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Many-body resonances and continuum states above many-body decay thresholds

K. Katō[1], T. Myo[2,3] and Y. Kikuchi[4]

[1] Nuclear Reaction Data Centre, Faculty of Science, Hokkaido University, Sapporo 060-0810, Japan
[2] General Education, Faculty of Engineering, Osaka Institute of Technology, Osaka, Osaka 535-8585, Japan
[3] Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan
[4] RIKEN Nishina Center, Wako, Saitama 351-0198, Japan

E-mail: kato@nucl.sci.hokudai.ac.jp

Abstract. We study the resonance spectroscopy of the He isotopes and their mirror, proton-rich nuclei with an $\alpha + N + N + N + N$ cluster model. Many-body resonances are treated with the correct boundary condition as the Gamow states using the complex scaling method. We obtain the resonances up to the five-body decaying states in ^{8}He and ^{8}C. The spectrum agrees with the recent experiment systematically for energies and decay widths and we predict several resonances. We also discuss the mirror symmetry breaking of He isotopes and their mirror nuclei for the configurations and the spatial distributions of the valence nucleons.

1. Introduction

Unstable nuclei have often been observed in unbound states beyond the particle thresholds due to the weak binding nature of valence nucleons. The resonance spectroscopy of unbound states in unstable nuclei has been developed using radioactive-beam experiments [1, 2]. In addition to the energies and decay widths of resonances, information on their configurations and spatial properties is important to understand the structures of unstable nuclei. In proton- and neutron-rich nuclei, the correlations between valence nucleons and those between a valence nucleon and a core nucleus lead to the exotic nuclear properties in resonances as well as in the weakly bound states. The comparison of the structures between proton- and neutron-rich nuclei is important to examine the mirror symmetry in unstable nuclei with a large isospin.

In this report, we show the results of the recent applications of the complex scaling method (CSM) to the spectroscopy of unstable nuclei. We focus on the structures of the neutron-rich He isotopes and their mirror proton-rich nuclei, most of the states of which are unbound owing to the weak binding of valence nucleons to the α particle. We investigate the many-body resonances observed in these nuclei, with the correct boundary condition for the multi-particle emission using the CSM beyond the two-body case.

2. Complex-scaled cluster orbital shell model

We briefly explain the method of describing the resonance and non-resonant continuum states of He isotopes and their mirror nuclei on the basis of the cluster model with an α core. To

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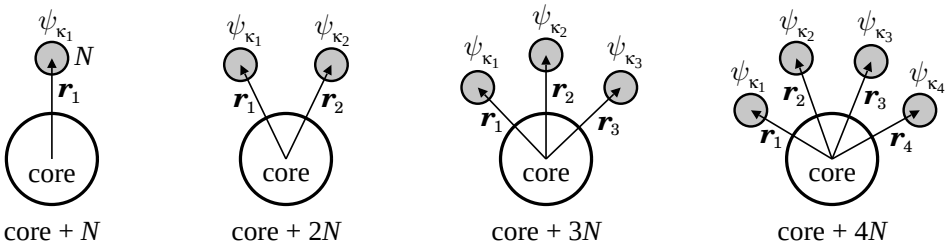


Figure 1. Sets of the spatial coordinates in CS-COSM.

describe the unbound states, it is essential to treat the boundary condition of the multi-particle emission and to solve the relative motion of each particle or cluster. For this purpose, we use the cluster orbital shell model with the complex scaling, called as CS-COSM, hereafter. We use the following Hamiltonian for the system consisting of α and valence nucleons [3, 4, 5, 6, 7]:

$$H_{\rm CS-COSM} = \sum_{i=1}^{N_{\rm v}} \left[\frac{\vec{p}_i^{\,2}}{2\mu} + V_i^{\alpha N} \right] + \sum_{i<j}^{N_{\rm v}} \left[\frac{\vec{p}_i \cdot \vec{p}_j}{(A_{\rm c}+1)\mu} + V_{ij}^{NN} \right], \quad (1)$$

where the operator $\vec{p}_i$ represents the relative momentum between a valence nucleon and α. The reduced mass μ is $mA_{\rm c}/(A_{\rm c}+1)$, where m is the nucleon mass and $A_{\rm c}=4$ of the α core. The number $N_{\rm v}$ is a valence nucleon number around the α core.

The total wave function $\Psi^{JT}_{\text{CS-COSM}}$ of the nucleus with a mass number A $(=A_{\rm c}+N_{\rm v})$, a spin J, and an isospin T, is represented by the superposition of the various configurations Φ_c^{JT} as

$$\Psi^{JT}_{\text{CS-COSM}} = \sum_c C_c^J \, \Phi_c^{JT}(A), \qquad \Phi_c^{JT}(A) = \prod_{i=1}^{N_{\rm v}} a^\dagger_{\kappa_i} |0\rangle, \quad (2)$$

where the vacuum $|0\rangle$ represents the α particle. The creation operator $a^\dagger_\kappa$ denotes the single-particle state of a valence nucleon above ^{4}He, with the quantum number $\kappa=\{n,\ell,j,t_z\}$ in a jj coupling scheme. Here, the index n is used to distinguish the different radial component of the single-particle state. The index c represents the set of κ as $c=\{\kappa_1,\cdots,\kappa_{N_{\rm v}}\}$. The expansion coefficients $\{C_c^J\}$ in Eq. (2) are determined by the diagonalization of the Hamiltonian matrix elements. The schematic illustration of the coordinate set is shown in Fig. 1 for a core + $N_{\rm v}N$ systems with $N_{\rm v}=1,\cdots,4$. We describe the radial component of the single nucleon wave function using the Gaussian expansion method [3, 8]. The antisymmetrization between a valence nucleon and α is treated on the orthogonality condition model [9], in which the single-particle state is imposed to be orthogonal to the $0s$ state occupied by an α core.

In the CS-COSM, the relative coordinates $\boldsymbol{r}_i$ between between a valence nucleon and α shown in Fig. 1, are transformed into $\boldsymbol{r}_i\, e^{i\theta}$ for $i=1,\cdots,N_{\rm v}$ to describe the many-body resonances and continuum states; here, θ is a scaling angle. Using the complex-scaled Hamiltonian $H^\theta_{\text{CS-COSM}}$, the complex-scaled Schrödinger equation is given as

$$H^\theta_{\text{CS-COSM}} \Psi^{JT,\theta}_{\text{CS-COSM}} = E^\theta_{JT} \Psi^{JT,\theta}_{\text{CS-COSM}}\,, \qquad \Psi^{JT,\theta}_{\text{CS-COSM}} = \sum_c C_c^{JT,\theta} \Phi_c^{JT}(A)\,. \quad (3)$$

The energy eigenvalues E^θ_{JT} are obtained on a complex energy plane. In the CS-COSM, we adopt a finite number of the radial basis states for valence nucleons, which provides a discretized representation of the continuum states in the CSM.

In Eq. (1), the α-N interaction $V^{\alpha N}$ consists of the KKNN potential [3, 10] for the nuclear part and the folded Coulomb part. We use the Minnesota potential [11] as the nucleon-nucleon

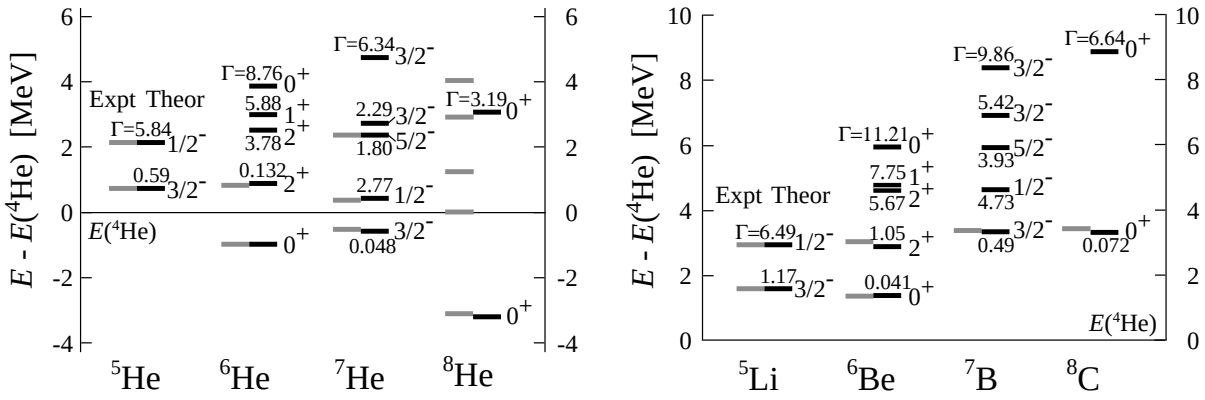


Figure 2. Energy levels of He isotopes (left) and their mirror nuclei (right) measured using the α energy. Units are in MeV. The black and gray lines correspond to the theoretical and experimental results, respectively. Small numbers denote theoretical decay widths.

interaction V^{NN} in addition to the point Coulomb interaction. For the single-particle states, we take the angular momenta $\ell \leq 2$ and adjust the two-neutron separation energy of ^{6}He to the experimental value of 0.975 MeV.

2.1. Results of He isotopes and their mirror nuclei

We show a systematics of energy levels observed experimentally and calculated by CS-COSM for He isotopes and their mirror nuclei in Fig. 2, measured from on the basis of the α particle. The small numbers near the levels represent the decay widths of the states. A good agreement is observed between the theoretically and experimentally obtained energy positions up to a five-body case of ^{8}He and ^{8}C. Further, there are several theoretical predictions of the excited states. The matter and charge radii of the ground states of ^{6}He and ^{8}He reproduce the recent experiments, as shown in Table 1. Hence, the CS-COSM wave functions describe the specially extended distributions of neutrons in the halo and skin structures observed in the He isotopes.

Firstly, we focus on the structures of ^{7}He and its mirror nucleus ^{7}B [6]. They are both unbound nuclei. Our CS-COSM predicts five resonances for each nucleus. For ^{7}He, the $3/2^-$ ground state is obtained as the two-body resonance located in the energy range between the thresholds of ^{6}He+n and α+3n. The other four states are four-body resonances above the α+3n threshold energy as shown in Fig. 2. For ^{7}B, the resonances are all located above the α+3p threshold energy and are interpreted as four-body resonances. There is no experimental data for the excited states of ^{7}B so far.

Table 1. Matter ($R_{\rm m}$) and charge ($R_{\rm ch}$) radii of ^{6}He and ^{8}He in comparison with the experimental values; a[12], b[13], c[14], d[15], and e[16]. Units are in fm.

		Present	Experiments
^{6}He	$R_{\rm m}$	2.37	2.33(4)[a] 2.30(7)[b] 2.37(5)[c]
	$R_{\rm ch}$	2.01	2.068(11)[d], 2.060(8)[e]
^{8}He	$R_{\rm m}$	2.52	2.49(4)[a] 2.53(8)[b] 2.49(4)[c]
	$R_{\rm ch}$	1.92	1.929(26)[d], 1.959(16)[e]

Table 2. S-factors of the ^{6}He-n components in ^{7}He.

	^{6}He(0_1^+)-n	^{6}He(2_1^+)-n
$3/2_1^-$	$0.63 + i0.08$	$1.60 - i0.49$
$3/2_2^-$	$0.00 - i0.01$	$0.97 + i0.01$
$3/2_3^-$	$0.01 + i0.00$	$0.04 - i0.01$
$1/2^-$	$0.95 + i0.03$	$0.07 - i0.02$
$5/2^-$	$0.00 + i0.00$	$1.00 + i0.01$

Table 3. S-factors of the ^{6}Be-p components in ^{7}B.

	^{6}Be(0_1^+)-p	^{6}Be(2_1^+)-p
$3/2_1^-$	$0.51 + i0.02$	$2.35 - i0.15$
$3/2_2^-$	$0.02 - i0.01$	$0.96 - i0.01$
$3/2_3^-$	$0.00 + i0.01$	$-0.01 - i0.06$
$1/2^-$	$0.93 - i0.02$	$0.10 - i0.01$
$5/2^-$	$0.00 + i0.00$	$1.04 - i0.01$

It is meaningful to discuss the mirror symmetry between ^{7}He and ^{7}B consisting of the α core and three valence neutrons or protons. We show the the spectroscopic factors (S-factors) of one-nucleon removal from each nucleus, the ^{6}He-n components of ^{7}He and the ^{6}Be-p components of ^{7}B. These quantities are important to examine the coupling behaviors between the $A = 6$ daughter nuclei and the last nucleon.

We list the results of the S-factors of ^{7}He and ^{7}B in Table 2 and 3, respectively. For the Gamow states, the S-factors can be complex numbers. In the results, most of the components are found to show almost the real values. A sizable difference is observed between the ground states of ^{7}He and ^{7}B in terms of the components including the $A = 6$ (2_1^+) states. The ^{6}Be(2_1^+)-p component in ^{7}B obtained as 2.35 (Table 3), is larger than the ^{6}He(2_1^+)-n component in ^{7}He as 1.60 (Table 2), by 47% for the real part. This difference indicates the breaking of the mirror symmetry in their ground states. The reason for the different values of the 2^+ couplings is that the ^{7}B ground state is located close to the ^{6}Be(2_1^+) state by 0.45 MeV in the energy as shown in Fig. 2. This situation does not occur in ^{7}He, in which the energy difference between ^{7}He$(3/2_1^-)$ and ^{6}He(2_1^+) is 1.46 MeV. The small energy difference between ^{7}B and ^{6}Be(2_1^+) enhances the ^{6}Be(2_1^+)-p component in ^{7}B because of the increase of the coupling to the open channel of the ^{6}Be(2_1^+)+p threshold. Contrastingly, the ^{6}Be(0_1^+)-p component in ^{7}B becomes smaller than that of ^{7}He, because the energy difference between the ground states of ^{7}B and ^{6}Be is 1.97 MeV, larger than that in the case of ^{7}He as 0.40 MeV. The difference between the S-factors in ^{7}He and ^{7}B originates from the Coulomb repulsion, which acts to shift the entire energy of the ^{7}B states upward with respect to the ^{7}He energy.

In conclusion, the mirror symmetry is largely broken only in the ground states of ^{7}He and ^{7}B, while the excited states of the two nuclei keep the symmetry approximately. In relation to the S-factors, we have calculated the one-neutron removal strength of ^{7}He to form ^{6}He as a function of the observed excitation energy of ^{6}He [4]. The dominant component comes from the 2^+ state , the strength of which peaks at the resonance energy of 0.84 MeV of ^{6}He(2_1^+).

Secondly, we compare the structures of ^{8}He and ^{8}C for their mirror 0^+ states as the five-body decaying states with an α core [7]. We investigate the role of the Coulomb interaction on the spatial properties of ^{8}C in comparison with ^{8}He. For this purpose, we calculate the various radii sizes of ^{8}He and ^{8}C, which reflect the spatial motion of four valence nucleons. We noted that the radius of Gamow states is obtained as a finite complex number, similar to the S-factors as was discussed. In the present analyses, most of the radii of resonances exhibit imaginary parts that are relatively smaller than the real ones, similar to case of the squared amplitudes [5, 7]. Hence, we discuss the spatial size of resonances using the real part of the complex radii.

The results of the radii of the $0_{1,2}^+$ states in ^{8}He and ^{8}C are listed in Table 4. We calculate the matter ($R_{\rm m}$), proton (R_p), neutron (R_n), and charge ($R_{\rm ch}$) radii, and the mean relative distances between α and a single valence nucleon ($r_{\text{c-}N}$) and between α and the center of mass of the four valence nucleons ($r_{\text{c-}4N}$).

Table 4. Various radii of the $0^+_{1,2}$ states of ^{8}He and ^{8}C in units of fm.

	$^8\mathrm{He}(0^+_1)$	$^8\mathrm{C}(0^+_1)$	$^8\mathrm{He}(0^+_2)$	$^8\mathrm{C}(0^+_2)$
R_m	2.52	$2.81 - 0.08i$	$7.56 + 2.04i$	$4.87 + 0.13i$
R_p	1.80	$3.06 - 0.10i$	$3.15 + 0.69i$	$5.46 + 0.15i$
R_n	2.72	$1.90 - 0.01i$	$8.53 + 2.32i$	$2.36 + 0.05i$
R_ch	1.92	$3.18 - 0.09i$	$3.22 + 0.67i$	$5.53 + 0.15i$
$r_{\text{c-}N}$	3.55	$4.05 - 0.12i$	$11.03 + 3.11i$	$7.21 + 0.21i$
$r_{\text{c-}4N}$	2.05	$2.36 - 0.03i$	$5.60 + 1.55i$	$3.68 + 0.13i$

For the 0^+_1 states, the matter radius of ^{8}C is larger than that of ^{8}He by about 12% in the real part. For 0^+_2 states, all the values of radii are complex numbers because of the resonances in the two nuclei. The imaginary parts are fairly large for $^8\mathrm{He}(0^+_2)$, but still smaller than the real ones. It is interesting that the matter radius of ^{8}C is smaller than that of ^{8}He in the 0^+_2 states. This relation is opposite to that observed for the ground states of ^{8}He and ^{8}C. For $^8\mathrm{He}(0^+_2)$, the observed large matter radius originates from the large neutron radius. For $^8\mathrm{C}(0^+_2)$, the large matter radius is due to the large proton radius, which is smaller than the neutron radius of ^{8}He.

We conclude that the relation of the spatial properties between ^{8}He and ^{8}C depends on the states, which can be explained in terms of the Coulomb interaction. The Coulomb interaction acts repulsively, thereby shifting the entire energy of ^{8}C upward with respect to the ^{8}He energy. In the ground state of ^{8}C, this repulsion extends the distances between α and a valence proton and between valence protons. On the other hand, the Coulomb interaction makes the barrier above the particle threshold in ^{8}C and the 0^+_2 resonance is affected by this barrier, the effect of which prevents the wave function of valence protons of ^{8}C from extending spatially. In ^{8}He, there is no Coulomb barrier for the four valence neutrons and the neutrons can extend to a large distance in the resonance. This role of the Coulomb interaction leads to the radius of $^8\mathrm{C}(0^+_2)$ being smaller than that of $^8\mathrm{He}(0^+_2)$.

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