

Bubbles Containing 4, 6 and 12 Electrons in Liquid Helium-3

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Abstract. We have performed calculations of the properties of bubbles in liquid helium containing small numbers of electrons. We use an iterative approach to estimate the energy of the electrons inside a bubble of given shape, and then vary the shape of the bubble to find the minimum energy. For helium-3 we show that at zero applied pressure bubbles containing 4 electrons are unstable against breakup into single electron bubbles, but are stable at pressures between -0.23 and -0.15 bars. Bubbles with 6 electrons are stable between -0.17 and -0.05 bars and bubbles with 12 electrons are stable over the pressure range -0.1 to 0.08 bars.

1. Introduction

It was first proposed by Ferrell [1] that an electron injected into liquid helium would force open a bubble in the liquid. Since then a large number of experiments have been performed to study these so-called electron bubbles. Measurements have been made of their optical properties [2], mobility [3], and effective mass [4]. Experimental [5] and theoretical studies [6] have also been made of multi-electron bubbles (MEB) containing a large number of electrons, e.g., 10^5 or more electrons. Here we report on calculations of the properties of bubbles containing just a small number of electrons.

Electron bubbles exist because there is an energy barrier (height approximately 1 eV in helium-4) preventing an electron from entering into bulk liquid helium. Because of this barrier, when a bubble contains an electron the wave function of the electron is close to zero at the surface of the bubble. Thus the total energy of a spherical bubble containing one electron is

$$E = \frac{h^2}{8mR^2} + 4\pi R^2 \alpha + \frac{4\pi}{3} R^3 P, \quad (1)$$

where R is the bubble radius, m is the mass of the electron, α is the surface tension, and P is the applied pressure. At zero pressure the energy is minimum for a radius of

$$R_0 = \left(\frac{h^2}{32\pi m \alpha} \right)^{1/4}. \quad (2)$$

Bubbles containing 2 electrons are known to be unstable against fission [7]. In a recent paper we have considered the shape and stability of bubbles containing 4, 6, and 12 electrons in liquid helium-4 [8]. Here we use the same method to obtain results for electron bubbles in helium-3.

As already mentioned in our earlier paper [8], calculations of the structure of bubbles containing a few electrons are of interest because these bubbles may be produced when a multi-electron bubble becomes unstable and collapses. Their presence could be detected through cavitation experiments.

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2. Calculation Method

To find the size and shape of a bubble containing Z electrons we need a method for calculating the energy of the electrons allowing for both quantum effects and the Coulomb repulsion. This method and its limitations are described in detail in ref. [8]. The first step is to consider the way to arrange Z electrons inside a sphere so as to minimize the Coulomb energy. There is an extensive literature on this problem (see [8]). For $Z = 2$ the electrons lie on the surface of the sphere at opposite ends of a diameter, and for $Z = 3$ they lie in a plane passing through the centre of the sphere. For $Z = 4$ they are at the vertices of a tetrahedron, for $Z = 6$ along Cartesian axes, and for $Z = 12$ they lie on the vertices of an icosahedron. However, for other values of Z the arrangement has lower symmetry and so we restrict attention to the special values just listed. To calculate the quantum corrections to the classical solution we place $Z - 1$ of the electrons on the surface of the sphere in the positions just mentioned, and then calculate the wave function and energy of the last electron in the potential provided by these electrons. From the wave function, we find the average distance $\langle \delta R \rangle$ of the last electron from the surface of the sphere. We then move each of the $Z - 1$ inwards from the surface of the sphere by this same amount from the surface and repeat the calculation until $\langle \delta R \rangle$ remains constant. Finally, the method is extended to consider a bubble shape which is non-spherical and has the symmetry of the classical solution.

3. Numerical Results

We specify the bubble shape by a radius function $R(\theta, \phi)$ which is a constant plus a term consisting of one linear combination of spherical harmonics giving a shape with the required symmetry. The linear combination is composed of a combination of $Y_{lm}(\theta, \phi)$, with different m but the same value of l . The value of l is chosen to be the smallest possible value which can give the desired symmetry. For $Z = 4$ this value of l is 3, for $Z = 6$ it is 4, and for $Z = 12$ it is 6. It is convenient to express the function $R(\theta, \phi)$ in terms of its maximum and minimum values. As an example, for $Z = 4$ this gives [8]

$$R(\theta, \phi) = \frac{R_{\max} + R_{\min}}{2} + \frac{R_{\max} - R_{\min}}{2} \left[\frac{5}{2} \cos^3 \theta - \frac{3}{2} \cos \theta + \frac{1}{\sqrt{2}} \sin^3 \theta \cos(3\phi) \right] \quad (3)$$

In Figs. 1 and 2 we show contour plots of the total energy of bubbles at $P = 0$ as a function of $R_{\max}$ and $R_{\min}$. For the surface tension we have taken the zero temperature value of $0.156 \text{ erg cm}^{-2}$ [9]. Just as for helium-4, we find that bubbles with $Z = 4$ or $Z = 6$ are unstable at zero pressure, i.e., there is no local minimum in the $R_{\max} - R_{\min}$ plane. Stable bubbles can be obtained at negative pressures. Bubbles with $Z = 4$ are found to be stable in the pressure range -0.23 to -0.15 bars, and for $Z = 6$ the range is -0.17 to -0.05 bars. At the lower end of the pressure range the bubble becomes unstable against explosion and at the upper end the bubble breaks up into single electron bubbles.

For $Z = 12$ the calculations predict that the bubble is stable over the pressure range -0.10 to 0.08 bars. The energy as a function of $R_{\min}$ and $R_{\max}$ is shown in Fig. 2. Figure 3 shows the variation of $R_{\min}$ and $R_{\max}$ as a function of pressure for the $Z = 12$ bubble. The shape of the $Z = 12$ bubble at three pressures is shown in Fig. 4.

It is interesting to compare these results with the calculations already performed for helium-4 [8]. For a single-electron bubble the radius varies with surface tension as $\alpha^{-1/4}$ and so a bubble in helium-3 should be larger than a helium-4 bubble by a factor of about 1.26. For multi-electron bubbles with Z large the radius varies as $\alpha^{-1/3}$. For the few-electron bubbles our results show that when the surface tension decreases the bubble size increases, but also the pressure range over which the bubble

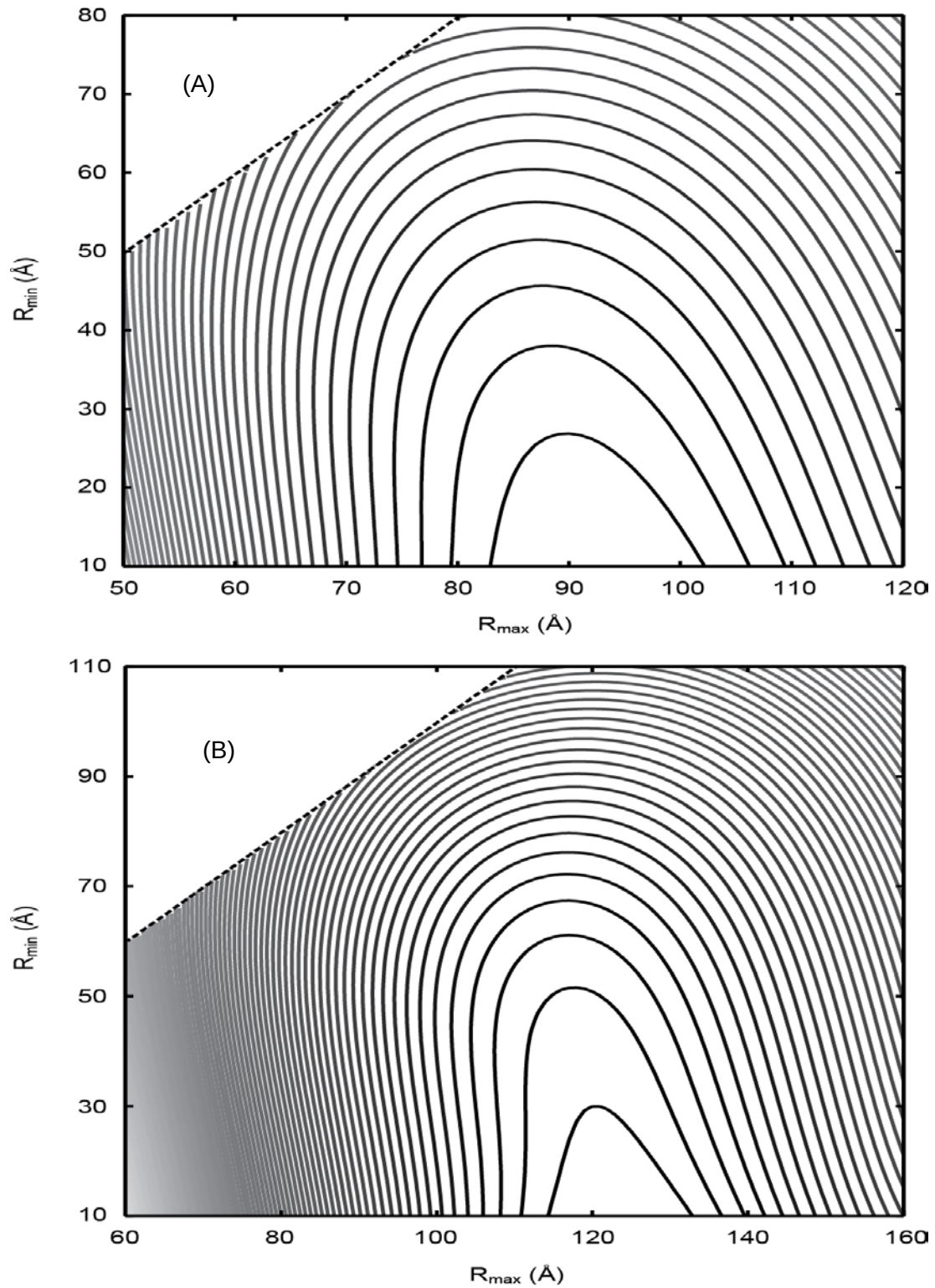


Fig. 1. Energy of bubbles at zero pressure containing (A) 4 electrons and (B) 6 electrons. The contour lines are spaced by increments of 3×10^{-14} ergs.

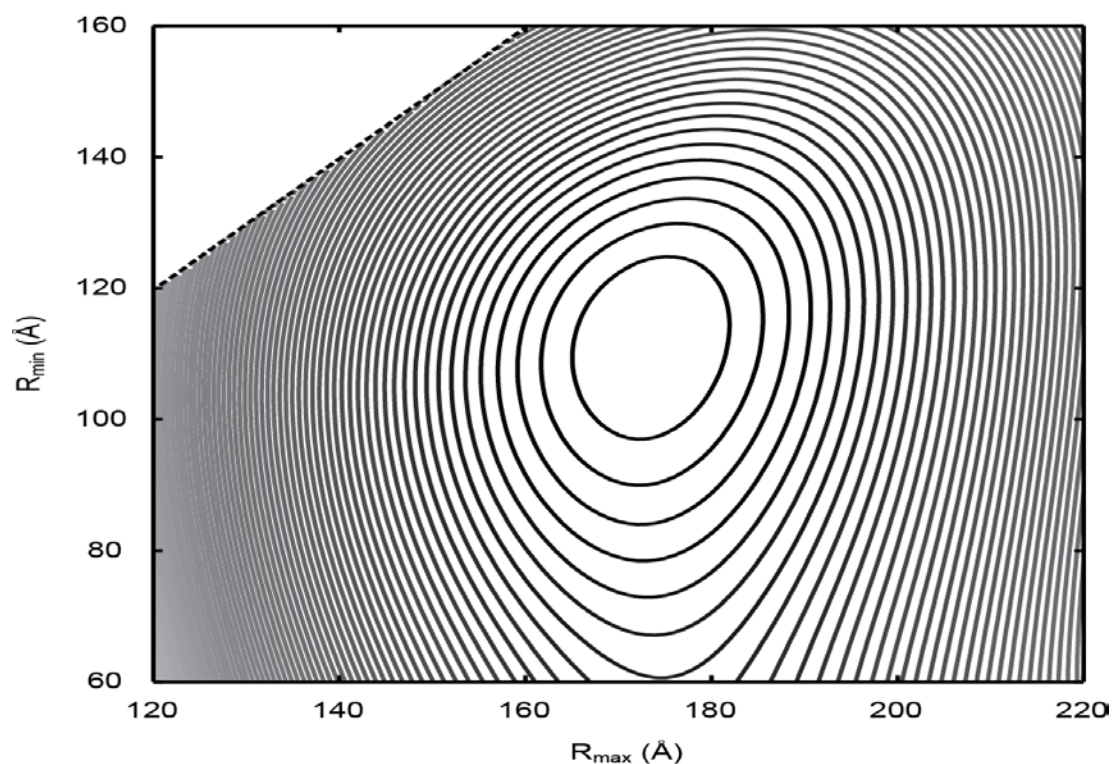


Fig. 2. Energy of bubbles at zero pressure containing 12 electrons. The contour lines are spaced by increments of 3×10^{-14} ergs.

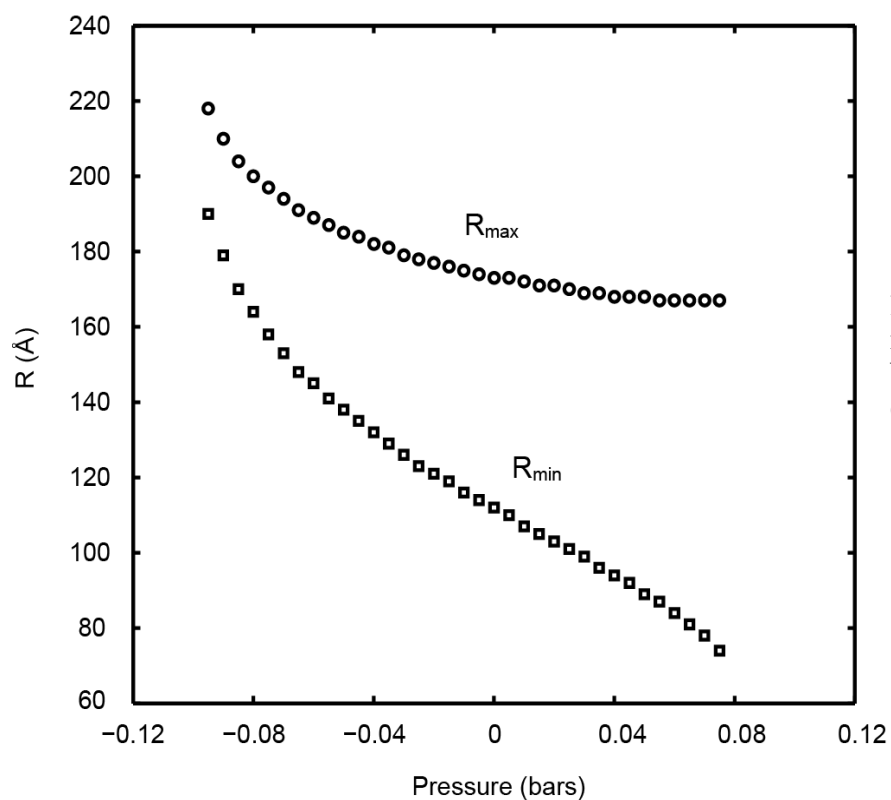


Fig. 3. Maximum and minimum radius for a bubble containing 12 electrons.

is stable decreases. This suggests that at some critical value of surface tension (different for each value of Z) there is no pressure at which the bubble is stable. Thus, since α decreases with increasing temperature and goes to zero at the critical point, there would be a critical temperature for each Z . However, it is probably not useful to calculate these temperatures with the current approach because when the temperature is raised to give a large decrease in α the vapor pressure will increase and it will be necessary to make allowance for the gas atoms inside the bubble.

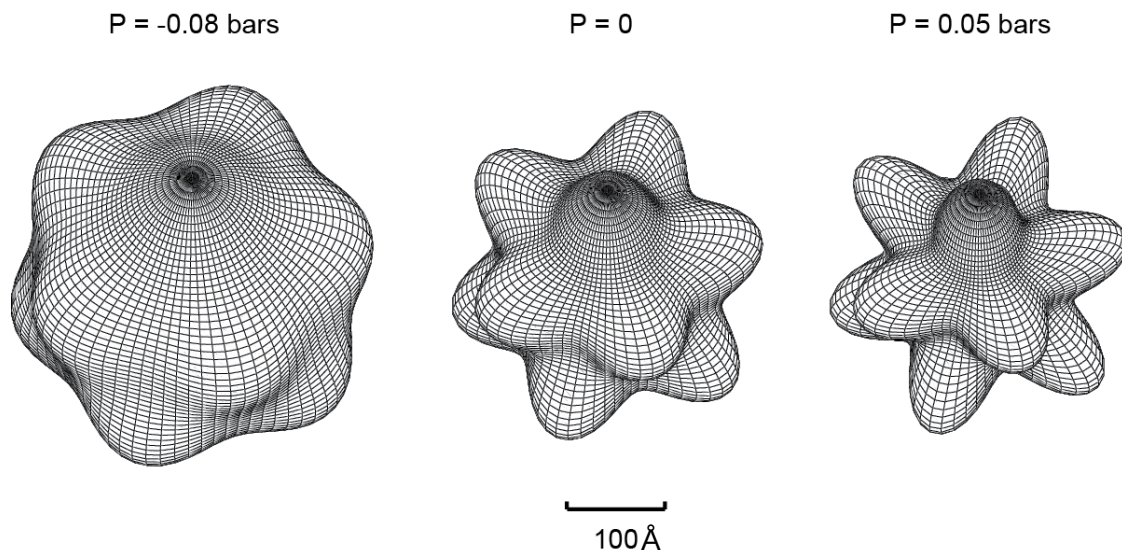


Fig. 4. Calculated shape of the $Z = 12$ bubble at pressures of -0.08, 0 and 0.05 bars.

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