

Spatial measure of reaction size in proton scattering

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Abstract. We introduce a scattering radius, which characterizes a spatial size of the scattering area, from partial wave decompositions of a cross section for a nucleon-nucleus scattering. The coupled-channel calculations of the $p + {}^{12}\text{C}$ scattering are performed in the range of $E_{\text{lab}} = 29.95$ MeV to 65 MeV, and the scattering radii for the elastic scattering and the various inelastic channels, which involve the rotational or vibrational excitations and the 3α excitations in ${}^{12}\text{C}$, are derived from the partial wave decomposition. We found that the scattering radii for the inelastic channels with a well developed 3α structure are strongly enhanced in comparison to the scattering radius for the elastic and collective channels. This enhancement of the scattering radius for the 3α channel strongly suggests that the scattering radius is sensitive to a size of the intrinsic structure of the finally excited state in the scattering process.

1. Introduction

Clustering phenomena, in which a nucleus is decomposed into some subunits, are well known to appear in the excited states of the lighter mass system [1]. In ${}^{12}\text{C}$, for instance, three α particles are strongly overlapped to each other in its ground 0_1^+ state, while they are weakly coupled in the excited 0_2^+ state, called Hoyle state, at $E_x = 7.65$ MeV [2,3]. One of characteristic features in cluster states is an extension of a matter radius. Previous theoretical calculations suggest that a radius of a cluster state is larger by about 50% than a radius of a ground state [1-3].

Unfortunately, a size of a cluster state is difficult to measure directly by experiments because a cluster state has a very short life-time. In a naive expectation, however, such a spatial extension may be reflected in observables of scattering phenomena. In recent studies, some enhancement factors originated from a spatial extension of a cluster structure are studied especially in the 3α state in ${}^{12}\text{C}$ [4, 5]. In Ref. [5], for instance, a diffraction radius of the Hoyle state in ${}^{12}\text{C}$ is speculated from the diffraction pattern observed in the differential cross section in the high energy proton scattering. In fact, a sign of an enhanced radius is confirmed for the Hoyle state of ${}^{12}\text{C}$.

Recently, we have proposed the method of the “scattering radius” on the basis of a partial wave decomposition of a scattering cross section [6]. The scattering radius is considered to characterize a spatial size of the reaction area for exclusive reactions. We can always derive the scattering radius just by doing the partial wave analysis of a scattering cross section. For the first time this method has been



applied to a simple two channel problem in the $p + {}^{12}\text{C}$ scattering going to the Hoyle state [6], where the prominent enhancement of the scattering radius in the Hoyle channel has been confirmed.

In this report, the method of the scattering radius is extended to the multi-channel problem in the $p + {}^{12}\text{C}$ scattering, going to the various inelastic channels, which contain the collective excitations of the 2_1^+ and 3_1^- states and the 3α excitations of the 0_2^+ and 2_2^+ states. The matter radius of the 2_2^+ state is expected to be strongly enhanced because the 2_2^+ state corresponds to the rotational excited states of the Hoyle state [3]. Thus, the scattering radius for the 2_2^+ state is very interesting in connection to the enhanced matter radius of this state.

2. Theoretical framework

2.1. Coupled-channel equations and nuclear interactions

We solve a set of the coupled-channel equation for the proton plus ${}^{12}\text{C}$ system, which is given in a symbolic form

$$(T_f(\mathbf{R}) + V_{f,f}(\mathbf{R}) - E_f)\chi_f(\mathbf{R}) = - \sum_{i \neq f} V_{f,i}(\mathbf{R}) \chi_i(\mathbf{R}) . \quad (1)$$

Here the subscripts of f and i design a channel. $T_f(\mathbf{R})$ represents the kinetic energy of the relative motion of the $p + {}^{12}\text{C}$ system with a relative coordinate $\mathbf{R}$, while $V_{f,i}(\mathbf{R})$ denotes the coupling potential for the transition from the channel i to the channel f . The total energy in the channel f , E_f , is given by the relation of $E_f = E - \varepsilon_f$ with a proton's incident energy E and an internal energy of ${}^{12}\text{C}$, ε_f . $\chi_f(\mathbf{R})$ is the proton- ${}^{12}\text{C}$ relative wave function for the channel f , which should be solved in the coupled-channel equations. In the present calculation, we include all the discrete states of ${}^{12}\text{C}$; the rotational state of 2_1^+ , the vibrational 3_1^- state, the 3α cluster states of 0_2^+ and 2_2^+ .

The coupling potential ($V_{f,i}(\mathbf{R})$) has a component of the real central ($V_{f,i}^{CE}(\mathbf{R})$) and the phenomenological imaginary potential ($-iU_{f,i}(\mathbf{R})$),

$$V_{f,i}(\mathbf{R}) = V_{f,i}^{CE}(\mathbf{R}) - iU_{f,i}(\mathbf{R})\delta_{f,i} . \quad (2)$$

The central potential, $V_{f,i}(\mathbf{R})$, is calculated by the folding model [7, 8]. We employ the DDM3Y (Density-Dependent Michigan 3-range Yukawa) [9, 10] effective nucleon-nucleon force and the internal wave function of ${}^{12}\text{C}$ calculated by the microscopic cluster model [3].

In addition to the folding potential, we introduce an imaginary potential $-iU_{f,i}(\mathbf{R})$ in the diagonal transition, which has a Woods-Saxon form factor [6], in order to reproduce the observed differential cross sections as much as possible. In the present calculation, we neglect the imaginary part of the off-diagonal potential and the spin-orbit potential although the latter potential was included in the analysis in Ref. [6].

Definition of the scattering radius

In the usual treatment, the coupled-channel equations (1) are solved by the partial wave decomposition. In the present calculation, the proton is treated as the spin-less particle, and the initial state of the target nucleus is also in the spin-less states. Thus, the differential cross section can be decomposed into the partial wave component, $\sigma(JLL'S)$, which is designed by the total spin J , the initial orbital spin L ($L=J$), the final spin of a target S , and the final orbital spin L' for the scattering two particles.

In order to characterize a size of scattering area, we simply define the effective orbital spin $\bar{L}$ from the partial cross section $\sigma(JLL'S)$ like

$$\bar{L} = \sqrt{\frac{\sum_{JL} \tilde{L}^4 \sigma(JL'S)}{\sum_{JL} \tilde{L}^2 \sigma(JL'S)}} \quad (3)$$

with $\tilde{L} = \sqrt{L(L+1)}$ which is calculated from the initial orbital spin L . The effective orbital spin is obtained by the fourth and second powers of the magnitude of the orbital spin, $\tilde{L} = \sqrt{L(L+1)}$. In Eq. (3), we simply assume the same functional form as the root-mean-squared radius for the matter density distribution. The scattering radius R_{SC} corresponding to $\bar{L}$ can be simply obtained from the relation of $\bar{L} = kR_{SC}$, where k denotes the wave number of the incident channel, measured in the laboratory system.

3. Results

We calculate the differential cross sections of the elastic and inelastic scattering in the energy range of $E_{lab}=29.95$ MeV to 65 MeV, and optimize the parameters in the imaginary potential for the diagonal transition so as to reproduce the observed cross sections. Figure 1 shows the comparison of the theoretical calculation with the experimental data at $E_{lab} = 65$ MeV. As can be clearly confirmed from Fig.1, the observed cross sections are nicely reproduced by the present coupled-channel calculation although the reproduction of the detailed oscillation is still insufficient in the 0_2^+ channel.

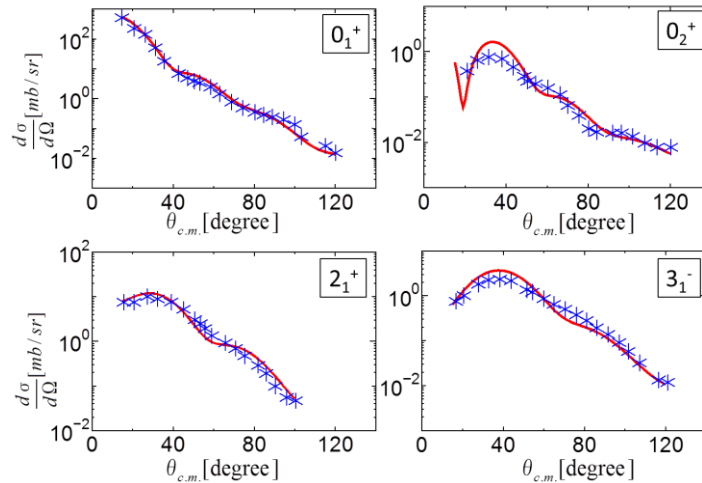


FIG. 1: Differential cross sections at $E_{lab} = 65$ MeV in the 0_1^+ (g.s.), 0_2^+ (3α state), 2_1^+ and 3_1^- channels. The stars and curves represent the experimental data and the theoretical calculation, respectively.

From the scattering calculation in Fig.1, we can obtain the partial cross section. In Fig.2, we show the distributions of the partial cross section $\sigma(JL)$ for the spin-less channels of the 0_1^+ (asterisks) and 0_2^+ (squares). In this figure, only the nuclear part of the partial cross sections is plotted, and the Coulomb part is subtracted in the elastic 0_1^+ channel. Here $\sigma(JL)$ is originally labeled by J and L , but the relation of $J=L$ is hold because the proton is treated as the spin-less particle. The values of $\sigma(J=L)$ are plotted in Fig.2, which are normalized by the total cross section of $\sigma = \sum_J \sigma(J)$. In this figure, we can clearly observe the partial cross section of the 0_2^+ channel has the extended distribution. This extension strongly suggests that a size of the scattering area for the inelastic 0_2^+ channel is

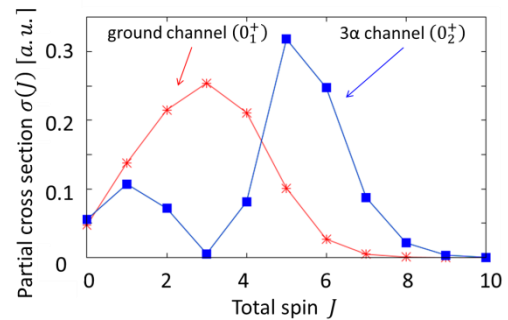


FIG. 2: Distributions of the partial cross sections at $E_{lab} = 65$ MeV. Both of the partial cross sections are divided by the total cross section.

larger than that for the elastic channel.

We have also observed the distributions of the partial cross sections of other inelastic channels. The distributions of the partial cross sections for the 3_1^- and 2_2^+ channels especially spread over a wide range of the total spin J . In order to characterize the spatial size of each reaction channel more clearly, the effective orbital spins $\bar{L}$ and the scattering radii, R_{SC} are calculated for all the channels from the partial cross section. The obtained effective orbital spins $\bar{L}$ and the scattering radii R_{SC} are summarized in table 1.

In this table, the matter radii of all the states ($\bar{r}$) [2] are also shown in the bottom row for comparison.

From table 1, we can see the enhancement of R_{SC} for the inelastic scattering going to the 0_2^+ , 2_2^+ and 3_1^- states, which are the resonance states just above the 3α threshold, in comparison to the scattering radius of the 0_1^+ and 2_1^+ states. The enhancement of the scattering radii in the 0_2^+ and 2_2^+ channels can be attributed to the spatial extension of the 3α structures of these states. On the contrary, there is an enhancement of the scattering radius in the 3_1^- channel although this state does not have the 3α -cluster structure but the spatially compact structure [1, 2]. The 3_1^- state is a resonant state existing above the 3α threshold of $E_{3\alpha}=7.27$ MeV ($E_x=9.6$ MeV). Therefore, the 3_1^- state has an extended tails in its radial wave function. This extension is considered to be one of the origins for the enhanced scattering radius of the 3_1^- state. In particular, the R_{SC} of the 2_2^+ is strongly enhanced. This is very interesting result, because the 2_2^+ state is 3α rotational state, which has much more extended matter radius, and this large radius has already been pointed out by Funaki et al. [3].

The scattering radius does not necessarily mean the matter radius of a target nucleus itself but our scattering radius is consistent to the matter radius of the 2_2^+ state. Thus, the scattering radius derived from the partial cross section can be used for the measure of the spatial size of the individual reaction channels.

Finally, we discuss the incident energy dependence of the scattering radii R_{SC} , which are shown in Fig.3. The results in Fig.3 represent that the scattering radius doesn't depend so much on the incident energy. Therefore, the qualitative features of the scattering radius are almost invariant with respective to the variation of the incident energy. In all the energy range considered here, the scattering radius of the 2_2^+ state (triangles) is the largest of all the channels, while the enhancements are commonly observed in the 0_2^+ (squares) and 3_1^- (open circles) channels. In contrast to these enhanced channels, the scattering radii of the ground 0_1^+ (asterisk) and its rotational state of 2_1^+ (black circles) are clearly suppressed. The magnitude correlation in the scattering radii observed in Fig.3 is the consistent to the matter radii predicted in the 3α cluster model shown in table 1 although the order of 0_2^+ and 3_1^- is inversed only at $E_{lab}=29.95$ MeV.

The error bars attached to individual symbols mean the theoretical ambiguity of the potential parameters. In the present coupled-channel calculation, a simultaneous reproduction of all the observed cross sections is a little difficult in a common parameter set of the imaginary potential and the strength of the folding potential. Thus, we have searched the several parameter sets of the potential, which can reproduce each of exit channels. For example, one parameter set is optimized for the reproduction of the elastic channel, while the other parameter set can reproduce only the Hoyle state, and so on. From the searched potential parameter sets, we can also evaluate the various scattering

	0_1^+	0_2^+	2_1^+	2_2^+	3_1^-
$\bar{L}$	4.64	6.37	5.65	6.82	5.96
R_{SC} [fm]	2.73	3.74	3.32	4.01	3.51
$\bar{r}$ [fm]	2.40	3.47	2.38	4.00	2.76

Table 1: The effective orbital spin $\bar{L}$ and the scattering radius R_{SC} at $E_{lab} = 65$ MeV. The matter radius $\bar{r}$ of the 0_1^+ , 0_2^+ , 2_1^+ , 2_2^+ and 3_1^- states are also shown in the bottom column [2]. R_{SC} and $\bar{r}$ are shown in units of fm.

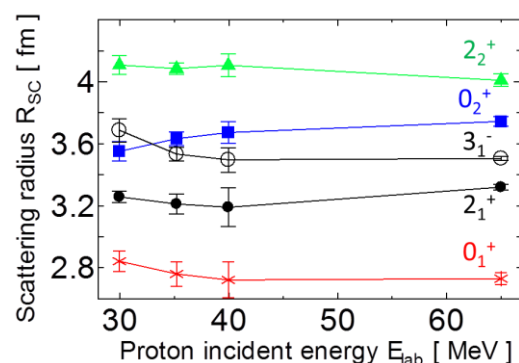


FIG. 3: The incident dependence of the scattering radius. Asterisk, black circle, open circle, square and delta are 0_1^+ , 2_1^+ , 3_1^- , 0_2^+ and 2_2^+ states respectively.

radii. The error bars mean the standard deviation of a set of the scattering radii, calculated from a set of the potential parameters. The resultant magnitude of the error reaches about $\pm 10\%$, at most, in all the exit channels although the error bars may be changed in the future parameter search. Thus, the qualitative features of the scattering radius are not changed so much even if much more tuned potentials in comparison to the present calculation are employed in the coupled-channels calculation.

Summery and discussion

In summary, we have introduced a scattering radius from the partial wave decomposition of the differential cross sections. The method of the scattering radius has been applied to the multi coupled-channel problem for the $p + {}^{12}\text{C}$ scattering, and the scattering radii of the elastic and inelastic scattering to 0_2^+ , 2_1^+ , 2_2^+ and 3_1^- states have been derived. We have found a clear enhancement of the scattering radius for the final inelastic channels in comparison to the elastic channel. In particular, we confirmed the enhanced scattering radius in the cluster channels (0_2^+ , 2_2^+). This enhanced scattering radius for the cluster channels is attributed to the spatial extension of the 3α structure.

The scattering radius is considered to give size information on a spatial area, where the elastic scattering or the transition to a final channel occurs. In principle, the scattering radius can always be defined when the partial wave analysis is possible for the differential cross section. In a recent study, effective radii in the deuteron induced reaction are derived from the continuum-discretized coupled-channel, and radii for the inclusive reactions are clearly defined [11]. In contrast to this approach, the present method is useful for the definition of the radius in an exclusive reaction, going to a specific channel.

In the present coupled-channel calculation, we have treated only the $p + {}^{12}\text{C}$ scattering system. The systematic studies are very important to establish the validity of the method of the scattering radius. For example, the analysis of the alpha and ${}^3\text{He}$ scatterings is also important because the previous studies have pointed out the evolution of the Airy structures in the angular distribution in connection to the enhanced radius of the Hoyle state [12, 13]. Furthermore, the ${}^{16}\text{O}$ nucleus is another typical example that the $\alpha + {}^{12}\text{C}$ [14, 15] and 4α [15] structures appear in the excited states. In particular, a recent study in Ref. [15] suggests that the 0_6^+ state corresponds to the 4α dilute state, and it is expected to have the extended matter radius. Thus, the analysis of the inelastic scattering of proton and alpha particle by the ${}^{16}\text{O}$ nucleus is very interesting. Systematic studies are now underway.

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