Spin-gap phenomenon in a strongly interacting ultracold Fermi gas

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Abstract. We investigate magnetic properties of a strongly interacting ultracold Fermi gas. Within the framework of an extended T-matrix approximation, we calculate the spin susceptibility χ in the unitarity limit. We show that effects of pairing fluctuations on this magnetic quantity are quite different in between the normal state and the superfluid phase. In the normal state, pairing fluctuations cause spin-gap phenomenon near the superfluid phase transition temperature T_c, where χ is anomalously suppressed. In the superfluid phase, on the other hand, the ordinary suppression of χ by the BCS energy gap is weakened by pairing fluctuations, because they induce finite density of states inside the gap. Our results indicate that the spin susceptibility is a useful quantity for the study of pairing fluctuations in the BCS-BEC crossover regime of an ultracold Fermi gas.

1. Introduction

An ultracold Fermi gas is now widely recognized as a useful quantum many-body system[1, 2]. Indeed, using a tunable pairing interaction associated with a Feshbach resonance, we can systematically examine how a weak-coupling BCS-type Fermi superfluid continuously changes into the Bose-Einstein condensation (BEC) of tightly bound molecules[3, 4, 5, 6, 7, 8, 9, 10, 11]. In the intermediate coupling regime where strong pairing fluctuations exist, we can study strong-coupling physics beyond the mean-field level.

While the cold Fermi gas system has the above mentioned advantage, compared with the electron system, there are not so many experimental techniques to observe various physical quantities. Although this weak point is gradually being overcome by recent extensive experimental efforts[12, 13, 14, 15, 16, 17, 18], the current stage of cold Fermi gas physics still needs ideas to observe strong coupling phenomena within the current experimental techniques.

In this paper, we investigate the uniform spin susceptibility χ in a unitary Fermi gas both above and below T_c. This magnetic quantity has recently become possible to observe in this field[15, 16, 17, 18]. (Note that spins $\sigma = \uparrow, \downarrow$ in this system are actually *pseudospins* describing two atomic hyperfine states.) Since χ is sensitive to the formation of (pseudo)spin-singlet pairs, the observation of this quantity is expected to give useful information about pairing fluctuations. In this paper, including strong-coupling effects within the framework of an extended T-matrix approximation (ETMA)[19, 20, 21], we calculate χ both above and below T_c. In the normal state, we show that pairing fluctuations cause the spin-gap phenomenon, being characterized by the suppression of χ. In the superfluid phase, on the other hand, pairing fluctuations are found to enhance χ, reflecting that they smear the BCS excitation gap in the density of states. We
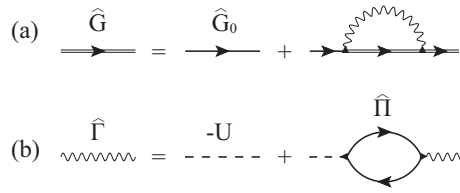
Figure 1. (a) Diagrammatic representation of the Dyson equation for the 2×2-matrix single-particle Green's function $\hat{G}$ (double-solid line). The solid line is the bare Green's function $\hat{G}^0$ with no self-energy correction. The wavy line is the particle-particle scattering matrix $\hat{\Gamma}$ which is diagrammatically given as panel (b). In panel (b), the dashed line is the pairing interaction $-U$, and $\hat{\Pi} = \{\Pi^{ij}\}$ is the pair-correlation function given in Eq. (5).

also compare our results with the recent experiment on a superfluid ^{6}Li Fermi gas. Throughout this paper, we take $\hbar = k_\mathrm{B} = 1$, and the system volume is taken to be unity, for simplicity.

2. Formulation: Extended T-matrix approximation

We consider a two-component Fermi gas described by the ordinary BCS model. Since we deal with both the normal and superfluid phases, it is convenient to employ the Nambu representation[22]. In this formalism, the BCS Hamiltonian is written as[23, 24]

$$H = \sum_{\mathbf{p}} \Psi_{\mathbf{p}}^{\dagger} \left[\xi_{\mathbf{p},\uparrow} \frac{\tau_3 + 1}{2} + \xi_{\mathbf{p},\downarrow} \frac{\tau_3 - 1}{2} - \Delta \tau_1 \right] \Psi_{\mathbf{p}} - U \sum_{\mathbf{q}} \rho_+(\mathbf{q}) \rho_-(-\mathbf{q}). \tag{1}$$

Here, $\Psi_{\mathbf{p}}^{\dagger} = (c_{\mathbf{p},\uparrow}^{\dagger}, c_{-\mathbf{p},\downarrow})$ is the Nambu field, where $c_{\mathbf{p},\sigma}$ is the annihilation operator of a Fermi atom with pseudospin $\sigma = \uparrow, \downarrow$, describing two atomic hyperfine states. $\xi_{\mathbf{p},\sigma} = p^2/2m - \mu - \sigma h$ is the kinetic energy of a Fermi atom, measured from the Fermi chemical potential μ (where m is an atomic mass). In this paper, although we consider the unpolarized case, we have introduced an infinitesimally small effective magnetic field h in $\xi_{\mathbf{p},\sigma}$ to calculate the spin susceptibility. $-U$ is a pairing interaction, which is related to the s-wave scattering length a_s as $4\pi a_s/m = -U/[1 - U \sum_{\mathbf{p}}^{\omega_\mathrm{c}} \frac{m}{p^2}]$ (where ω_c is a cut-off energy)[6]. In Eq. (1), $\rho_{\pm}(\mathbf{q}) = [\rho_1(\mathbf{q}) \pm i\rho_2(\mathbf{q})]/2$, where $\rho_j(\mathbf{q}) = \sum_{\mathbf{p}} \Psi_{\mathbf{p}+\mathbf{q}/2}^{\dagger} \tau_j \Psi_{\mathbf{p}-\mathbf{q}/2}$ $(j = 1, 2)$ are the generalized density operators[23, 24] (where τ_j $(j = 1 \sim 3)$ are the Pauli matrices acting on the particle-hole space). In Eq. (1), the superfluid order parameter $\Delta = U \sum_{\mathbf{p}} \langle c_{-\mathbf{p},\downarrow} c_{\mathbf{p},\uparrow} \rangle$ is taken to be real and is proportional to the τ_1 component without loss of generality. In this case, ρ_1 and ρ_2 physically describe amplitude and phase fluctuations of the order parameter Δ, respectively[23]. In this paper, we ignore effects of a harmonic trap and consider a uniform Fermi gas, for simplicity.

Strong-coupling effects on single-particle excitations are taken into account by the self-energy $\hat{\Sigma}(\mathbf{p}, i\omega_n)$ in the 2×2-matrix single-particle thermal Green's function,

$$\hat{G}(\mathbf{p}, i\omega_n) = \begin{pmatrix} G_{11}(\mathbf{p}, i\omega_n) & G_{12}(\mathbf{p}, i\omega_n) \\ G_{21}(\mathbf{p}, i\omega_n) & G_{22}(\mathbf{p}, i\omega_n) \end{pmatrix} = \frac{1}{i\omega_n - \xi_{\mathbf{p},\uparrow} \frac{\tau_3+1}{2} - \xi_{\mathbf{p},\downarrow} \frac{\tau_3-1}{2} + \Delta \tau_1 - \hat{\Sigma}(\mathbf{p}, i\omega_n)}, \tag{2}$$

where ω_n is the fermion Matsubara frequency. In the extended T-matrix approximation (ETMA), the Dyson equation for the Green's function in Eq. (2) is diagrammatically described as Fig.1. The resulting 2×2-matrix self-energy is given by

$$\hat{\Sigma}(\mathbf{p}, i\omega_n) = -T \sum_{\mathbf{q}, i\nu_l} \sum_{i,j=\pm} \Gamma^{ij}(\mathbf{q}, i\nu_l) \tau_{-i} \hat{G}(\mathbf{p} - \mathbf{q}, i\omega_n - i\nu_l) \tau_{-j}. \tag{3}$$

Here, ν_l is the boson Matsubara frequency, and

$$\hat{\Gamma}(\mathbf{q}, i\nu_l) = \begin{pmatrix} \Gamma^{-+}(\mathbf{q}, i\nu_l) & \Gamma^{--}(\mathbf{q}, i\nu_l) \\ \Gamma^{++}(\mathbf{q}, i\nu_l) & \Gamma^{+-}(\mathbf{q}, i\nu_l) \end{pmatrix} = -U \left[1 + U \begin{pmatrix} \Pi^{-+}(\mathbf{q}, i\nu_l) & \Pi^{--}(\mathbf{q}, i\nu_l) \\ \Pi^{++}(\mathbf{q}, i\nu_l) & \Pi^{+-}(\mathbf{q}, i\nu_l) \end{pmatrix} \right]^{-1}, \quad (4)$$

is the particle-particle scattering matrix, where

$$\Pi^{ij}(\mathbf{q}, i\nu_l) = -T \sum_{\mathbf{p}, i\omega_n} \mathrm{Tr} \left[\tau_i \hat{G}^0(\mathbf{p} + \mathbf{q}, i\omega_n + i\nu_l) \tau_j \hat{G}^0(\mathbf{p}, i\omega_n) \right] \quad (5)$$

is the lowest-order pair-correlation function. $\hat{G}^0(\mathbf{p}, i\omega_n)$ is the bare Green's function given by Eq.(2) with $\hat{\Sigma}=0$.

The superfluid order parameter Δ is determined from the condition, $\det[\hat{\Gamma}(\mathbf{q} = \mathbf{0}, i\nu_l = 0)]^{-1}_{h=0} = 0$[24], which gives

$$1 = U \sum_{\mathbf{p}} \frac{1}{2E_{\mathbf{p}}} \tanh \frac{E_{\mathbf{p}}}{2T}, \quad (6)$$

where $E_{\mathbf{p}} = \sqrt{(p^2/2m - \mu)^2 + \Delta^2}$. Although Eq. (6) has the same form as the ordinary BCS gap equation, the Fermi chemical potential μ in this equation is known to remarkably deviate from the Fermi energy ε_{F} in the unitarity limit[6]. This strong-coupling correction can be conveniently incorporated into the theory by solving the gap equation (6), together with the equation for the number N of Fermi atoms, $N = 2T \sum_{\mathbf{p}, i\omega_n} G_{11}(\mathbf{p}, i\omega_n)|_{h=0}$. In the normal state ($\Delta = 0$), we only solve the number equation to determine μ as a function of the temperature.

The uniform spin susceptibility $\chi(T)$ is calculated from

$$\chi(T) = \left. \frac{\partial \Delta N}{\partial h} \right|_{h \to 0} = T \sum_{\mathbf{p}, i\omega_n} \mathrm{Tr} \left. \frac{\partial \hat{G}(\mathbf{p}, i\omega_n)}{\partial h} \right|_{h \to 0}. \quad (7)$$

Here, $\Delta N = N_\uparrow - N_\downarrow$, where N_σ is the number of Fermi atoms in the σ component. In this paper, we numerically evaluate Eq. (7) by taking a small but finite value of h.

We note that the self-energy $\hat{\Sigma}_{\mathrm{TMA}}$ in the ordinary T-matrix approximation (TMA)[24, 25, 27, 28, 29, 30] is obtained by replacing the renormalized Green's function $\hat{G}$ in Eq. (2) with the bare one $\hat{G}^0$. The strong-coupling theory developed by Nozières and Schmitt-Rink (NSR)[5] is also reproduced by expanding the Green's function in Eq. (2) with the self-energy being replaced by Σ_{TMA} to $O(\hat{\Sigma}^{\mathrm{TMA}})$. While the TMA and the NSR theory have been extensively used to clarify various BCS-BEC crossover physics in ultracold Fermi gases[8, 9, 24, 25, 26, 27, 28, 29, 30], they are known to unphysically give negative spin susceptibility in the crossover region[19, 31, 32]. On the other hand, it has recently been shown that the ETMA can correctly describe the spin susceptibility in the whole BCS-BEC crossover region[19, 21, 33], and the results agree well with the recent experiment on a ^{6}Li normal Fermi gas in the BCS regime[19]. Thus, in this paper, we also employ the ETMA to examine strong-coupling corrections to χ in the unitarity limit.

3. Spin susceptibility in a unitary Fermi gas
Figure 2(a) shows the spin susceptibility χ in a unitary Fermi gas ($(k_{\mathrm{F}} a_s)^{-1} = 0$). In the normal state, the ETMA spin susceptibility is smaller than the mean-field result ('MF' in this figure), and decreases with decreasing the temperature when $T/T_{\mathrm{F}} \leq 0.37$. Since pairing fluctuations are completely ignored in the mean-field theory, this spin-gap behavior of χ originates from strong pairing fluctuations near T_{c}.

We briefly note that the present ETMA gives the first-order phase transition as shown in the inset in Fig. 2(a), which is, however, an artifact of this strong-coupling theory. This leads to the
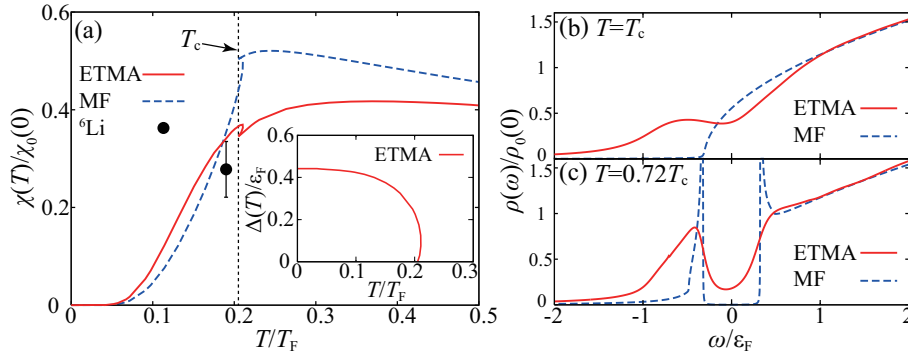
Figure 2. (a) Calculated ETMA spin susceptibility χ in a unitary Fermi gas ($(k_F a_s)^{-1} = 0$), normalized by the spin susceptibility $\chi_0(0)$ of a free Fermi gas at $T = 0$. The temperature is normalized by the Fermi temperature T_F. 'MF' is the mean-field BCS result in Eq. (8) where μ and Δ evaluated in the ETMA are used. (Note that the expression is reduced to that for a free Fermi gas above T_c, except that the ETMA result for μ is used.) The solid circle shows the recent experiment on a ^{6}Li Fermi gas[15]. The inset shows $\Delta(T)$. (b) Single-particle density of states $\rho(\omega)$ at T_c. $\rho_0(0)$ is the density of states at the Fermi level for a free Fermi gas. 'MF' is the density of states for a free Fermi gas where μ obtained in the ETMA is used. (c) $\rho(\omega)$ in the superfluid state at $T = 0.72T_c$. 'MF' is the superfluid density of states in the mean-field BCS theory where μ and Δ obtained in the ETMA are used.

singular behavior of χ near T_c seen in Fig. 2(a). In this regard, we recall that the same problem exists in the TMA[24], as well as the NSR theory[34]. Although the improvement of the ETMA so as to recover the expected second-order superfluid phase transition is a crucial issue[35, 36], we leave this as a future problem.

In the superfluid phase, apart from the above mentioned singularity just below T_c, χ decreases with decreasing the temperature, as shown in Fig. 2(a). Although this behavior is already seen in the mean-field case, the decrease is more remarkable in the latter. In the mean-field BCS theory, the spin susceptibility is given by[38]

$$\chi = \frac{1}{2T} \int_{-\infty}^{\infty} d\omega \rho(\omega) \text{sech}^2 \frac{\omega}{2T}, \tag{8}$$

so that the decrease of the spin susceptibility below T_c is found to be deeply related to the development of the BCS gap in the superfluid density of states $\rho(\omega)$. Evaluating ETMA density of states using the formula,

$$\rho(\omega) = -\frac{1}{\pi} \sum_{\mathbf{p}} \text{Im} G_{11}(\mathbf{p}, i\omega_n \to \omega + i\delta), \tag{9}$$

we find from Fig. 2(c) that pairing fluctuations make $\rho(\omega)$ gapless even below T_c. As a result, compared with the mean-field case, the suppression of χ is less remarkable. Thus, although pairing fluctuations suppress χ above T_c, they enhance χ below T_c.

We briefly note that our result is close to the recent experiment on a ^{6}Li Fermi gas (solid circle in Fig. 2(a))[15]. Our results are also consistent with the recent quantum Monte-Carlo simulation[39], as well as the results in the self-consistent T-matrix approximation[40].

4. Summary

To summarize, we have discussed strong-coupling corrections to the spin susceptibility χ in a unitary Fermi gas. Within the framework of an extended T-matrix approximation, we showed

that effects of pairing fluctuations on this magnetic quantity are different in between the normal state and the superfluid phase. In the normal state, pairing fluctuations suppress χ, leading to the spin-gap phenomenon near T_c. In the superfluid phase, pairing fluctuations enhance χ, because they induce finite density of states inside the BCS gap. Since the spin susceptibility is observable in ultracold Fermi gases, our results would be useful for the study of strong-coupling physics in the BCS-BEC crossover region using the uniform spin susceptibility.

Acknowledgments

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[33] We note that the origin of the difference between the ETMA- and TMA-susceptibility can be simply understood when we approximate $\hat{\Gamma}(\mathbf{q}, i\nu_l)$ to the value at $\mathbf{q} = \nu_l = 0$ ($\equiv \Gamma$)[21]. In this case, while the dressed Green's function in the ETMA self-energy in Eq.(3) leads to the RPA (random phase approximation) type susceptibility $\chi_{\text{ETMA}} \simeq \chi_{\text{DOS}}/[1 + \Gamma\chi_{\text{DOS}}]$ (where $\chi_{\text{DOS}} = -(T/2)\sum_{\mathbf{p},i\omega_n} \text{Tr}\hat{G}(\mathbf{p}, i\omega_n)^2$), the TMA (where $\hat{G}$ in Eq. (3) is replaced by $\hat{G}^0$) gives $\chi_{\text{TMA}} \simeq \chi_{\text{DOS}}[1 -$

$\Gamma_0\chi_{\mathrm{DOS}}$], so that the latter becomes negative when $\Gamma\chi_{\mathrm{DOS}} > 1$. We also note that the so-called GG_0 theory[9] uses $\hat{G}^0$ for $\hat{G}$ in Eq.(3), although the dressed Green's function is partially used in $\Pi^{ij}(\mathbf{q}, i\nu_l)$. Thus, the RPA-type series is truncated as in the TMA case, so that this approach is considered to underestimate χ compared to the ETMA.

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