

Signal analysis and characterization of a micro-electro-mechanical oscillator for the study of quantum fluids

C S Barquist¹, J Bauer¹, T Edmonds¹, P Zheng¹, W G Jiang¹, M Gonzalez¹, Y Lee¹ and H B Chan²

¹ Department of Physics, University of Florida, Gainesville, Florida, USA

² Department of Physics, Hong Kong University of Science and Technology, HK

Abstract. We present the full characterization of a micro-electro-mechanical system (MEMS) based probe developed for the study of quantum fluids. The device consists of a pair of parallel plates ($200 \times 200 \mu\text{m}^2$) with a well-defined gap ($1.25 \mu\text{m}$) in which a fluid film forms when immersed in liquid. The mobile plate is suspended above the fixed plate (substrate) by four serpentine springs. This geometry allows for the study of the properties of the surrounding liquid through the resonant behavior of the mobile plate. This device demonstrated its potential as a high precision probe suitable for the study of films of quantum fluids and also of quantum turbulence. In this work we present our detailed analysis of the device signal in our current measurement scheme. Based on our analysis, we determined the transduction factor of a device at room temperature and 4 K, which allows us to convert the measured quantities to physical quantities such as displacement, velocity, and force.

1. Introduction

Micro-Electro-Mechanical Systems or MEMS have a wide variety of uses in the engineering and scientific worlds. In our laboratory we have developed a MEMS-based probe for the study of quantum fluids functioning in the ultra low temperature regime [1]. The device is a comb-drive resonator designed specifically to form a thin film of superfluid ^3He and also to perform interface sensitive measurements. An overview of the MEMS device is shown in Fig. 1. The device is composed of the center plate which is suspended above the substrate by the four serpentine springs and the two sets of integrated capacitors on each side. The gap between the center plate and the substrate which defines a slab when immersed in liquid could be 0.75 , 1.25 , or $2.00 \mu\text{m}$. The device used in this work has a $1.25 \mu\text{m}$ gap and a $200 \times 200 \mu\text{m}^2$ center plate.

Similar devices have been tested in liquid ^3He down to $300 \mu\text{K}$ and in liquid ^4He down to 50 mK and demonstrated their potential as a unique and versatile experimental probe for quantum fluids [1]. It is essential to calibrate a device to be able to convert the measured quantity (electrical voltage) to physically relevant quantities such as displacement and force to accomplish quantitative understanding of the results and phenomena. This conversion is done through the transduction factor, $\tilde{\beta}$ which is device specific and independent of the surrounding fluid. In this paper we present a full theoretical analysis of our measurement scheme. We demonstrate the validity of our analysis through various measurements and finally present the measured transduction factor at room temperature and 4.2 K in vacuum.



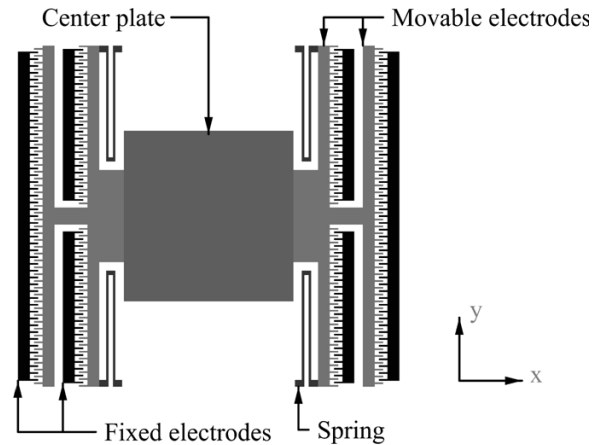


Figure 1. Top view of the MEMS device. The darker colored regions are fixed, they are where the carrier signal and actuating voltages are applied. The lighter colored regions are movable and interact with the fluid. The movable parts are actuated by the capacitive interaction between the teeth of the electrodes.

2. Measurement scheme

Investigation of fluid properties using the MEMS oscillator is carried out by observing how the fluids affect the resonant properties of the device. Because the properties of the fluid change with controllable parameters such as temperature and pressure so do those of the device in the fluid. The resonant properties of the device – resonance frequency, resonance width or quality factor, and resonant peak amplitude – can be measured by driving the device in the linear regime and sweeping the frequency through the resonance. These resonant properties can then be related back to physical quantities such as the damping coefficient and the velocity.

The MEMS device is driven by the capacitive interaction between the fixed and movable electrodes. The inter-locking teeth of the electrodes form an array of parallel plate capacitors, with the capacitance being dependent on the overlap of the teeth of the movable electrode with those of the fixed electrode as shown in Fig. 1. Because the capacitance depends on the amount of overlap between the teeth, the energy stored in the field between the teeth depends on this overlap. The overlap between the teeth depends on the displacement of the movable structure, therefore there is a position dependent energy in the system and consequently a force. It is this force that is used to actuate the device. Choosing the frequency of the oscillating force to be that of the resonant motion in a desired direction insures that the motion of the device is in the desired direction. The displacement of the device is detected by measuring the capacitance change, differential capacitance to be more specific. We adopted a capacitance bridge to perform high resolution differential capacitance measurement. Our measurement scheme is displayed in Fig. 2.

Our scheme is basically an amplitude modulation. The high frequency (hf) carrier signal labeled as V_{hf} is fed into the circuit through an isolation transformer for the bridge. The carrier signal is then split using an inductive ratio transformer. The ratio then is adjusted to balance out any asymmetries in the differential capacitance when the device is in its equilibrium position in the presence of a DC bias. The actuation of the device is preformed by the low frequency (lf) signal labeled as V_{lf} which is swept through the mechanical resonance. The lf signal is added to the hf carrier signal via a splitter used in reverse as a signal adder. The DC is added after a blocking capacitor in order to isolate the rest of the circuit from the DC voltage. A summing amplifier and blocking capacitor are also placed on the other side of the circuit for

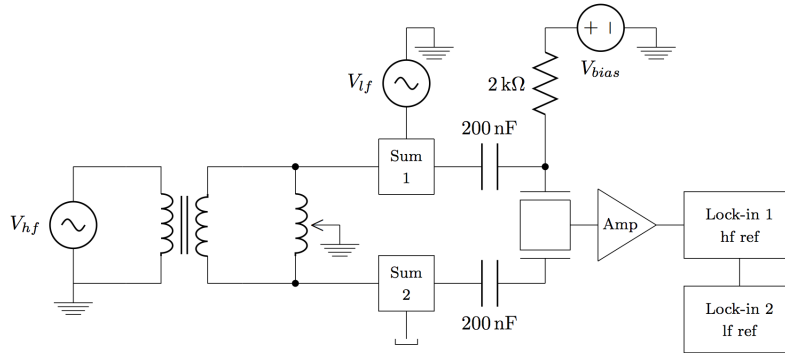


Figure 2. Measurement Scheme. The movable portion of the MEMS device is represented by the box connected to the ‘Amp’ with the fixed electrodes on either side of it represented by one plate of a capacitor.

symmetry, although they serve no functional purpose. When the device is actuated from the balance point, a net charge is induced on the center plate and is detected using an Amptek A250 charge sensitive amplifier. The signal from the amplifier is then demodulated by two lock-in amplifiers: first high frequency and then low frequency.

3. Analysis

Consider the voltage output A of the charge-sensitive amplifier when the plate has a charge q . The amplifier has a charge-to-voltage conversion factor α which is determined by a feedback capacitor. The amplitude A in volts is given by:

$$A = \alpha q. \quad (1)$$

The total charge comes from the capacitive interaction between the fixed and the movable electrodes, and the net charge q can be split into the charge $q_{r(l)}$ coming from the interaction between electrodes on the right (left) side of the device:

$$q_l = C_l V_l, \quad (2)$$

$$q_r = C_r V_r. \quad (3)$$

C_l and C_r are the capacitances between the fixed and the movable electrodes on the left and right side, respectively. V_l and V_r are the voltages between the fixed and the center electrodes. For small displacements, x , from the equilibrium position, x_0 , the capacitance changes linearly:

$$C_l = \tilde{\beta}(x_0 + x), \quad (4)$$

$$C_r = \tilde{\beta}(x_0 - x). \quad (5)$$

$\tilde{\beta}$ is a geometrical parameter that gives the capacitance between the fixed and moving electrodes as a function of the overlap of the comb teeth. The voltages applied to each side are:

$$V_L = \frac{V_{hf}}{2} + V_{lf} + V_{bias}, \quad (6)$$

$$V_R = -\frac{V_{hf}}{2}. \quad (7)$$

For brevity, the sinusoidal time dependence of V_{hf} and V_{lf} are implied. Using Eqs. (2) - (7) provides a relationship between the applied voltages, the displacement, and the charge on the plate:

$$q = \tilde{\beta}x[V_{hf} + V_{bias} + V_{lf}]. \quad (8)$$

In Eq. (8) terms that depend only on x_0 and not on x are neglected since these terms will be filtered by our detection scheme. The displacement x is the solution to the equation of motion for a damped driven harmonic oscillator with the natural frequency ω_0 and damping coefficient γ :

$$\ddot{x} + \frac{\gamma}{m}\dot{x} + \omega_0^2 x = \frac{F}{m}, \quad (9)$$

where m is the mass of the oscillator (center plate). The solution then takes the form:

$$x = Be^{-i\omega t}, \quad (10)$$

where B is the amplitude of oscillation and ω is the frequency of the driving force:

$$B = \frac{F}{m} \frac{(\omega_0^2 - \omega^2) + i(\frac{\gamma}{m}\omega)}{(\omega_0^2 - \omega^2)^2 + (\frac{\gamma}{m}\omega)^2}. \quad (11)$$

On the other hand the force on the plate is found by differentiating the capacitive energy with respect to the overlap:

$$\vec{F}_j = -\frac{d}{dx}[\frac{1}{2}C_j V_j^2]\hat{x} = -\frac{1}{2}\frac{dC_j}{dx}V_j^2\hat{x}, \quad (12)$$

where j denotes the left or right side. The net force is given by:

$$\vec{F} = \vec{F}_L + \vec{F}_R = \frac{\tilde{\beta}}{2}[V_{lf}^2 + V_{bias}^2 + V_{hf}V_{lf} + V_{hf}V_{bias} + 2V_{lf}V_{bias}]\hat{x}. \quad (13)$$

Each term in Eq. (13) contains its own respective frequency. The only relevant force term is the one that drives the oscillator through the resonance. All other terms are off resonance or filtered out during detection. Therefore, the force can then be written as an effective force:

$$F = \tilde{\beta}V_{lf}V_{bias}. \quad (14)$$

Upon substituting Eq. (14) into (11) one has a solution for the amplitude of oscillation as a function of frequency, which can be linked to the detected output voltage with the aid of Eqs. (8) & (1). It is worth mentioning that the driving force does not depend on V_{hf} .

One needs a way to determine $\tilde{\beta}$ experimentally. For this consider the signal at resonance corresponding to the imaginary (absorption) term in Eq. (11). Since $\omega = \omega_0$ at resonance and $\frac{\gamma}{m} = \Delta\omega$, where $\Delta\omega$ is the full-width-half-maximum (FWHM) of the resonance [2], the out of phase amplitude is given by:

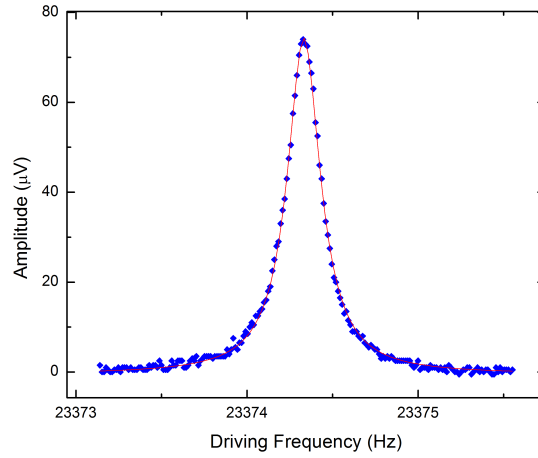


Figure 3. Absorption curve, the red line is the fitted Lorentzian curve. $V_{lf} = 274 \mu V_{peak}$, $V_{bias} = 10 V$, $V_{hf} = 1.41 V_{peak}$

$$\Im m(B) = \frac{\tilde{\beta} V_{lf} V_{bias}}{m \Delta \omega \omega_0}. \quad (15)$$

An expression is obtained for the detected voltage A after the charge sensitive amplifier by combining (15), (8), and (1):

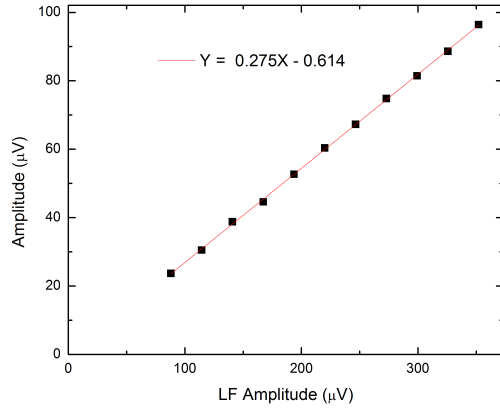
$$A = \frac{\alpha \tilde{\beta}^2 V_{bias} V_{hf} V_{lf}}{m \Delta \omega \omega_0}. \quad (16)$$

In this expression only the transduction factor is unknown. All other quantities can be measured experimentally or are determined by the known geometry of the device and material parameters. Unlike the driving force, the signal amplitude is proportional to all three excitation voltages. By measuring the signal amplitude at resonance as a function of the applied voltage one can be sure not to include any intrinsic offset not accounted for in Eq. (16). Doing this one uses the slope of the amplitude vs. applied voltage to determine the transduction factor. Experimentally, the transduction factor can be determined through:

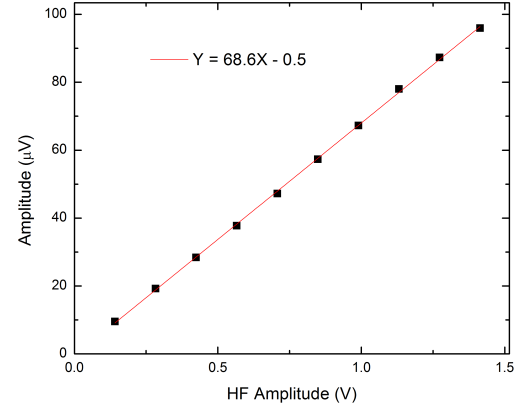
$$\tilde{\beta} = \sqrt{\frac{dA}{dV_{lf}} \frac{m \Delta \omega \omega_0}{\alpha V_{bias} V_{hf}}}. \quad (17)$$

4. Measurement

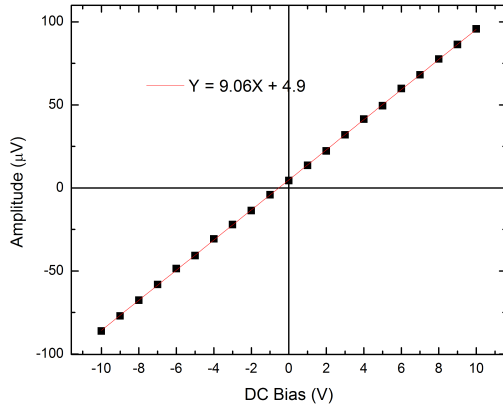
To test the validity of Eq. (16) the device signal is measured as a function of the three voltages: V_{lf} , V_{hf} , and V_{bias} . The transduction factor is calculated using Eq. (17) and compared with the theoretical value. The amplitude, A , is measured by sweeping the frequency of V_{lf} through the resonance. Each sweep contained 200 data points over a frequency range 5 times the FWHM (see Fig. 3). All the measurements were done in the linear regime of the oscillator where the resonance curves fall on a single Lorentzian when scaled by the excitation. The data is then fitted to a Lorentzian to obtain the peak amplitude at resonance.



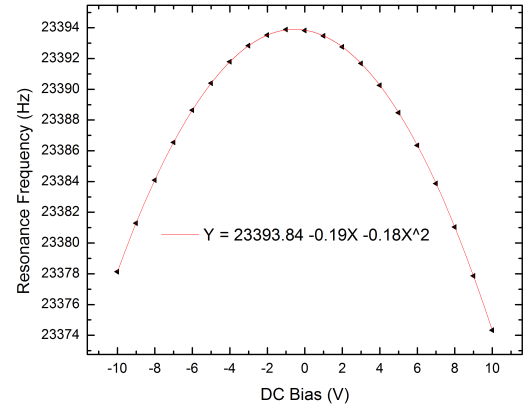
(a) Amplitude vs. V_{lf} . $V_{hf} = 1.41 V_{peak}$ & $V_{bias} = 10 V$



(b) Amplitude vs. V_{hf} . $V_{lf} = 353 \mu V_{peak}$ & $V_{bias} = 10 V$



(c) Amplitude vs. DC bias. $V_{hf} = 1.41 V_{peak}$ & $V_{lf} = 353 \mu V$



(d) Resonance frequency vs. DC bias. $V_{hf} = 1.41 V_{peak}$ & $V_{lf} = 353 \mu V$

Figure 4. MEMS response to excitation voltages

The signal amplitudes as a function of V_{lf} , V_{hf} , and V_{bias} are shown in Fig. 4. For each of these measurements the other two excitations were held constant. For example, V_{hf} was held at $1.41 V_{peak}$ and V_{bias} at $10 V$ while V_{lf} was varied in Fig. 4(a). These measurements confirm the linear dependences on all three excitations as expected in Eq. (16). Altering the DC bias has an interesting effect on the MEMS device. V_{bias} affects the resonance frequency of the device (see Fig. 4(d)). The shape of the bias dependence indicates softening of the device at higher bias. It might be caused by levitation or tilting of the center plate caused by asymmetric electric fields due to the presence of the substrate.

The transduction factor, $\tilde{\beta}$, is calculated from the slope in Fig. 4(a), $\tilde{\beta} = 1.36 \times 10^{-9} Fm^{-1}$. The transduction factor can be estimated by computing the capacitance of the overlapping comb-drive teeth in the parallel plate approximation:

$$\tilde{\beta} = \frac{N\epsilon\epsilon_0 t}{d}, \quad (18)$$

where N is the number of capacitors made by the teeth, ϵ is the relative permittivity, ϵ_0 is the

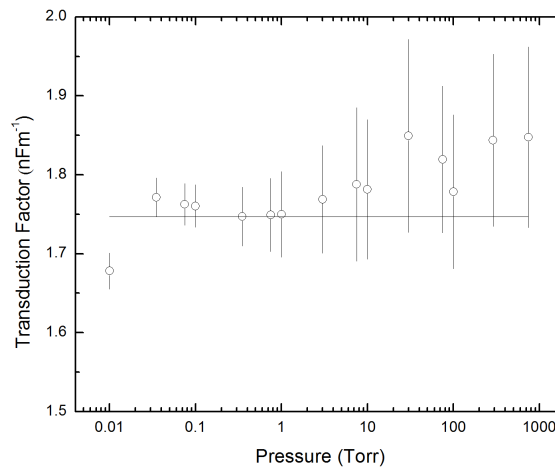


Figure 5. Pressure dependence of another MEMS device of the same type at room temperature in air. The solid line indicates the weighted mean of the transduction factor.

permittivity of free space, t is the height of the teeth perpendicular to the line pointing from one tooth to the other, and d is the distance between teeth. This formula is the simplest form of parallel plate capacitance ignoring the fringe field effect and gives $\tilde{\beta} = 1.68 \times 10^{-9} \text{ Fm}^{-1}$, which is in good agreement with our measurement. We also performed multiple measurements of $\tilde{\beta}$ for the same device after thermal cycling and at RT. The transduction factor at room temperature is found to be $\tilde{\beta} = 1.36 \times 10^{-9} \text{ Fm}^{-1}$ and remained the same after thermal cycling, verifying that $\tilde{\beta}$ is a device specific geometric parameter. We also measured the pressure dependence of the transduction factor for another device of the same design at room temperature in air (Fig 5). At higher pressures the uncertainties become large due to higher damping. The solid line indicates the weighted average of the transduction factor, $\tilde{\beta} = 1.75 \times 10^{-9} \text{ Fm}^{-1}$. Within the uncertainty, no significant pressure dependence is present.

5. Conclusions & Discussion

We performed a full circuit analysis of our measurement scheme involving three excitation voltages and a capacitance bridge. The transduction factor $\tilde{\beta}$, introduced in our analysis, is a device specific geometrical parameter that links electrical quantities to physically meaningful quantities. Our analysis was experimentally verified and $\tilde{\beta}$ was determined experimentally and compared with the theoretical value.

Acknowledgments

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References

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