Equilibrium axial current due to a static localized spin in Weyl semimetals

Katsuhisa Taguchi

Department of Applied Physics, Nagoya University, Nagoya, 464-8603, Japan

E-mail: `taguchi@rover.nuap.nagoya-u.ac.jp`

Abstract. We theoretically study the equilibrium axial current j_5^{eq}, which is the difference between the charge current of left-handed and right-handed helicity of the Weyl fermions, in the junction of a magnetic insulator attached to Weyl semimetals with time-reversal breaking and inversion symmetry. The charge current of each helicity $j_{\gamma=\pm}^{\mathrm{eq}}$ is induced by the localized spin S of the magnetic insulator. $j_{\gamma=\pm}^{\mathrm{eq}}$ is proportional to $\nabla \times S$ and is independent of the sign of γ. As a result, the difference between $j_{\gamma=+}^{\mathrm{eq}}$ and $j_{\gamma=-}^{\mathrm{eq}}$ are canceled out, and j_5^{eq} is zero.

1. Introduction

The axial current is defined by the difference between the charge current of right- and left-handed helicity in Dirac electron systems [1, 2, 3, 4, 5, 6, 7]. Although this concept was initially devised in elementary particle physics, currently, it strongly affects condensed matter physics as well because candidate materials of massless Dirac material, such as Weyl semimetal (WS) has been reported [8, 9, 10, 11, 12, 13, 14, 15]. It is noted that the axial current can flow without an accompanying charge current [1, 2, 3, 4, 5, 6, 7]. The property of the axial current is similar to pure spin current in spintronics[16]. Therefore, one can expect that the axial current can be applied to a method of low-consumption electronics.

The axial current is induced when we apply a static magnetic field H[1, 2, 3, 4, 5, 6, 7]. This phenomenon is called as the chiral separation effect. Its origin lies in the difference of helicity between right-handed and left-handed fermions in the WS. The helicity $\gamma = \hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{p}}$ indicates the relative angle between the direction of the spin $\hat{\boldsymbol{\sigma}}$ and that of the momentum $\hat{\boldsymbol{p}}$ of Weyl fermions. The helicity of right-handed fermions is $\gamma = +1$, whereas that of left-handed ones is $\gamma = -1$, but both spins are parallel to each other along the applied magnetic field (Figure. 1(a)). Here the axial current can be regard as the equilibrium flow. Recently, the nonequilibrium axial current has been also studied in the WS / a magnetic insulator (MI) junction. The nonequilibrium axial current is driven by the spin transfer from the spin angular momentum of the the localized spin into the conduction electrons spin in the WS[16]. The preexisting work considered only the nonequilibrium axial current due to the spin transfer. The equilibrium axial current due to the spin transfer has not been discussed, so far.

In this paper, we study the equilibrium axial current j_5^{eq} due to the spin transfer in the WS / MI junction, where WS is time reversal breaking and inversion symmetry within a diffusive regime. Using the Green's functions techniques, we calculate the charge current of each helicity, $j_{\gamma=\pm}^{\mathrm{eq}}$ and obtain the axial current $j_5^{\mathrm{eq}} \equiv j_+^{\mathrm{eq}} - j_-^{\mathrm{eq}}$ in the linear response of the localized spin$S$ in the MI.
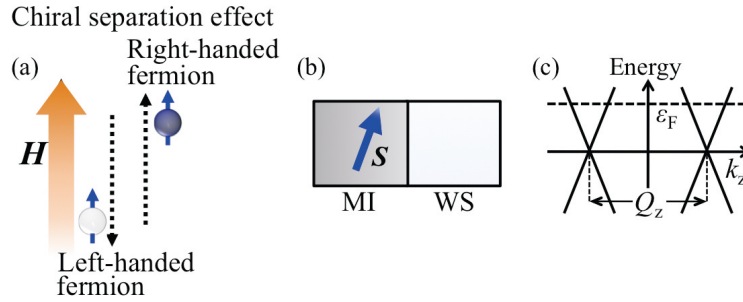
Figure 1. (a) Schematic illustration of the chiral separation effect. When a magnetic field $\boldsymbol{H}$ is applied, right-handed and left-handed fermions are separated along the $\boldsymbol{H}$ direction. (b) MI/WS junction with the chiral separation effect resulting from the static localized spin $\boldsymbol{S}$. (c) Schematic illustration of the energy dispersion of the WS with time-reversal symmetry breaking and inversion symmetry.

2. Charge current of each helicity due to the localized spin

We will consider how the localized spin contributes to $\boldsymbol{j}_5^{\mathrm{eq}}$ in the WS / MI junction (represented in the Figure. 1 (b)). The total Hamiltonian we consider is given by

$$\mathcal{H} = \mathcal{H}_{\mathrm{W}} + \mathcal{H}_{\mathrm{ex}} + V_{\mathrm{i}}, \tag{1}$$

where the first term,

$$\mathcal{H}_{\mathrm{W}} = \sum_{\gamma=\pm} \mathcal{H}_{\mathrm{W},\gamma} = \sum_{\gamma=\pm} \left\{ \sum_{\boldsymbol{k}} \psi_{\boldsymbol{k},\gamma}^{\dagger} [\hbar v_{\mathrm{F},\gamma} (\boldsymbol{k} - \gamma \boldsymbol{Q}/2) \cdot \hat{\boldsymbol{\sigma}} - \epsilon_{\mathrm{F}}] \psi_{\boldsymbol{k},\gamma} \right\}, \tag{2}$$

is the Hamiltonian describing the doped WS. Here $\psi_{\boldsymbol{k},\gamma}^{\dagger} = (\psi_{\boldsymbol{k},\gamma,\uparrow}^{\dagger} \ \psi_{\boldsymbol{k},\gamma,\downarrow}^{\dagger})$, and $\psi_{\boldsymbol{k},\gamma}$ are the creation and annihilation operators of the Weyl fermions of each helicity sector γ, respectively (where indices $\uparrow$ and $\downarrow$ represent spin), ϵ_{F} is the Fermi energy [Figure. 1(c)], and $v_{\mathrm{F},\gamma} = \gamma v_{\mathrm{F}}$ is the Fermi velocity. We assume that a single pair of Dirac cones exists in the WS with inversion-symmetry and time-reversal-symmetry breaking with nonzero $\boldsymbol{Q}$. The parameter $\boldsymbol{Q}$ of Eq. (2) denotes the position of the Weyl node with $\gamma \boldsymbol{Q}/2$ and its magnitude $|\boldsymbol{Q}|$ is the distance between two Dirac cones. The second term in Eq. (1),

$$\mathcal{H}_{\mathrm{ex}} = \sum_{\gamma=\pm} \mathcal{H}_{\mathrm{ex},\gamma} = -\sum_{\gamma=\pm} J_{\mathrm{ex}} \int d\boldsymbol{x} \, \boldsymbol{S} \cdot (\psi_{\gamma}^{\dagger} \hat{\boldsymbol{\sigma}} \psi_{\gamma}), \tag{3}$$

describes the interaction between the localized spin in the MI and the spin in the WS, where $J_{\mathrm{ex}} > 0$ is the exchange coupling constant, $\boldsymbol{S} = S\boldsymbol{n}(\boldsymbol{x})$ is the classical vector representing the spin structure, S is its magnitude, and $\boldsymbol{n}$ is the unit vector representing the direction. Finally, the last term in Eq. (1), V_{i}, represents nonmagnetic impurity scattering, which causes a relaxation time τ of the transport of conduction electrons in the WS.

In the following calculation, $\boldsymbol{Q}$ is chosen to be parallel to the quantization axis of the localized spin (z axis) as $\boldsymbol{Q} = Q_z \boldsymbol{z}$ and Q_z is independent of time. In addition, we incorporate the term proportional to $\boldsymbol{Q}$ in $H_{\mathrm{W},\gamma}$ into $\mathcal{H}_{\mathrm{ex},\gamma}$ by using the following transformation: $\boldsymbol{S} \to \boldsymbol{S}' = (S_x, S_y, S_z - \frac{\hbar v_{\mathrm{F}}}{2 J_{\mathrm{ex}}} Q_z)$. This transformation enables us to calculate the axial current rather easily. Then, we assume that the effect of $\mathcal{H}_{\mathrm{ex},\gamma}$ is weak and can be treated as a perturbation. This condition is satisfied by $J_{\mathrm{ex}} |\boldsymbol{S}'| \tau / \hbar \ll 1$ within the diffusive transport regime.

To consider the axial current, we will calculate the current $\boldsymbol{j}_{\gamma}$ using the above assumptions. We define the charge current of each helicity sector γ as $\boldsymbol{j}_{\gamma} = -e v_{\mathrm{F},\gamma} \langle \psi_{\gamma}^{\dagger} \boldsymbol{\sigma} \psi_{\gamma} \rangle$ from the

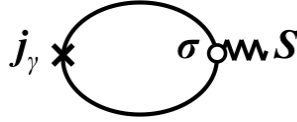**Figure 2.** Feynman diagram of $\boldsymbol{j}_\gamma$. The bold line shows the Green's function of $\mathcal{H}_{\mathrm{W},\gamma}$ and the wavy line represents the exchange interaction $\mathcal{H}_{\mathrm{ex},\gamma}$.

conservation law $\dot{\rho}_\gamma = -\boldsymbol{\nabla}\cdot\boldsymbol{j}_\gamma$, where $\rho_\gamma \equiv -e\langle\psi_\gamma^\dagger\psi_\gamma\rangle$ is the charge density of helicity γ. The current is represented by using the same space and time of lesser Green's functions $\hat{G}_\gamma^< = \langle\psi_\gamma^\dagger\psi_\gamma\rangle/(-i\hbar)$ as

$$j_{i,\gamma}(\boldsymbol{x},t) = i\hbar e v_{\mathrm{F},\gamma}\mathrm{tr}[\hat{\sigma}_i\hat{G}_\gamma^<(\boldsymbol{x},t:\boldsymbol{0},0)]. \tag{4}$$

$j_{i,\gamma}$ due to the linear response of the localized spin is described diagrammatically in Figure. 2. We note that in this paper, we consider the equilibrium charge current $j_{i,\gamma}^{\mathrm{eq}} \equiv j_{i,\gamma}^{\mathrm{eq}}(\boldsymbol{x},0)$. Here $j_{i,\gamma}^{\mathrm{eq}}$ is derived from the spin-spin response function $\Pi_{ij,\gamma}(\boldsymbol{q},\Omega=0) \equiv \Pi_{ij,\gamma}(\boldsymbol{q},0)$ and is given by

$$j_{i,\gamma}^{\mathrm{eq}}(\boldsymbol{x},0) = \frac{-i\hbar J_{\mathrm{ex}}e v_{\mathrm{F},\gamma}}{V}\sum_{\boldsymbol{q}}e^{-i\boldsymbol{q}\cdot\boldsymbol{x}}\Pi_{ij,\gamma}(\boldsymbol{q},\Omega=0)S_{\boldsymbol{q},\Omega=0}^{\prime j}, \tag{5}$$

$$\Pi_{ij,\gamma}(\boldsymbol{q},0) = \frac{1}{V}\sum_{\boldsymbol{k},\omega}\mathrm{tr}[\hat{\sigma}_i\hat{g}_{\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\omega,\gamma}\hat{\Lambda}_{j,\gamma}\hat{g}_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\omega,\gamma}]^<, \tag{6}$$

where $\hat{\Lambda}_{j,\gamma}$ is the vertex function of V_{i} and V is the system volume in the WS. The vertex function $\hat{\Lambda}_{j,\gamma}$ is represented from the 4×4 matrix $\hat{\Lambda}_\gamma$ and the Pauli matrix $\boldsymbol{\sigma}$ as $\hat{\Lambda}_{j,\gamma} = [\hat{\Lambda}_\gamma]_{j\nu}\hat{\sigma}_\nu$. Here the matrix $\hat{\Lambda}_\gamma$ is defined by $\hat{\Lambda}_\gamma = \sum_{n=0}^\infty(\hat{\Gamma}_\gamma)^n$, where 4×4 matrix $\hat{\Gamma}_\gamma$ satisfies $\hat{\Gamma}_{\gamma,\nu} = [\hat{\Gamma}_\gamma]_{\nu\mu}\hat{\sigma}_\mu$. This $\hat{\Gamma}_{\gamma,\nu}$ is represented as

$$\hat{\Gamma}_{\mu,\gamma}(\boldsymbol{q},0) = \frac{1}{V}\sum_{\boldsymbol{k}}n_{\mathrm{i}}u_{\mathrm{i}}^2\hat{g}_{\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\omega,\gamma}\hat{\sigma}_\mu\hat{g}_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\omega,\gamma}. \tag{7}$$

In the Eqs. (6) and (7), $\hat{g}_{\boldsymbol{k},\omega,\gamma}$ is the Green's function of $\mathcal{H}_{\mathrm{W},\gamma}$ including V_{i}. The retarded (advanced) Green's function $g^r(g^a)$ in $\mathcal{H}_{\mathrm{W},\gamma}$ is given by

$$\hat{g}_{\boldsymbol{k},\omega,\gamma}^r = [\hbar\omega + \epsilon_{\mathrm{F}} - \hbar v_{\mathrm{F},\gamma}\boldsymbol{k}\cdot\hat{\boldsymbol{\sigma}} + i\eta]^{-1}, \tag{8}$$

where $\eta \equiv \hbar/(2\tau) = n_{\mathrm{i}}u_{\mathrm{i}}^2\nu_e/4$ is the self-energy of V_{i} and is obtained in the Born approximation. Here n_{i}, u_{i}, and ν_e are the concentration of impurities, the potential energy of impurities, and the density of states at ϵ_{F}, respectively. We calculate $\Pi_{ij,\gamma}$ by using $\hat{g}_{\boldsymbol{k},\omega,\gamma}^< = f_\omega(\hat{g}_{\boldsymbol{k},\omega,\gamma}^a - \hat{g}_{\boldsymbol{k},\omega,\gamma}^r)$ [17], where f_ω is the Fermi distribution function. Then, the response function is represented by

$$\Pi_{ij,\gamma}(\boldsymbol{q},0) = \sum_{\boldsymbol{k},\omega}f_\omega\mathrm{tr}[\hat{\sigma}_i\hat{g}_{\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\omega,\gamma}^a\hat{\Lambda}_{j,\gamma}^{\mathrm{aa}}\hat{g}_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\omega,\gamma}^a] - \mathrm{h.c}, \tag{9}$$

where $\hat{\Lambda}_{j,\gamma}^{\mathrm{nm}} = [\hat{\Lambda}_\gamma^{\mathrm{nm}}]_{j\nu}\hat{\sigma}_\nu$ is constructed by the advanced Green's function $(n,m = a)$ or the retarded Green's function $(n,m = r)$. $\hat{\Lambda}_{j,\gamma}^{aa} = \hat{\sigma}_j + \hat{\Gamma}_{j,\gamma}^{aa} + \cdots$ is described by $\hat{\Gamma}_{j,\gamma}^{aa} = \frac{1}{V}\sum_{\boldsymbol{k}}n_{\mathrm{i}}u_{\mathrm{i}}^2\hat{g}_{\boldsymbol{k}-\frac{\boldsymbol{q}}{2},\omega,\gamma}^a\hat{\sigma}_\mu\hat{g}_{\boldsymbol{k}+\frac{\boldsymbol{q}}{2},\omega,\gamma}^a$. Here $\hat{\Lambda}_{j,\gamma}^{aa}$ can be approximately estimated by $\hat{\Lambda}_{j,\gamma}^{aa} \simeq \hat{\sigma}_j + o(q)$. Then, $\Pi_{ij,\gamma}$ is obtained by expanding with $q/k_{\mathrm{F}} \ll 1$ within $\hbar/(\epsilon_{\mathrm{F}}\tau) \ll 1$ as

$$\Pi_{ij,\gamma}(\boldsymbol{q},0) = \frac{\nu_e}{4\epsilon_{\mathrm{F}}}v_{\mathrm{F},\gamma}q_\zeta\epsilon_{j\zeta a}\mathrm{tr}[\hat{\sigma}_i\hat{\sigma}_a] + o(q^2). \tag{10}$$

Substituting Eq. (10) into Eq. (5), we finally obtain $j_{i,\gamma}^{\mathrm{eq}}$ in the lowest order of $\boldsymbol{q}$ as

$$j_{i,\gamma}^{\mathrm{eq}}(\boldsymbol{x},0) = \frac{-i\hbar\nu_e J_{\mathrm{ex}} e v_{\mathrm{F},\gamma}^2}{2\epsilon_{\mathrm{F}} V} \sum_{\boldsymbol{q}} e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} q_\zeta \epsilon_{j\zeta i} S_{\boldsymbol{q},\Omega=0}^{\prime j} + o(q^2) \tag{11}$$

$$= -\frac{\hbar\nu_e J_{\mathrm{ex}} e v_{\mathrm{F}}^2}{2\epsilon_{\mathrm{F}}} \epsilon_{i\zeta j} \nabla_\zeta S^{\prime j}(\boldsymbol{x}). \tag{12}$$

In the above equation, we use $v_{\mathrm{F},\gamma}^2 = v_{\mathrm{F}}^2$. From Eq. (12), $j_{i,\gamma}^{\mathrm{eq}}$ is induced by $\boldsymbol{\nabla} \times \boldsymbol{S}$ and is independent of γ.

3. Discussion of axial current, charge current, and spin polarization density

Using Eq. (12), we discuss the axial current $\boldsymbol{j}_5^{\mathrm{eq}} \equiv \boldsymbol{j}_+^{\mathrm{eq}} - \boldsymbol{j}_-^{\mathrm{eq}}$. Because $j_{i,\gamma}^{\mathrm{eq}}$ in the Eq. (12) is independent of γ, the difference between $\boldsymbol{j}_+^{\mathrm{eq}}$ and $\boldsymbol{j}_-^{\mathrm{eq}}$ are canceled out each other. Therefore, the axial current $\boldsymbol{j}_5^{\mathrm{eq}}$ becomes

$$\boldsymbol{j}_5^{\mathrm{eq}} = 0. \tag{13}$$

However, the total charge current, which is defined by $j_i^{\mathrm{eq}} = j_{i,+}^{\mathrm{eq}} + j_{i,-}^{\mathrm{eq}}$, is given by

$$\boldsymbol{j}^{\mathrm{eq}}(\boldsymbol{x},0) = -\frac{\hbar\nu_e J_{\mathrm{ex}} e v_{\mathrm{F}}^2}{\epsilon_{\mathrm{F}}} \boldsymbol{\nabla} \times \boldsymbol{S}'. \tag{14}$$

From the above result, $\boldsymbol{j}^{\mathrm{eq}}$ is proportional to $\boldsymbol{\nabla} \times \boldsymbol{S}' = \boldsymbol{\nabla} \times (\boldsymbol{S} - \frac{\hbar v_{\mathrm{F}}}{2J_{\mathrm{ex}}}\boldsymbol{Q})$. When $\boldsymbol{Q}$ is also independent of the space in the WS, the charge current becomes $\boldsymbol{j}^{\mathrm{eq}} = -\frac{\hbar\nu_e J_{\mathrm{ex}} e v_{\mathrm{F}}^2}{\epsilon_{\mathrm{F}}} \boldsymbol{\nabla} \times \boldsymbol{S}$. Here $\boldsymbol{j}^{\mathrm{eq}}$ can be regard as the magnetization current density as $\boldsymbol{j}^{\mathrm{eq}} \propto \boldsymbol{\nabla} \times \boldsymbol{M}$, because the localized spin $\boldsymbol{S}$ can be replaced with the magnetization $\boldsymbol{M} = -g\mu_B \boldsymbol{S}/a^3$, where g is the Landé factor, μ_B is the Bohr magneton, a is the lattice constant.

We next discuss the spin density $\boldsymbol{s}^{\mathrm{eq}}$, which is defined by $\boldsymbol{s}^{\mathrm{eq}} = \boldsymbol{s}_+^{\mathrm{eq}} + \boldsymbol{s}_-^{\mathrm{eq}}$, in the WS from Eq. (12). Here $\boldsymbol{s}_{\gamma=\pm}^{\mathrm{eq}} = \frac{1}{2}\langle\psi_\gamma^\dagger \hat{\sigma} \psi_\gamma\rangle$ is the spin density in each helicity. Because the direction of the spin and the momentum of right (left)-handed are parallel (antiparallel) to each other in the WS, $\boldsymbol{s}_\gamma^{\mathrm{eq}}$ is represented by $\boldsymbol{s}_\gamma^{\mathrm{eq}} = \boldsymbol{j}_\gamma^{\mathrm{eq}}/(-2e v_{\mathrm{F},\gamma})$. From Eq. (12), $\boldsymbol{s}_\gamma^{\mathrm{eq}}$ is given by

$$\boldsymbol{s}_\gamma^{\mathrm{eq}}(\boldsymbol{x},0) = \frac{\hbar\nu_e J_{\mathrm{ex}} v_{\mathrm{F}_\gamma}}{2\epsilon_{\mathrm{F}}} \boldsymbol{\nabla} \times \boldsymbol{S}'. \tag{15}$$

From the above equation, $\boldsymbol{s}_\gamma^{\mathrm{eq}}$ is induced by $\boldsymbol{\nabla} \times \boldsymbol{S}'$ and is proportional to the sign of $\gamma = \pm$ as $\boldsymbol{s}_+^{\mathrm{eq}} = -\boldsymbol{s}_-^{\mathrm{eq}}$. Therefore, total spin density $\boldsymbol{s}^{\mathrm{eq}}$ vanishes as

$$\boldsymbol{s}^{\mathrm{eq}} = 0. \tag{16}$$

From the above equation, there is no spin polarization due to the static localized spin in the WS. Equations (13), (14), and (16) are the main results of this section.

Next, we will compare the property of $\boldsymbol{j}_5^{\mathrm{eq}}$ and the nonequilibrium axial current $\boldsymbol{j}_5^{\mathrm{neq}}$. $\boldsymbol{j}_5^{\mathrm{eq}}$ is zero, but $\boldsymbol{j}_5^{\mathrm{neq}}$ is induced by the dynamical localized spin [16]. The summary of the difference is shown in Table I. In a similar way, there is the difference between $\boldsymbol{j}^{\mathrm{eq}}$ and the nonequilibrium charge current $\boldsymbol{j}^{\mathrm{neq}}$. $\boldsymbol{j}^{\mathrm{eq}}$ is generated by $\boldsymbol{\nabla} \times \boldsymbol{S}$ as $\boldsymbol{j}^{\mathrm{eq}} \propto \boldsymbol{\nabla} \times \boldsymbol{S}$, but $\boldsymbol{j}^{\mathrm{neq}}$ is zero. The property of $\boldsymbol{s}^{\mathrm{eq}}$ and the nonequilibrium spin density $\boldsymbol{s}^{\mathrm{neq}}$ are difference. $\boldsymbol{s}^{\mathrm{eq}}$ is zero and $\boldsymbol{s}^{\mathrm{neq}}$ is driven by the dynamics of the localized spin $\partial_t \boldsymbol{S}$. Therefore, there is no spin polarization in the WS unless the localized spin depends on time.

Table 1. Summary of the axial current, the charge current, and the spin density due to the localized spin.

	Axial current	Charge current	Spin density
Static $\boldsymbol{S}$	$\boldsymbol{j}_5^{\mathrm{eq}} = 0$	$\boldsymbol{j}^{\mathrm{eq}} \propto \boldsymbol{\nabla} \times \boldsymbol{S}$	$\boldsymbol{s}^{\mathrm{eq}} = 0$
Dynamical $\boldsymbol{S}$	$\boldsymbol{j}_5^{\mathrm{neq}} \propto \partial_t \boldsymbol{S}$	$\boldsymbol{j}^{\mathrm{neq}} = 0$	$\boldsymbol{s}^{\mathrm{neq}} \propto \partial_t \boldsymbol{S}$

Finally, we discuss the gauge invariance in the presence of the localized spin in the WS. Owing to spin-momentum locking, $\boldsymbol{S}$ plays a role like the electromagnetic vector potential as $\mathcal{H}_{\mathrm{W},\gamma} + \mathcal{H}_{\mathrm{ex},\gamma} \propto \boldsymbol{\sigma} \cdot (\boldsymbol{k} - \frac{e}{\hbar}\boldsymbol{\mathcal{A}}_\gamma)$, where the vector potential $\boldsymbol{\mathcal{A}}_\gamma = J_{\mathrm{ex}}\boldsymbol{S}/(ev_{\mathrm{F},\gamma})$ is conjugate to $\boldsymbol{j}_\gamma^{\mathrm{eq}}$. Therefore, the observable quantity should be proportional to the gauge invariant form as $-\partial_t \boldsymbol{\mathcal{A}}_\gamma \equiv \boldsymbol{\mathcal{E}}_\gamma$ or $\boldsymbol{\nabla} \times \boldsymbol{\mathcal{A}}_\gamma \equiv \boldsymbol{\mathcal{B}}_\gamma$. From viewpoint of the effective electromagnetic field, $\boldsymbol{j}^{\mathrm{eq}}$ can be driven by the effective magnetic field $\boldsymbol{\mathcal{B}}_\gamma$ from Eq. (14).

4. Conclusion

We study the equilibrium axial current $\boldsymbol{j}_5^{\mathrm{eq}}$ due to $\boldsymbol{S}$ in the WS / MI junction. Using the Green's function, we calculate $j_{i,\gamma}^{\mathrm{eq}}$ in the linear response of $\boldsymbol{S}$. From the results, $j_{i,\gamma}^{\mathrm{eq}}$ is proportional to $\boldsymbol{\nabla} \times \boldsymbol{S}'$ and $v_{\mathrm{F},\gamma}$ as you can see Eq. (12). Then, $\boldsymbol{j}_5^{\mathrm{eq}}$ becomes zero because of the cancelation between $j_{i,+}^{\mathrm{eq}}$ and $j_{i,-}^{\mathrm{eq}}$. In addition, the total spin density $\boldsymbol{s}^{\mathrm{eq}}$ is also zero, but the total charge current is induced by the localized spin from Eq. (14).

References

[1] Vilenkin A 1980 *Phys. Rev. D* **22**, 3080.
[2] Metlitski A M and Zhitnitsky R A 2005 *Phys. Rev. D* **72**, 045011.
[3] Newman M G and Son T D 2006 *Phys. Rev. D* **73**, 045006.
[4] Kharzeev D and Zhitnitsky A 2007 *Nucl. Phys. A* **797**, 67.
[5] Kharzeev E D, McLerran D L, and H. J. Warringa 2008 *Nucl. Phys. A* **803**, 227.
[6] Gorbar V E, Miransky A V, Shovkovy A I, and Wang X 2013 *Phys. Rev. D* **88**, 025025.
[7] Kharzeev D, Landsteiner K, Schmitt A, Yee H 2013 *Lect. Notes Phys.* **871**, 241.
[8] Wan X, Turner M A, Vishwanath A, and Savrasov Y S 2011 *Phys. Rev. B* **83** 205101.
[9] Balents L 2011 *Physics* **4**, 36.
[10] Burkov A A and Balents L 2011 *Phys. Rev. Lett.* **107**, 127205.
[11] Halász B G and Balents L 2012 *Phys. Rev. B* **85**, 035103.
[12] Xu G, Weng H, Wang Z, Dai X, and Fang Z 2011 *Phys. Rev. Lett.* **107**, 186806.
[13] Liu C, Ye P, and Qi X 2013 *Phys. Rev. B* **87**, 235306.
[14] Hosur P and Qi X 2013 *C. R. Physique* **14**, 857.
[15] Tominaga J, Kolobov V A, Fons P, Nakano T, and Murakami S 2014 *Adv. Mater. Interfaces* **1**, 1300027.
[16] Taguchi K and Tanaka Y, 2014 Axial Current driven by Magnetization Dynamics in Weyl Semimetals *Preprint* 1406.4636.
[17] Haug H and Jauho P A 2007 *Quantum Kinetics in Transport and Optics of Semiconductors* (Springer, New York, 2nd ed.), pp. 45–46.