorg. Nucl. Chem.* **28** 741
- [5] Ren Y and Ren Z 2012 *Phys. Rev. C* **85** 044608
- [6] Buck B, Merchant A C, and Perez S M 1993 *At. Data Nucl. Data Tables* **54** 53
- [7] Royer G 2000 *J. Phys. G* **26** 1149
- [8] Poenaru D N, Plonski I H, and Greiner W 2006 *Phys. Rev. C* **74** 014312
- [9] Xu C and Ren Z 2006 *Phys. Rev. C* **73** 041301(R) Xu C and Ren Z 2006 *Phys. Rev. C* **74** 014304
- [10] Denisov V Yu and Ikezoe H 2005 *Phys. Rev. C* **72** 064613
- [11] Mohr P 2006 *Phys. Rev. C* **73** 031301(R)
- [12] Kelkar N G and Castañeda H M 2007 *Phys. Rev. C* **76** 064605
- [13] Chowdhury P R, Samanta C, and Basu D N 2008 *Phys. Rev. C* **77** 044603
- [14] Delion D S, Liotta R J, Schuck P, Astier A and Porquet M G 2012 *Phys. Rev. C* **85** 064306
- [15] Ni D and Ren Z 2009 *Phys. Rev. C* **80** 051303(R) Ni D and Ren Z 2010 *Phys. Rev. C* **81** 024315
- [16] Qian Y, Ren Z and Ni D 2011 *Phys. Rev. C* **83** 044317
- [17] Xu C and Ren Z 2006 *Nucl. Phys. A* **778** 1
- [18] Denisov V Yu and Khudenko A A 2009 *Phys. Rev. C* **80** 034603
- [19] Santhosh K P, Sahadevan S, and Joseph J G 2011 *Nucl. Phys. A* **850** 34
- [20] Delion D S, Peltonen S, and Suhonen J 2006 *Phys. Rev. C* **73** 014315
- [21] Peltonen S, Delion D S, and Suhonen J 2008 *Phys. Rev. C* **78** 034608
- [22] Ni D and Ren Z 2010 *Phys. Rev. C* **81** 064318 Ni D and Ren Z 2011 *Phys. Rev. C* **83** 067302
- [23] Ni D and Ren Z 2012 *Phys. Rev. C* **86** 054608
- [24] Ni D and Ren Z 2013 *Phys. Rev. C* **87** 027602
- [25] Hagino K, Rowley N, and Kruppa A T 1999 *Comput. Phys. Comm.* **123** 143
- [26] Kruppa A T, Barmore B, Nazarewicz W, and Vertse T 2000 *Phys. Rev. Lett.* **84** 4549 Barmore B, Kruppa A T, Nazarewicz W, and Vertse T 2000 *Phys. Rev. C* **62** 054315
- [27] Suzuki Y and Ohkubo S 2010 *Phys. Rev. C* **82** 041303(R)
- [28] Wildermuth K and Tang Y C 1994 *A Unified Theory of the Nucleus* (Academic Press, New York)
- [29] Einstein A 1917 *Phys. Z* **18** 121
- [30] Stewart T L, Kermode M W, Beachey D J *et al* 1996 *Phys. Rev. Lett.* **77** 36
- [31] Delion D S 2009 *Phys. Rev. C* **80** 024310
- [32] Hodgson P E and B  t  k E 2003 *Phys. Rep.* **374** 1
- [33] Iriondo M, Jerrestam D and Liotta R J 1986 *Nucl. Phys. A* **454** 252
- [34] NNDC of the Brookhaven National Laboratory, <http://www.nndc.bnl.gov>.