Journal of Physics: Conference Series **569** (2014) 012020 doi:10.1088/1742-6596/569/1/012020

Treatment of the Mirror ^{3}H(α,γ)^{7}Li and ^{3}He(α,γ)^{7}Be Reactions in the Algebraic Version of the Resonating Group Model

A S Solovyev[1], S Yu Igashov[1], and Yu M Tchuvil'sky[2]

[1] All-Russia Research Institute of Automatics (VNIIA), 127055 Moscow, Russia
[2] Scobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, 119991 Moscow, Russia

E-mail: alexander.solovyev@mail.ru

Abstract. A unified microscopic approach based on the algebraic version of the resonating group model has been realized for description of the radiative capture reactions ^{3}H(α, γ)^{7}Li and ^{3}He(α, γ)^{7}Be, which play an important role for modern nuclear astrophysics. The astrophysical *S*-factors of the reactions and branching ratios between capture to the ground and first excited states of the ^{7}Li and ^{7}Be nuclei have been calculated. The comparison with the most recent experimental data demonstrates a good agreement.

1. Introduction

The mirror ^{3}H(α, γ)^{7}Li and ^{3}He(α, γ)^{7}Be reactions of the radiative capture are rather significant for the nuclear astrophysics [1, 2]. Both of these reactions are important for the primordial nucleosynthesis investigation, in particular, for solution of the so-called ^{7}Li problem [3]. Moreover, the ^{3}He(α, γ)^{7}Be reaction is very interesting for study of the stellar nucleosynthesis including kinetics of processes at the Sun since it is the starting point for the second and third chains of the pp-cycle of hydrogen burning. For that reasons, the experimental interest, which arose since the middle of the last century [4–6], does not decrease up till now [7–19].

The radiative capture cross section is exponentially small at low sub-barrier energies because of Coulomb repulsion between colliding nuclei. That is why the experimental measurement of the cross section turns out to be extremely hard (or even impossible in some cases) task at the energies which are important for astrophysics. Thus microscopic theoretical calculation tuned by comparison with the experimental results obtained at higher energies occurs the most justified and promising approach for determination of the cross section or astrophysical *S*-factor at low energies.

The resonating group model (RGM) [20] is one of the most consistent realizations of the cluster model. Pioneering RGM-calculation of the ^{3}He(α, γ)^{7}Be and ^{3}H(α, γ)^{7}Li reactions was performed in 1981 [21]. More recent RGM-calculation is presented, for example, in [22]. Most part of calculations of the ^{3}He(α, γ)^{7}Be reaction was made before appearance of the modern experimental data [16–18] covering the middle energies. The data [5] were the sole ones presented in the last century which cover the middle energies. Therefore, the calculations were focused on the description of experiment [5] and extrapolation the data to the low energy domain. Modern data [16–18] occurred in quantitative and qualitative discrepancy with [5] in the middle energy region. As a result, the questions concerning energy dependence of the *S*-factor of the ^{3}He(α, γ)^{7}Be reaction appeared again.

The present work is focused on the microscopic treatment of the ^{3}H(α, γ)^{7}Li and ^{3}He(α, γ)^{7}Be reactions in the framework of the algebraic version (AV) [23] of the RGM which looks promising for these purposes. The astrophysical *S*-factors of the ^{3}H(α, γ)^{7}Li and ^{3}He(α, γ)^{7}Be reactions and branching ratios between capture to the ground and first excited states of the ^{7}Li and ^{7}Be nuclei have been calculated. Parameters of the dynamic model are tuned to reproduce the results of new experiments.

2. Ingredients of the theoretical model

2.1. The AVRGM basis wave functions

The main idea of the AVRGM consists in expansion of the relative motion wave function of colliding nuclei (clusters) over the oscillator basis

$$f_{\nu lm}(\mathbf{q}) = (-1)^{(\nu-l)/2}\left(\frac{2\Gamma\left((\nu-l+2)/2\right)}{r_0^3\,\Gamma\left((\nu+l+3)/2\right)}\right)^{1/2}\left(\frac{q}{r_0}\right)^l L_{(\nu-l)/2}^{(l+1/2)}\left(\frac{q^2}{r_0^2}\right)\exp\left(-\frac{q^2}{2r_0^2}\right)Y_{lm}(\mathbf{n_q}), \qquad (1)$$

where Γ, $L_n^{(\beta)}$, Y_{lm} are gamma-function, generalized Laguerre polynomial, and spherical harmonic respectively, ν is number of oscillator quanta, *l*, *m* are relative orbital momentum and its projection, **q** is Jacobi vector of the relative motion of clusters, r_0 is oscillator radius. As a result, the total wave function of the system is sought in the form of expansion over the AVRGM basis

$$\Psi_{J^\pi Mls\nu} = N_{J^\pi ls\nu}\,\mathrm{A}\left\{\sum_{m+\sigma=M} C_{lm\ s\sigma}^{JM}\left[\phi_{s_1}^{(1)}\phi_{s_2}^{(2)}\right]_{s\sigma} f_{\nu lm}(\mathbf{q})\right\}. \qquad (2)$$

Here $N_{J^\pi ls\nu}$ is the norm, A is antisymmetrization operator, $\phi_{s_1}^{(1)}$, $\phi_{s_2}^{(2)}$ are internal wave functions of clusters with spins s_1, s_2 coupled to the channel spin *s*, σ – its projection, *J*, *M* are total angular momentum and its projection, π is parity of system, $C_{lm\ s\sigma}^{JM}$ is Clebsch–Gordan coefficient. In this study the functions $\phi_{s_1}^{(1)}$ and $\phi_{s_2}^{(2)}$ are chosen in the form of the lowest compatible with the Pauli exclusion principle wave functions of the translationally invariant oscillator shell model.

2.2. The AVRGM equations sets

Substituting the total wave function expansion over the basis (2) into the projected many-particle Schrödinger equation we can obtain the set of the AVRGM equations for unknown expansion coefficients. As a result, for the continuum states we arrive the nonhomogeneous set

$$\sum_{\nu=\nu_0}^{\nu_{\mathrm{as}}-2}\left(\left\langle J^\pi Mls\mu\middle|H\middle|J^\pi Mls\nu\right\rangle - E\delta_{\mu\nu}\right)C_{J^\pi Mls\nu}^{(\mathrm{C})} = F_{J^\pi Mls\mu},\quad \mu=\nu_0,\nu_0+2,\ldots,\nu_{\mathrm{as}}, \qquad (3)$$

$$F_{J^\pi Mls\mu} = -\sum_{\nu=\nu_{\mathrm{as}}}^{\nu_{\max}}\left\langle J^\pi Mls\mu\middle|H\middle|J^\pi Mls\nu\right\rangle C_{J^\pi Mls\nu}^{(\mathrm{as})}. \qquad (4)$$

Here ν_0 is the minimal compatible with the Pauli exclusion principle number of oscillator quanta, and the expansion coefficients $C_{J^\pi Mls\nu}^{(\mathrm{C})}$ are replaced by their asymptotic values $C_{J^\pi Mls\nu}^{(\mathrm{as})}$ [24] starting from the sufficiently large ν_{as}. The value $\nu_{\max}$ is large enough to neglect in (4) the terms with $\nu > \nu_{\max}$. For the bound states equations set is similar but homogeneous (with zero right side of the equations). The modified Hasegawa–Nagata potential [25] was applied as the strong NN-potential in the calculations.

2.3. Astrophysical S-factor and branching ratio of the radiative capture

Astrophysical *S*-factor $S(E_{\mathrm{c.m.}}) = E_{\mathrm{c.m.}}\,\sigma(E_{\mathrm{c.m.}})\exp(2\pi\eta)$ has more smooth energy dependence than

cross section since the exponential smallness caused by Coulomb barrier penetrability is explicitly extracted from it.

Assuming predominance of the E1-transition the partial cross section of the discussed reactions in the AVRGM basis (2) can be written as

$$\sigma_{i\to f}(E_{\text{c.m.}}) = \frac{8\pi}{9\hbar(2l_i+1)}\left(\frac{E_\gamma}{\hbar c}\right)^3 \left| \sum_{\nu_i,\nu_f} C^{(\text{D})}_{J_f^{\pi_f} l_f s \nu_f} \left\langle J_f^{\pi_f} l_f s \nu_f \left\| M_1^{\text{E}} \right\| J_i^{\pi_i} l_i s \nu_i \right\rangle C^{(\text{C})}_{J_i^{\pi_i} l_i s \nu_i} \right|^2, \qquad (5)$$

where the indices i, f denote initial and final states of the system respectively, E_γ is energy of the emitted photon, M_1^{E} is electric dipole operator (its matrix elements have been calculated in our work [26]). In the expression (5) the initial wave function is assumed to be a partial wave of a unit-flux scattering wave function and the channel spin s equals 1/2.

Branching ratio of the ^{3}H(α, γ)^{7}Li (^{3}He(α, γ)^{7}Be) reaction is determined by the expression $R(E_{\text{c.m.}}) = \sigma_1(E_{\text{c.m.}})/\sigma_0(E_{\text{c.m.}})$, where σ_0, σ_1 are the capture cross sections of the ^{3}H + ^{4}He (^{3}He + ^{4}He) system to the ground and first excited states of the ^{7}Li (^{7}Be) nucleus. Cross section σ_0 is the sum of three partial ones (5) with the following values of the quantum numbers (J, l): (J_f, l_f) = (3/2, 1); (J_i, l_i) = (1/2, 0), (3/2, 2), (5/2, 2). In turn, cross section σ_1 is the sum of two partial ones (5) with (J, l) possessing the following values: (J_f, l_f) = (1/2, 1); (J_i, l_i) = (1/2, 0), (3/2, 2). The total cross section is the sum of σ_0 and σ_1.

3. Results of calculations

The comparison of the calculated astrophysical S-factor and branching ratio of the ^{3}H(α, γ)^{7}Li reaction with experimental data [6–9] is presented in figure 1.

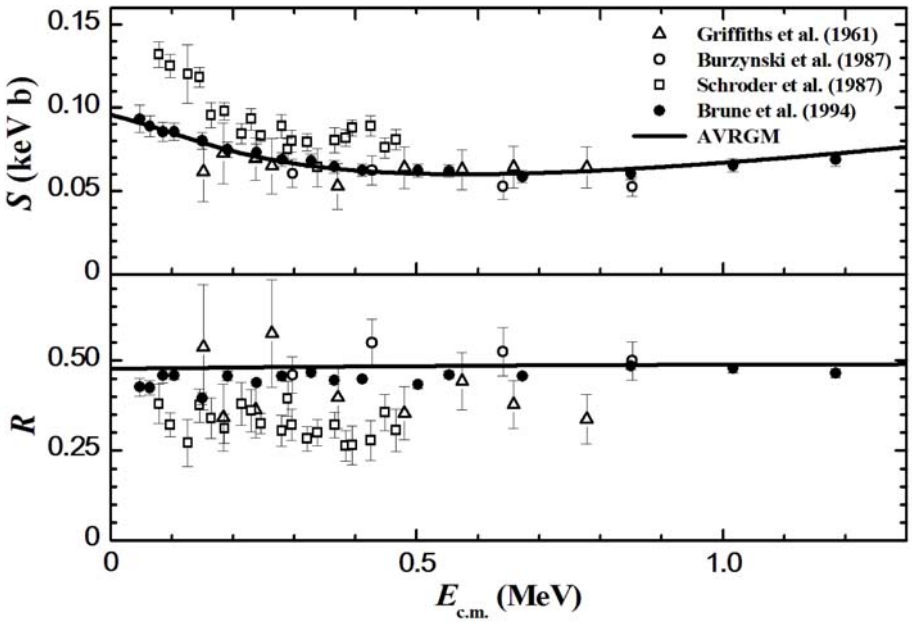

Figure 1. Astrophysical S-factor and branching ratio of the ^{3}H(α, γ)^{7}Li reaction.

Figure 1 demonstrates rather good agreement with the data [6, 7, 9]. It seems to be essential that the most recent data [9] with relatively small errors covering sufficiently wide energy range are well-described by the theoretical curves. There are only two adjustable parameters in this study: r_0 and the Majorana forces intensity g_c, which is introduced into the central part

$$V_{ij} = \sum_{n=1}^{3} V_n \left(w_n + (1 - g_c) m_n - g_c m_n P_{ij}^{\sigma} P_{ij}^{\tau} + b_n P_{ij}^{\sigma} - h_n P_{ij}^{\tau} \right) \exp(-\mu_n r_{ij}^2) \quad (6)$$

of the nucleon-nucleon potential (see [25]). The following values $r_0 = 1.386$ fm and $g_c = 1.021$ were adopted here. The calculated astrophysical *S*-factor and branching ratio of the ^{3}He(α, γ)^{7}Be reaction and the experimental data from [10–19] are presented in figure 2.

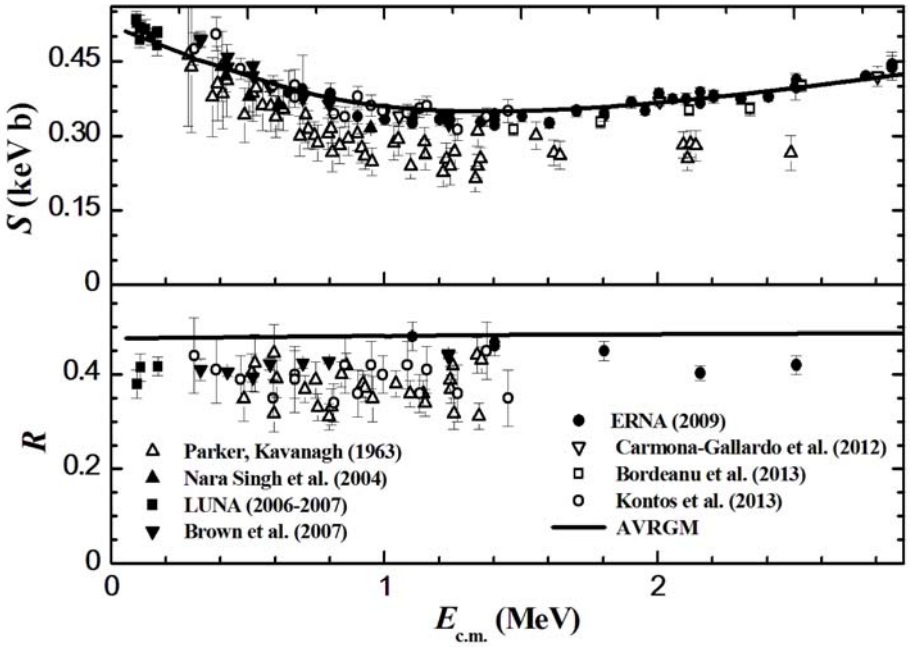

Figure 2. Astrophysical *S*-factor and branching ratio of the ^{3}He(α, γ)^{7}Be reaction.

The sole 'old' data [5] covering the middle energies (references to other data obtained in the last century can be found, for example, in [2]) are also marked in figure 2. It is very important that many early *S*-factor calculations were aimed at description of data [5] which, as it is seen from figure 2, do not explicitly agree with the newest data [16–18] in the middle energies region. The calculated *S*-factor of the ^{3}He(α, γ)^{7}Be reaction is in very good agreement with modern data [10–19]. The calculated branching ratio also demonstrates reasonable fit of the data. In the calculations we chose the following values of the parameters: $r_0 = 1.2$ fm, $g_c = 1.035$. It should be noted, that the use of these r_0 and g_c values for calculation of ^{3}H(α, γ)^{7}Li reaction provides satisfactory but worse results.

Table 1. Energies of the ^{4}He, ^{3}H, ^{7}Li nuclei.

	$E(^4\text{He})$	$E(^3\text{H})$	$E(^7\text{Li})$	$E(^7\text{Li}^*)$
Experiment	–28.296	–8.482	–39.244	–38.766
Calculation	–28.296	–6.467	–35.997	–35.930

Table 2. Energies of the ^{4}He, ^{3}He, ^{7}Be nuclei.

	$E(^4\text{He})$	$E(^3\text{He})$	$E(^7\text{Be})$	$E(^7\text{Be}^*)$
Experiment	–28.296	–7.718	–37.600	–37.171
Calculation	–26.703	–2.073	–29.746	–29.638

The calculated with the corresponding set of adjustable parameters and experimental [27, 28] values of the binding energies of the ^{4}He, ^{3}He, and ^{3}H nuclei, as well as the ground and first excited states energies of the ^{7}Li and ^{7}Be nuclei are presented in the tables. As it can be seen from the tables, the calculated energies are smaller than the corresponding experimental ones in magnitude. The model demonstrates reasonable sensitivity to the adjustable parameters: the variation of the r_0 within 1% influences upon S and R within 3% and 0.1% respectively. The similar variation of g_c results in variation S and R which not exceeds 18% and 1%.

4. Conclusion

In the present work the investigation of the isospin mirror ^{3}H(α, γ)^{7}Li and ^{3}He(α, γ)^{7}Be reactions was performed. Fully antisymmetrized microscopic wave functions of the single-channel AVRGM were used for the calculations of astrophysical S-factors and branching ratios of these reactions. Comparison with the most recent data shows well enough agreement. At the same time the calculated S-factors for the ^{3}H(α, γ)^{7}Li and ^{3}He(α, γ)^{7}Be reactions differ quantitatively and qualitatively from 'old' data [5] and [8] respectively. In fact, it turns out to be impossible to reproduce these data using reasonable values of the adjustable parameters. Obtained values of the S-factors show more smooth energy dependence than those data. On the whole, taking into account remaining underbinding of the clusters and the compound nuclei, one can conclude that further more comprehensive theoretical investigations of these reactions keep actuality.

References

[1] Adelberger E G *et al* 2011 *Rev. Mod. Phys.* **83** 195
[2] Xu Y *et al* 2013 *Nucl. Phys. A* **918** 61
[3] Iocco F *et al* 2009 *Phys. Rep*. **472** 1
[4] Holmgren H D and Johnston R L 1959 *Phys. Rev.* **113** 1556
[5] Parker P D and Kavanagh R W 1963 *Phys. Rev.* **131** 2578
[6] Griffiths G M, Morrow R A, Riley P J and Warren J B 1961 *Can. J. Phys.* **39** 1397
[7] Burzyński S, Czerski K, Marcinkowski A and Zupranski P 1987 *Nucl. Phys. A* **473** 179
[8] Schröder U *et al* 1987 *Phys. Lett. B* **192** 55
[9] Brune C R, Kavanagh R W and Rolfs C 1994 *Phys. Rev. C* **50** 2205
[10] Nara Singh B S, Hass M, Nir-El Y and Haquin G 2004 *Phys. Rev. Lett.* **93** 262503
[11] Bemmerer D *et al* 2006 *Phys. Rev. Lett.* **97** 122502
[12] Gyürky Gy *et al* 2007 *Phys. Rev. C* 75 035805
[13] Confortola F *et al* 2007 *Phys. Rev. C* **75** 065803
[14] Costantini H *et al* 2008 *Nucl. Phys. A* **814** 144
[15] Brown T A D *et al* 2007 *Phys. Rev. C* **76** 055801
[16] Di Leva A *et al* 2009 *Phys. Rev. Lett.* **102** 232502; **103** 159903(E)
[17] Carmona-Gallardo M *et al* 2012 *Phys. Rev. C* **86** 032801(R)
[18] Bordeanu C *et al* 2013 *Nucl. Phys. A* **908** 1
[19] Kontos A *et al* 2013 *Phys. Rev. C* **87** 065804
[20] Wildermuth K and Tang Y C 1977 *A Unified Theory of the Nucleus* (Braunschweig: Vieweg)
[21] Liu Q K K, Kanada H, Tang Y C 1981 *Phys. Rev. C* **23** 645
[22] Arai K, Baye D and Descouvemont P 2002 *Nucl. Phys. A* **699** 963
[23] Filippov G F and Okhrimenko I P 1980 *Phys. Atom. Nucl.* **32** 480
[24] Igashov S Yu 2008 *The J-Matrix Method. Developments and Applications* ed A D Alhaidari, E J Heller, H A Yamani and M S Abdelmonem (Berlin: Springer) part 2 pp 49–66
[25] Kanada H, Kaneko T, Nagata S and Nomoto M 1979 *Progr. Theor. Phys.* **61** 1327
[26] Solovyev A S, Igashov S Yu and Tchuvil'sky Yu M 2014 *Bull. Russ. Acad. Sci. Phys.* **78** 433
[27] Tilley D R *et al* 2002 *Nucl. Phys. A* **708** 3
[28] Audi G, Wapstra A H and Thibault C 2003 *Nucl. Phys. A* **729** 337