Hetero-pairing and pseudogap phenomena in an ultracold Fermi gas with mass imbalance

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Abstract. We investigate strong-coupling properties of a unitary Fermi gas consisting of two different species with different masses. Including pairing fluctuations within the self-consistent T-matrix approximation, we calculate the single-particle density of states in the normal state. We show that the pseudogap phenomenon, which is characterized by a dip structure in the density of states around $\omega = 0$, appears more remarkably in the light mass component than in the heavy mass component. As a result, the pseudogap temperature, which is determined as the temperature at which the pseudogap disappears in the density of states, is higher in the former than in the latter. We also find that this different pseudogap temperatures lead to the existence of two kinds of pseudogap regions. That is, one is the ordinary pseudogap regime where the pseudogap appears in both the component, and the other case is that the light mass component only exhibits the pseudogap phenomenon. As the origin of these component-dependent pseudogap phenomena, we point out the importance of different Fermi temperatures between the two components. Since the formation of hetero-Cooper pairs and their condensation are expected in various systems, such as a ^{6}Li-^{40}K Fermi gas mixture, an exciton (polariton) gas, as well as color superconductivity, our results would be useful for the understanding of strong-coupling properties of these novel Fermi condensates.

1. Introduction

Although superfluid ^{40}K[1] and ^{6}Li[2] Fermi gases have attracted much attention in cold Fermi gas physics especially on the viewpoint of the BCS-BEC crossover[3, 4], they still belong to the same category as metallic superconductivity in the sense that Cooper pairs are formed between the same species. Thus, since the realization of these atomic Fermi superfluids, further possibilities beyond the homo-pairing have been explored in cold Fermi gas physics. At present, a tunable interaction associated with a Feshbach resonance has been realized in a ^{6}Li-^{40}K Fermi mixture[5, 6, 7, 8, 9], and ^{6}Li-^{40}K hetero-molecules have been observed[7], although their superfluid phase transition has not been reported yet. Once the superfluid phase transition is achieved in this system, we would be able to systematically study the BCS-BEC crossover physics in this hetero-pairing state by adjusting the interaction strength associated with a Feshbach resonance. Since the so-called Sarma state[10] is predicted in a mass-imbalanced Fermi gas[11], the assessment of this prediction would also become possible. In addition, since hetero-pairs have been also discussed in various fields, such as semiconductor physics (excitons [12, 13], exciton-polaritons [14, 15]), as well as high-energy physics (color superconductivity [16, 17]), the realization of a hetero-superfluid Fermi gas would widely contribute to the understanding of these novel Fermi condensates.

In the current stage of research for a superfluid Fermi gas with mass imbalance, the achievement of the superfluid phase transition temperature T_c is a crucial issue. In this regard, the so-called pseudogap phenomenon, which is characterized by a gap-like structure in the single-particle excitations above T_c, would be helpful, because this precursor phenomenon of the superfluid phase transition gradually becomes remarkable as one approaches T_c. Thus, using this, one can evaluate to what extend the system is close to the superfluid instability. Recently, a photoemission-type experiment has become possible[18], and pseudogapped single-particle excitation spectra have been observed in a mass-balanced ^{40}K Fermi gas[18]. The observed pseudogap structures agree well with strong-coupling calculations[19, 20, 21, 22, 23]. Thus, this experimental technique would also be useful for the observation of the pseudogap phenomenon in the presence of mass imbalance.

In this paper, we investigate single-particle excitations and strong-coupling effects in a unitary Fermi gas with mass imbalance. In our previous paper[24], we showed that the ordinary T-matrix approximation (TMA), which has been extensively used to successfully explain various BCS-BEC crossover physics in the mass-balanced case, breaks down in the presence of mass imbalance. To overcome this difficulty, we have extended the TMA to include higher-order pairing fluctuations[24, 25]. Although this extended T-matrix approximation (ETMA) can partially solve this difficulty, it was found to still give unphysical results in the highly mass-imbalanced regime, originating from an inconsistent treatment of the single-particle Green's function. Thus, in this paper, we further extend the ETMA to completely eliminate this inconsistency. For this purpose, we employ the self-consistent T-matrix approximation[26, 27], to examine the pseudogap phenomenon and effects of mass imbalance. From the temperature dependence of the pseudogapped density of states, we determine the pseudogap temperatures in both the light mass and the heavy mass components, each of which is defined as the temperature at which a dip structure in the density of states disappears. Throughout this paper, for simplicity, we put $\hbar = k_B = 1$, and the system volume V is taken to be unity.

2. Formalism
We consider a two-component Fermi gas with mass imbalance, described by the Hamiltonian,

$$H = \sum_{\boldsymbol{p},\sigma=\mathrm{L},\mathrm{H}} \xi_{\boldsymbol{p}\sigma} c_{\boldsymbol{p}\sigma}^{\dagger} c_{\boldsymbol{p}\sigma} - U \sum_{\boldsymbol{q}} \sum_{\boldsymbol{p},\boldsymbol{p}'} c_{\boldsymbol{p}+\boldsymbol{q}/2,\mathrm{L}}^{\dagger} c_{-\boldsymbol{p}+\boldsymbol{q}/2,\mathrm{H}}^{\dagger} c_{-\boldsymbol{p}'+\boldsymbol{q}/2,\mathrm{H}} c_{\boldsymbol{p}'+\boldsymbol{q}/2,\mathrm{L}}, \qquad (1)$$

where $c_{\boldsymbol{p}\sigma}$ is the annihilation operator of a Fermi atom with the kinetic energy $\xi_{\boldsymbol{p}\sigma} = p^2/(2m_\sigma) - \mu_\sigma$, measured from the Fermi chemical potential μ_σ. The pseudospin σ describes the light mass component ($\sigma = \mathrm{L}$) with an atomic mass m_L, and the heavy mass component ($\sigma = \mathrm{H}$) with an atomic mass m_H. $-U$ is a tunable pairing interaction, which is related to the s-wave scattering length a_s as $4\pi a_s/m = -U/[1 - U\sum_{\boldsymbol{p}}(m/p^2)]$ (where $2m^{-1} = m_\mathrm{L}^{-1} + m_\mathrm{H}^{-1}$ is twice the reduced mass). In this paper, we consider the unitarity limit $(k_\mathrm{F} a_s)^{-1} = 0$ (where $k_\mathrm{F} = (3\pi^2 N)^{1/3}$ is the Fermi momentum, with N being the total number of Fermi atoms). For simplicity, we consider a uniform Fermi gas, ignoring effects of a harmonic trap.

Strong-coupling effects are conveniently described by the self-energy $\Sigma_\sigma(\boldsymbol{p}, i\omega_n)$ in the single-particle thermal Green's function,

$$G_\sigma(\boldsymbol{p}, i\omega_n) = \frac{1}{i\omega_n - \xi_{\boldsymbol{p}\sigma} - \Sigma_\sigma(\boldsymbol{p}, i\omega_n)}, \qquad (2)$$

where ω_n is the fermion Matsubara frequency. In the self-consistent T-matrix approximation (SCTMA)[26, 27], the self-energy has the form

$$\Sigma_\sigma(\boldsymbol{p}, i\omega_n) = T \sum_{\boldsymbol{q}, \nu_n} \Gamma(\boldsymbol{q}, i\nu_n) G_{-\sigma}(\boldsymbol{q} - \boldsymbol{p}, i\nu_n - i\omega_n). \qquad (3)$$

Figure 1. Particle-particle scattering matrix Γ in the SCTMA. The double solid line is the dressed Green's function G in Eq. (2). The dotted line is the pairing interaction $-U$.

Here, ν_n is the boson Matsubara frequency, and $-\sigma$ means the opposite component to σ. In Eq. (3), the particle-particle scattering matrix $\Gamma(\boldsymbol{q}, i\nu_n)$ is diagrammatically given as Fig.1, which gives $\Gamma(\boldsymbol{q}, i\nu_n) = -U/[1 - U\Pi(\boldsymbol{q}, i\nu_n)]$, where

$$\Pi(\boldsymbol{q}, i\nu_n) = T \sum_{\boldsymbol{k}, i\omega_n} G_{\mathrm{L}}(\boldsymbol{k} + \boldsymbol{q}/2, i\nu_n + i\omega_n) G_{\mathrm{H}}(-\boldsymbol{k} + \boldsymbol{q}/2, -i\omega_n) \tag{4}$$

is the pair-correlation function.

As usual, the superfluid phase transition temperature T_{c}, as well as the chemical potentials μ_{L} and μ_{H}, are self-consistently determined by solving the Thouless criterion [28]

$$[\Gamma(\boldsymbol{q} = 0, i\nu_n = 0)]^{-1} = 0, \tag{5}$$

together with the number equations

$$N_{\mathrm{L}} = N_{\mathrm{H}} = \frac{N}{2}, \tag{6}$$

where

$$N_\sigma = T \sum_{\boldsymbol{p}, i\omega_n} G_\sigma(\boldsymbol{p}, i\omega_n). \tag{7}$$

In the normal state above T_{c}, we only solve the number equation (6), to determine μ_{L} and μ_{H}. The single-particle density of states $\rho_\sigma(\omega)$ is then calculated from the analytic continued Green's function as

$$\rho_\sigma(\omega) = -\frac{1}{\pi} \sum_{\boldsymbol{p}} \mathrm{Im}[G_\sigma(\boldsymbol{p}, i\omega_n \to \omega + i\delta)]. \tag{8}$$

We briefly note the difference between the present SCTMA and the previous ETMA used in Refs.[24, 25]. The SCTMA is reduced to the ETMA formalism, when the dressed Green's function G in the pair-correlation function $\Pi(\boldsymbol{q}, i\nu_n)$ in Eq. (4) is simply replaced by the bare one,

$$G_\sigma^0(\boldsymbol{p}, i\nu_n) = \frac{1}{i\omega_n - \xi_{\boldsymbol{p}\sigma}}. \tag{9}$$

Thus, the ETMA involves the internal inconsistency that the bare Green's function G_0 in Eq. (9) is used in $\Pi(\boldsymbol{q}, i\nu_n)$, although the dressed Green's function G is used in the self-energy in Eq. (3). As shown in Ref.[24], this inconsistency becomes crucial in the highly mass imbalanced regime, leading to the unphysical vanishing T_{c} in the BCS region. This problem is avoided in the present SCTMA because of the fully self-consistent treatment of the dressed Green's function[26], so that we can safely discuss strong-coupling physics in the whole region $0 \leq m_{\mathrm{L}}/m_{\mathrm{H}} \leq 1$. For more details about the difference between the two approximations, we refer to Refs.[24, 26].
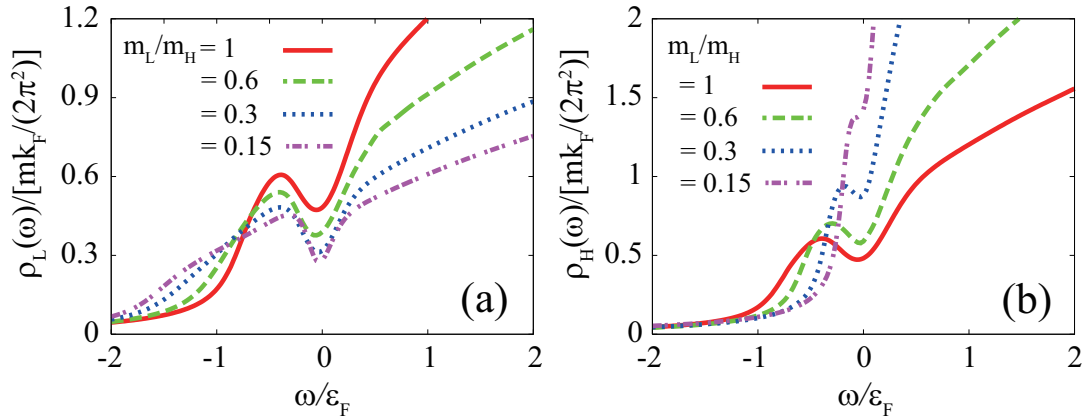
Figure 2. Calculated density of states $\rho_\sigma(\omega)$ in a mass-imbalanced unitary Fermi gas at T_c. (a) Light mass component. (b) Heavy mass component. $\varepsilon_F = k_F^2/(2m)$, where $k_F = (3\pi^2 N)^{1/3}$, and m is twice the reduced mass.
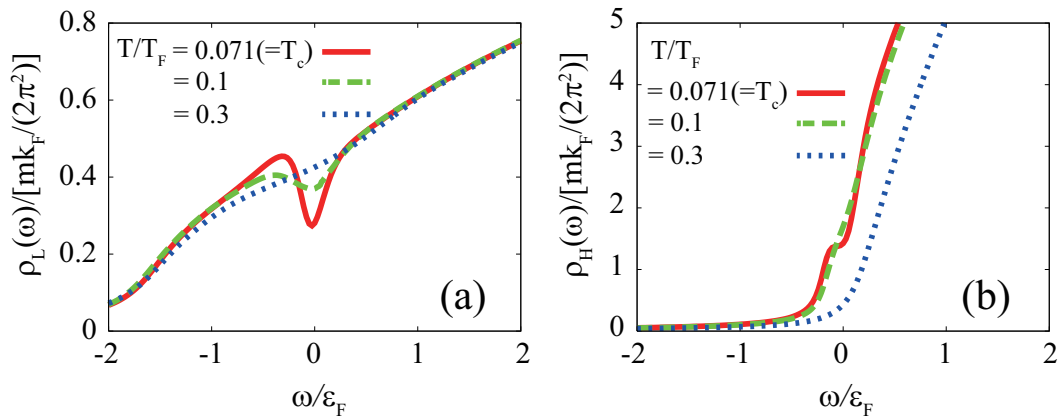

Figure 3. Calculated density of states $\rho_\sigma(\omega)$ in a unitary Fermi gas when $T \geq T_c$. (a) Light mass component. (b) Heavy mass component. We take $m_L/m_H = 0.15$. $T_F = k_F^2/(2m)$, where $k_F = (3\pi^2 N)^{1/3}$, and m is twice the reduced mass.

3. Pseudogapped density of states in a mass-imbalanced Fermi gas

Figure 2 shows the density of states in a unitary Fermi gas. In the mass-balanced case ($m_L/m_H = 1$), one sees a dip structure around $\omega = 0$ in each the component. Since the superfluid order parameter vanishes at T_c, this is the pseudogap originating from strong pairing fluctuations[22]. In the light mass component (Fig.2(a)), the pseudogap continues to exist even in the highly mass imbalanced regime ($m_L/m_H = 0.15 \ll 1$). In contrast, the dip structure in Fig. 2(b) gradually becomes obscure with decreasing the ratio m_L/m_H of the mass imbalance, to eventually disappear when $m_L/m_H = 0.15$. Thus, although the pseudogap originates from the formation of preformed Cooper pairs consisting of atoms in both the components, the appearance of this phenomenon is very different in between the two components.

This component-dependent pseudogap phenomenon is due to the fact that the Fermi temperature $T_F^L = k_F^2/(2m_L)$ in the light mass component is higher than the Fermi temperature $T_F^H = k_F^2/(2m_H)$ in the heavy mass component. The *scaled* temperature T/T_F^H in the heavy
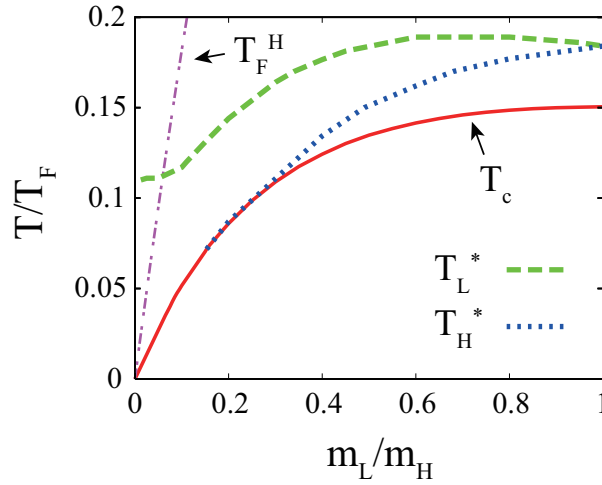
Figure 4. Pseudogap temperature in the light mass component T_L^* and that in the heavy mass component T_H^* in a unitary Fermi gas. T_F^H is the Fermi temperature of the heavy mass component.

mass component then becomes higher than T/T_F^L in the light mass component, so that thermal excitations smear out the pseudogapped density of states more remarkably in the former. In the highly mass imbalanced case ($m_L/m_H = 0.15$), when we raise the temperature from T_c, while the pseudogap structure is still seen at $T/T_F = 0.1$ (where $T_F = k_F^2/(2m)$) in the light mass component (Fig.3(a)), Fig.3(b) shows that even the shoulder structure seen at T_c completely disappears in the heavy mass component when $T/T_F = 0.1$. At this temperature, one finds,

$$\frac{T}{T_F^H} = 0.38 > \frac{T}{T_F^L} = 0.06, \tag{10}$$

which means that the heavy mass component is effectively at much higher temperature than the light mass component.

4. Component-dependent pseudogap temperatures

To identify the pseudogap regime in a quantitative manner, we conveniently introduce the pseudogap temperature T_σ^* ($\sigma =$ L, H) in each component, as the temperature at which the dip structure disappears in the density of states $\rho_\sigma(\omega)$. As expected from the discussions in Sec.3, Fig. 4 shows $T_L^* > T_H^*$ when $m_L/m_H < 1$. When $m_L/m_H \lesssim 0.15$, the pseudogap no longer exists in the heavy mass component even at T_c (See Fig. 2(b)), so that the pseudogap temperature T_H^* is absent there.

The component-dependence on the pseudogap phenomenon naturally leads to two kinds of pseudogap regions. That is, when $T_c \leq T \leq T_H^*$, the pseudogap appears in the both components. On the other hand, the pseudogap can be seen only in the light mass component when $T_H^* \leq T \leq T_L^*$[30].

In the highly mass imbalanced regime, Fig.4 shows that the pseudogap temperature T_L^* in the light mass component becomes higher than the Fermi temperature T_F^H in the heavy mass component. In this regard, we recall that the superfluid phenomenon is purely a quantum phenomenon. Thus, this result indicates that, although the pseudogap phenomenon is a precursor of this quantum phenomenon, the pseudogap is still seen in the light mass component even when the heavy mass component is in the classical regime ($T/T_F^H \gtrsim 1$).
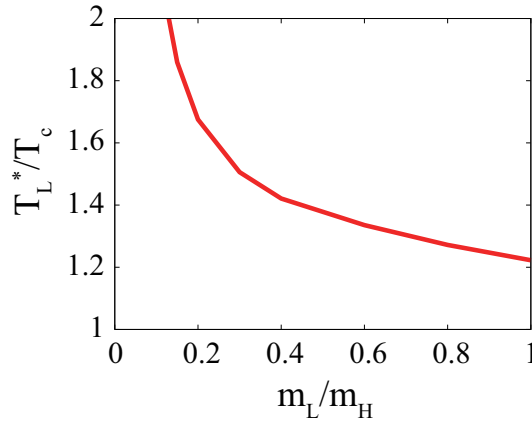
Figure 5. Calculated ratio $T_{\mathrm{L}}^*/T_{\mathrm{c}}$ of the pseudogap temperature in the light mass component to the superfluid phase transition temperature, as a function of $m_{\mathrm{L}}/m_{\mathrm{H}}$.

Figure 5 shows the ratio $T_{\mathrm{L}}^*/T_{\mathrm{c}}$ as a function of $m_{\mathrm{L}}/m_{\mathrm{H}}$. Since the superfluid phase transition is a quantum phenomenon, both the components should be in the Fermi degenerate regime at T_{c}, which leads to $T_{\mathrm{c}} < T_{\mathrm{F}}^{\mathrm{H}}$ ($< T_{\mathrm{F}}^{\mathrm{L}}$). On the other hand, one finds $T_{\mathrm{F}}^{\mathrm{H}} < T_{\mathrm{L}}^*$ in the highly mass imbalanced regime, as shown in Fig.4. Reflecting these, the ratio $T_{\mathrm{L}}^*/T_{\mathrm{c}}$ becomes large when $m_{\mathrm{L}}/m_{\mathrm{H}}$ becomes small, as shown in Fig. 5. This means that, even in the highly mass imbalanced regime where T_{c} is very low, one can observe the pseudogap in the light mass component in the relatively wide temperature region, $T_{\mathrm{c}} \le T \le T_{\mathrm{L}}^*$.

5. Summary
To summarize, we have investigated strong-coupling properties of a unitary Fermi gas consisting of two different species with different masses. Within the framework of the self-consistent T-matrix approximation, we calculated the single-particle density of states in the normal state, to examine effects of mass imbalance on the pseudogap phenomenon originating from pairing fluctuations.

We showed that the pseudogap structure is more clearly seen in the light mass component than in the heavy mass component. When we determine the pseudogap temperature in each component (which is evaluated as the temperature at which the pseudogap structure disappears in the density of states), it is higher in the light mass component (T_{L}^*) than in the heavy mass component (T_{H}^*). As a result, we obtain two kinds of pseudogap region. In the region $T_{\mathrm{c}} \le T \le T_{\mathrm{H}}^*$, the pseudogap appears in both the components. When $T_{\mathrm{H}}^* \le T \le T_{\mathrm{L}}^*$, on the other hand, the pseudogap only appears in the light mass component. In the highly mass imbalanced regime, the pseudogap phenomenon does not occur in the heavy mass component, so that the pseudogap temperature in this component is absent there.

The component-dependent pseudogap phenomena originates from the different Fermi temperatures between the two components. The Fermi temperature $T_{\mathrm{F}}^{\mathrm{L}}$ in the light mass component is higher than the Fermi temperature $T_{\mathrm{F}}^{\mathrm{H}}$ in the heavy mass component, so that the scaled temperature T/T_{F}^σ is always higher in the latter than in the former. As a result, thermal effects in the heavy mass component is more remarkable for a given temperature T, so that the pseudogap structure is easily smeared out thermally in this component, compared with the light mass component.

In the case of a ^{6}Li-^{40}K Fermi gas mixture, one finds $m_{\mathrm{L}}/m_{\mathrm{H}} = 6/40 = 0.15 \ll 1$. Thus, in this case, the light mass component (^{6}Li) is suitable for the observation of the pseudogap

phenomenon. Since the pseudogap is a precursor of the superfluid phase transition, toward the realization of the superfluid phase transition of this Fermi gas mixture, our results would be helpful to assess to what extent the system is close to the superfluid instability using the pseudogap phenomenon. In addition, since Fermi superfluids with hetero-Cooper pairs have also been discussed in other fields, such as semiconductor physics (exciton gas and exciton-polariton condensate), as well as high-energy physics (color superconductivity), our results would also contribute to the study of these unconventional Fermi condensates.

Acknowledgments

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