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# The quantum Hall effect in quantum dot systems

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**Abstract.** It is proposed to use quantum dots in order to increase the temperatures suitable for observation of the integer quantum Hall effect. A simple estimation using Fock-Darwin spectrum of a quantum dot shows that good part of carriers localized in quantum dots generate the intervals of plateaus robust against elevated temperatures. Numerical calculations employing local trigonometric basis and highly efficient kernel polynomial method adopted for computing the Hall conductivity reveal that quantum dots may enhance peak temperature for the effect by an order of magnitude, possibly above 77 K. Requirements to potentials, quality and arrangement of the quantum dots essential for practical realization of such enhancement are indicated. Comparison of our theoretical results with the quantum Hall measurements in $InAs$ quantum dot systems from two experimental groups is also given.

## 1. Introduction

The integer quantum Hall effect has been a subject for intense research in the two-dimensional systems, quantum wells and heterojunctions, since the moment of its discovery in 1980 [1], while the quantum dot systems were undeservedly overlooked, except for a few papers [2],[3]. It is traditional that the quantum Hall effect is a part of the low-temperature physics since it demands both strong magnetic fields $B$ and liquid helium temperatures $T$ to become visible in the conventional two-dimensional systems. So hard experimental conditions reflect fundamental physics of the effect, which is interplay between localization due to random disorder potential and dephasing processes due to electron-phonon interaction [4],[5]. Experiments show that the plateau widths scale with a temperature like $\Delta B \propto T^{\kappa}$ with $\kappa \approx 0.4$, and this non-trivial value of the scaling index $\kappa$ (substantially different from unity) proves that influence of electron-phonon interaction does not reduce to temperature broadening of the distribution function. Strictly speaking, all the electron states in a quantum Hall system are localized at $T = 0$ except for a null set [4], and the electron-phonon interaction cannot be treated in the same simple manner as for zero magnetic field. Therefore, a resulting "degree of delocalization", described quantitatively by the plateau widths, is governed by characteristics of disorder potential (its amplitude and correlation length), providing that the parameters of electron-phonon interaction are fixed. Of course, it is desirable to have a deep and smooth disorder potential for better localization of carriers, which guarantees higher temperatures for observation of the quantum Hall effect. Using modulation doping, it is possible to make disorder potential really smooth (as compared to a scale of the magnetic length) and observe the quantum Hall plateaus for high Landau levels (up to $n \sim 10$), however, amplitude of the potential produced by the remote dopants is not enough

to overcome $T \sim 10K$, even for $n = 0$. Evidently, the quantum dots produce a potential which is both deep and smooth, and it is worth to apply them for the quantum Hall effect.

We develop a theory of the integer quantum Hall effect in the quantum dot systems at finite temperatures, where electron-phonon interaction is taken into account by artificial broadening of the spectral functions $\mathcal{D}(\varepsilon)$ in Eq. (6) for the quantum correlator of velocities $w_{xy}(\varepsilon_1, \varepsilon_2)$. This approach allows us to reproduce low temperature behaviour of the transition width ($\Delta\varepsilon \propto T^\kappa$ with $\kappa \approx 0.4$) and carry out numerical calculations of the Hall conductivity efficiently. The latter is achieved by using a compact form of the one-particle hamiltonian (local trigonometric basis [6]) and the kernel polynomial method [7]. We have studied temperature dependencies of the magnetotransport curves between 4 K and 80 K as well as influence of the quantum dot parameters (size and potential depth), their density and arrangement, and our calculations show that the temperatures higher than $40K$ can be achieved for observation of the quantum Hall effect with a quantum dot array with a density of $1.4 \cdot 10^{10}$ cm$^{-2}$ at magnetic field $B = 10$ T. However, it is vital to preserve the short-range component of disorder small enough, the condition hardly satisfied in the available experimental papers [2],[3]. So, finally we give specific requirements essential for practical observation of the integer quantum Hall effect at elevated temperatures.

## 2. Theory: the model, spectrum and Hall conductivity

Basically, we consider rather simple model of the two-dimensional electron gas at strong magnetic field and an external potential, described in detail in Ref. [8]. To take into account both the quantum dots and disorder, we present the potential in the form $V(\mathbf{r}) = V_{\rm QD}(\mathbf{r}) + V_{\rm dis}(\mathbf{r})$, where potentials of the individual quantum dots are modelled by the gaussians,

$$V_{\rm QD}(\mathbf{r}) = -\sum_i V_i \exp\left(-\frac{(\mathbf{r}-\mathbf{r}_i)^2}{2d_i^2}\right), \tag{1}$$

which gives parabolic expansion near potental minima, a simple and practical approximation used in many papers, including Refs. [9],[10] (here, we have projected the task on plane of a structure by integrating out the third dimension). Physically, it is a very common case of the quantum dots composed due to variation of the alloy composition, for example $InGaAs$ quantum dots [11]. Though some researches call those quantum dots $InAs$, it is very unlikely that the effects of size quantization and strain alone are so strong and homogeneous to produce $\sim$ 1eV blueshift of the optical spectra, almost equal to difference between the bandgaps of $InAs$ and $GaAs$, and reproduced in all the papers on this issue. Thus, we estimate quantum dot potential amplitude as tens of meV instead of hundreds of meV, and their lateral dimensions as tens of nm. Such magnitudes are comparable with the cyclotron energy and magnetic length at magnetic field $B = 10$ T, which are $\hbar\omega_c = \hbar eB/m^*c \approx 30$ meV, $l_B = \sqrt{\hbar c/eB} \approx 8$ nm when considering electrons with the effective mass $m^* = 0.04m_0$ (something between $InAs$ and $GaAs$), meaning that magnetoconfined states are formed [12], and spectrum of carriers can be described by the Fock-Darwin solution [13]:

$$\varepsilon_{n,m} = \frac{\hbar\omega_c}{2}\left[2n + 1 + (n+m+1)(\sqrt{1+(2\omega_0/\omega_c)^2} - 1)\right], \tag{2}$$

where $\hbar\omega_0 = \sqrt{V_0\hbar^2/m^*d_0^2} \approx 20$ meV for $V_0 = 45$ meV, $d_0 = 14$ nm, $m^* = 0.04m_0$, and distance between the successive magnetoconfined levels of a given Landau level $n$ is about 10 meV, thus compared to the cyclotron energy of about 30 meV. As a result, we get $N = 3$ magnetoconfined levels from each quantum dot for each Landau level, these states are strongly localized and form special intervals of the quantum Hall plateaus robust to elevated temperatures. It is easy to estimate ratio of these states for a given density of the quantum dots (say, $n_{QD} = 1.4 \cdot 10^{10}$ cm$^{-2}$),

$$\mathcal{R} = 2\pi l_B^2 n_{QD} N \approx 20\%. \tag{3}$$

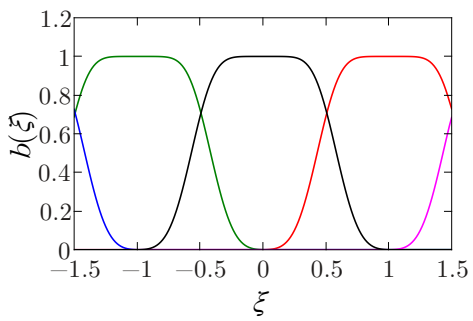


**Figure 1.** Optimal window function $b(x-q)$ is shown for odd and even $q$.

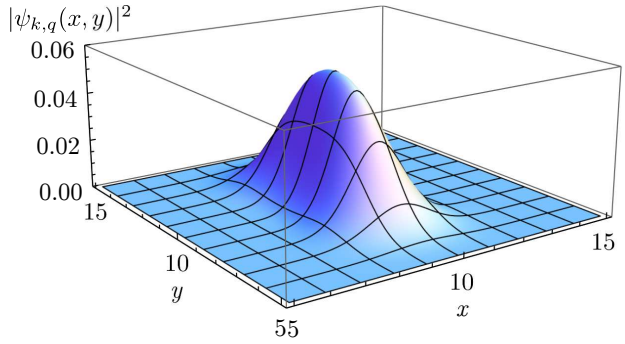


**Figure 2.** One of two peaks of the wave function $|\psi_{k,q}(x,y)|^2$ at $q=2$, $k=16$, $N=16^2$, $M=2\sqrt{N}$.

However, the estimation given by Eq. (3) does not imply that these special parts of plateaus are *absolutely* robust against temperature, and to find the maximum temperature maintaining the quantum Hall effect, we have carried out numerical calculations of the Hall conductivity treating electron-phonon interaction in an approximate manner, by smearing delta-functions in definition of the quantum correlator of velocities $w_{xy}$, so that

$$\sigma_{xy}(\varepsilon) = \frac{2e^2\hbar}{S} \int\limits_{\varepsilon_1=-\infty}^{\varepsilon} \int\limits_{\varepsilon_2=\varepsilon}^{\infty} \frac{w_{xy}(\varepsilon_1,\varepsilon_2)}{(\varepsilon_1-\varepsilon_2)^2} d\varepsilon_1 d\varepsilon_2, \tag{4}$$

$$w_{xy}(\varepsilon_1,\varepsilon_2) = \sum_{\alpha,\beta} \mathrm{Im}\{(v_x)_{\alpha\beta}v_y)_{\beta\alpha}\}\mathcal{D}(\varepsilon_1-\varepsilon_\alpha)\mathcal{D}(\varepsilon_2-\varepsilon_\beta), \tag{5}$$

$$\mathcal{D}(\varepsilon) = \frac{1}{\pi}\frac{\delta}{\delta^2+\varepsilon^2}, \tag{6}$$

where $S = L_x \times L_y$ is a sample area, $\alpha$ and $\beta$ count eigenstates of the one-particle hamiltonian [8], and $\delta$ is a temperature induced broadening, which describes the low temperature behaviour $\Delta\varepsilon \propto T^\kappa$ ($\kappa \approx 0.4$) best when defined as $\delta(T) = \mathcal{C}T^2$. Since to reveal the effects of a temperature we need to model rather extended samples, more than $100 l_B \sim 1\,\mu$m long, it is important to optimize computing by means of compact representation of the one-particle hamiltonian as described in Sec. 3 and avoiding explicit hamiltonian diagonalization by use of the kernel polynomial method [7], whose implementation to current problem will be published elsewhere.

## 3. Local trigonometric basis

In order to write the one-particle hamiltonian in a compact form, we start from its representation in basis of the "Landau stripes" [8], which we denote as $\Psi_k(x,y)$ (let us fix the Landau level number here, say $n=0$, and set $l_B=1$). These elements are extended over $x$ and localized near $y = L_y k/N_L$, where $N_L = S/2\pi$ is number of the electron states on each Landau level. Alternatively, we can apply fourier transform with respect to the quantum number $k$, resulting in the same set of functions but of the exchanged coordinates $x \leftrightarrow y$. To construct the *localized* basis instead, it is needed to appy the *short-time* Fourier transform,

$$\varphi_{k,q}(x,y) = \frac{1}{\sqrt{M}} \sum_{l=1}^{N_L} w\left(l/M - q\right) \exp(-2\pi i l k/M) \Psi_l(x,y), \tag{7}$$

where $w(\xi)$ is a window function, vanishing outside the interval $[-\frac{1}{2},\frac{1}{2}]$, and $M$ is the integer of the order of $\sqrt{N}$. Though it is expected that the functions $\varphi_{k,q}(x,y)$ are localized near the

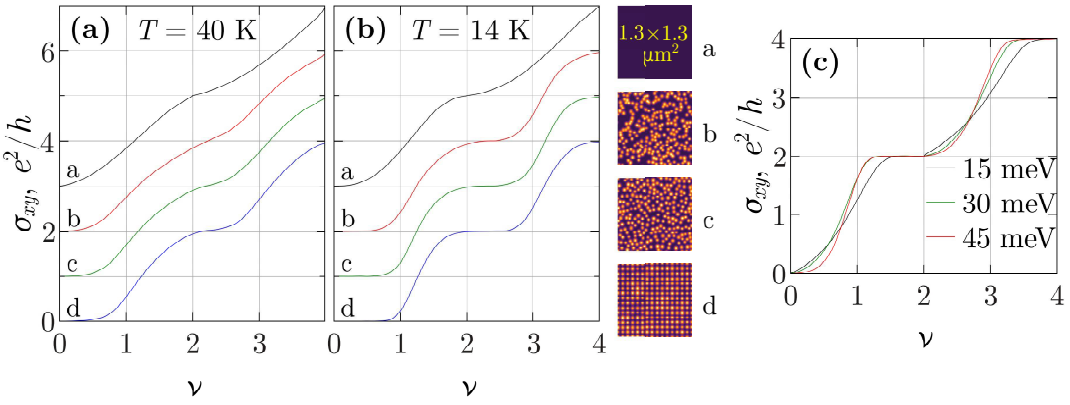


**Figure 3.** Picture of the quantum Hall effect in the quantum dot systems with different degree of arrangement presented for the temperatures (a) 40 K and (b) 14 K. Plot (c) demonstrates influence of the quantum dot potential amplitude for the model d ("crystal" of quantum dots). Filling factor $\nu = 2\pi l_B^2 n_e$ is proportional to electron concentration $n_e$ and inversely proportional to magnetic field $B$ (which was fixed during this set of calculations).

point $(L_x k/M, L_y qM/N_L)$, the Balian-Low theorem [14] tells us that $(\Delta x)^2 = \infty$ or $(\Delta y)^2 = \infty$ due to restrictions imposed on the window function $w(\xi)$ in order to keep orthogonality of the transformation. However, this recipe can be happily improved [6], and the functions

$$\psi_{k,q}(x,y) = \sqrt{\frac{2}{M}} \sum_{l=1}^{N_L} b\,(l/M - q)\cos[\pi(k-1/2)(l/M - q + 1/2)]\Psi_l(x,y) \qquad (8)$$

form the *local trigonometric basis*. Speaking formally, the function given by Eq. (8) are not localized at all, however, each of them consists of the two well localized functions, one situated at $(L_x k/2M, L_y qM/N_L)$, and another one at $(L_x(1 - k/2M), L_y qM/N_L)$, but this is enough to form *sparse* representation of the one-particle hamiltonian. Surely, the window function $b(\xi)$ still obeys some relations protecting orthogonality of the transformation [6], and it is practical to find specific shape of $b(\xi)$ supporting best localization within these restrictions numerically. Shape of the optimized window function and a hump of one of the functions (8) are shown in Figs. 1, 2. As a result, the hamiltonian matrix has fixed number ($\sim 100$) of non-vanishing elements in each row, providing for efficient modelling of the systems larger than $1\,\mu\mathrm{m}^2$.

## 4. Numerical results and discussion

We have carried out numerical calculations of the Hall conductivity following the approach described in previous sections, and our results are shown in Fig. 3 for three types of the quantum dot arrangement in comparison with case of a quantum well, as illustrated between the plots (b) and (c) of Fig. 3 (the potentials produced by quantum dot systems are shown there). Let us indicate these models as quantum dot "vacuum" (a), "gas" (b), "liquid" (c) and "crystal" (d). We used parameters typical for strong magnetic field measurements in *InGaAs* quantum dot systems, as expained in Sec. 2: $B = 10\,\mathrm{T}$ ($l_B \approx 8\,\mathrm{nm}$), $m^* = 0.04 m_0$ ($\hbar\omega_c \approx 30\,\mathrm{meV}$), $g^* = 0$ [15], $n_{QD} = 1.4 \cdot 10^{10}\,\mathrm{cm}^{-2}$, $d_0 = 14\,\mathrm{nm}$, $V_0 = 45\,\mathrm{meV}$ (except for the two curves in Fig. 3(c) given for $V_0$ of 15 and 30 meV). Relative scattering of the quantum dot potential depths $V_i$ and radiuses $d_i$ was taken as $\pm 20\%$, and the short-range disorder potential producing $2\Gamma = 4\,\mathrm{meV}$ broadening of the Landau levels was included. Figs. 3(a, b) demonstrate developing picture of the integer quantum Hall effect in the quantum dot systems at $T = 40\,\mathrm{K}$ (for the model "d"), while only a

hint of quantization is seen for the quantum well model "a" at $T = 14\,\mathrm{K}$ (where all the quantum dot models exhibit really wide plateaus). Thus, quantum dots are shown theoretically to increase maximum temperature allowed for observation of the quantum Hall effect by almost an order of magnitude, especially when they are properly arranged, a property expected from the Stranski-Krastanov quantum dots. However, it is vital to protect quantum dots from the pernicious short-range potential, its effective amplitude $\Gamma$ must be much smaller than the quantum dot potential depth $V_0$ and cyclotron energy $\hbar\omega_c$. Once this condition is not entirely satisfied, like for one of the curves in Fig. 3 corresponding to $V = 15\,\mathrm{meV}$, we notice considerable narrowing of the plateaus, and at $2\Gamma/V_0 \sim 0.5$ they fully disappear.

Experimentally, the quantum Hall effect in quantum dot systems has been studied in just a few papers [2],[3], and experimental results are not so optimistic as our theoretical predictions. The phenomenon has been observed only at liquid helium temperatures ($T \lesssim 4\,\mathrm{K}$ in Ref. [2] and $T \lesssim 0.3\,\mathrm{K}$ in Ref. [3]), only the fundamental plateau (corresponding to filling factor $\nu = 2$) was pronounced (as expected for the short-range disorder [8]), and quantization precision was not high (about 1%). This should not be perceived as a surprise since quantum dots enhance the quantum Hall effect only if they have enough power for this, in particular, their potential amplitude is much larger than the effective amplitude of the short-range disorder potential. We attribute unsatisfactory experimental results to *insufficient quality* of the investigated samples in the sense that they contain enough short-range (as compared to $l_B$) variations of composition and *insufficient electron concentration*, which drives the fundamental quantum Hall plateau ($\nu = 2$) at the moderate magnetic fields, corresponding to (relatively) large values of magnetic length ($B = 3\,\mathrm{T}$, $l_B = 15\,\mathrm{nm}$ in Ref. [2] and $B = 1.5\,\mathrm{T}$, $l_B = 21\,\mathrm{nm}$ in Ref. [3]). Taking into account $\sim 1\,\mathrm{eV}$ difference between $InAs$ and $GaAs$ band gaps, it is easy to estimate that short-range compositional fluctuations of the order of 1% produce random potential amplitude and Landau level width $2\Gamma$ of the order of $10\,\mathrm{meV}$, which is by no means smaller than the cyclotron energy corresponding to magnetic fields mentioned above ($\hbar\omega_c \approx 10\,\mathrm{meV}$ in Ref. [2] and $\hbar\omega_c \approx 5\,\mathrm{meV}$ in Ref. [3]). However, since nanotechnology is developing quite fast, we expect observation of the predicted effect of the *quantum Hall enhancement by quantum dots* in future quantum dot structures with smoothed compositional fluctuations and enhanced electron concentration.

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