Theoretical study of ghost imaging with cold atomic waves under the condition of partial coherence

Jun CHEN[1,2] and Yun-xian LIU[1]

[1]College of Optical and Electronic Technology, China Jiliang University, Hangzhou 310018, China

E-mail:chenjun.sun@gmail.com

Abstract. A matter wave ghost imaging mechanism is proposed and demonstrated theoretically. This mechanism is based on the Talbot-Lau effect. Periodic gratings of matter wave density, which appear as a result of interference of atoms diffracted by pulses of an optical standing wave, are utilized to produce the reference wave and the signal wave simultaneously for the ghost imaging. An advantage of this mechanism is that during the imaging process, the beam-splitter is not needed, which highly simplifies the experimental setup and makes the ghost imaging possible in the field of matter wave.

1. Introduction

Ghost imaging (GI), also known as correlated imaging has attracted a lot of interest for its great applications in quantum metrology, lithography, and holography [1]. It is a method to obtain the image of an object by correlation measurements between two optical beams [2], [3]. One optical beam passes through the object and illuminates a single-pixel (bucket) detector which provides no spatial resolution. The other optical beam does not interact with the object, but it impinges on a high-resolution point-like detector. The name "ghost" comes from the fact that the information about the spatial distribution of the object is retrieved, through the intensity correlation measurements by scanning the point-like detector in the reference path, although the beam in this path never interacts with the object to be imaged.

In recent years, the development of matter wave optics has attracted a lot of interest. Many optical effects have found their counterparts in the field of matter wave. The realization of Hanbury- Brown Twiss (HBT) effect with cold atoms opened the possibility for multi-bit information processing with matter waves [4]. It is known that, the essential physics behind the HBT effect and the GI effect is the same [5]. Both effects arise as a result of the intensity fluctuations of the stochastic wave source. The difference is that, in the GI, many bits of information of the object are treated together, whereas, in the HBT, only one bit information is required to be obtained. A natural question arises that whether it is possible to realize the GI with a matter wave. In a traditional GI experiment, a beam-splitter is an indispensable optical component to create point-to-point correlation [6]. In 2008, the computational GI was proposed, where the beam splitter disappears and a light modulator was used [7]. In 2012, an interesting GI without beam splitter was proposed, in which a periodic diffraction grating was applied [8]. In the field of matter wave, a large-area beam-splitter is quite a difficulty for experiments [9][10],

[2] To whom any correspondence should be addressed.

which limits the extension of GI from the field of optics to the field of matter wave. In this manuscript, we propose a mechanism which utilizes a periodic atomic grating to produce the point-to-point correlation of the matter wave field, and present a theoretical framework to demonstrate the GI with matter waves.

2. Matter wave Talbot-Lau interferometer

We propose a mechanism for the matter wave ghost imaging (GI) as shown in Fig.1, which can be divided into two parts. One is designed for *Talbot-Lau interferometer*, which generates periodic density patterns of the matter wave. It comprises two absorptive gratings which are standing waves of resonant optical fields (also called optical masks) [11]. The other part of the GI scheme is for *imaging*, where the correlation detection is performed at a detection plane by a bucket detector D_2 and a high-resolution detector D_1. The object to be imaged is placed behind one of the periodic density patterns on the detection plane. The matter wave transmitted from the object is the so called signal wave, and is collected by the detector D_2. Since D_2 is a bucket (single-pixel) detector, no spatial resolution of the object is gathered by D_2. The high-resolution detector D_1 is placed behind another periodic density pattern which is the so called reference wave, and collects the information of the density distribution of this pattern. For the location of D_1, it is worth noting that detector D_1 can be placed at any density pattern other than the one for D_2. According to the density correlation (the second order correlation) of the matter wave between D_1 and D_2, the image of the object would be retrieved. We would first present the theoretical derivations of the periodic density patterns in the *Talbot-Lau interferometer*, and then the *imaging* process.

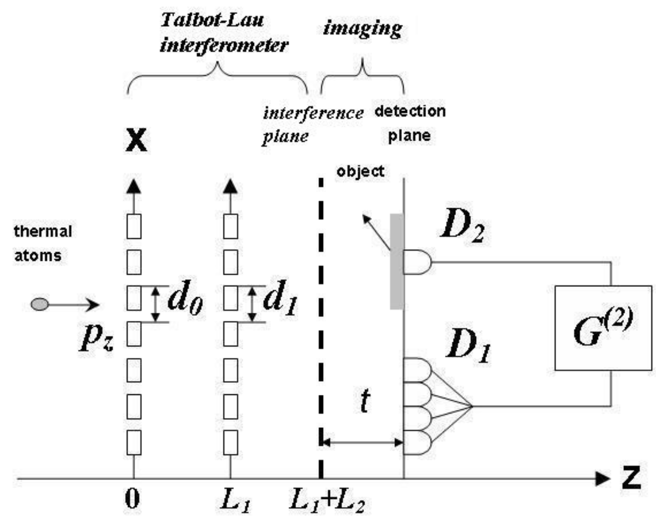

Figure 1. Scheme for the Talbot-Lau effect based ghost imaging with thermal atoms.

Talbot-Lau effect is a self-imaging of a periodic structure illuminated by partially coherent or incoherent light. In Talbot-Lau interferometer, monochromatic spatially incoherent light passes through a set of two parallel gratings separated by a known distance. At particular separations, illumination of the second grating is coherent and, subsequently, the second grating's image forms in the diffraction zone. For matter waves, the Talbot-Lau effect can be realized with atom beams manipulated by solid gratings and also optical fields [11]. Utilizing Talbot-Lau effect, point-to-point correlation of the matter wave field can be produced without beam splitters.

In the above setup of Fig.1, we assume that the matter wave source is a monochromatic incoherent atom beam. A flux of atoms enters the plane at $z=0$ with a longitudinal momentum p_z which is much greater than its transverse components. The Wigner function of the matter wave at $z=0$, is

Journal of Physics: Conference Series **568** (2014) 012002 doi:10.1088/1742-6596/568/1/012002

$$W_0(x,p)=1. \tag{1}$$

The matter wave beam passes through the first atomic grating which is located at *z=0* and is perpendicular to the propagation direction. After the first grating, the Wigner function turns out to be

$$W_1(x,p)=\int H_0(x,q)W_0(x,p-q)\mathrm{d}q=\int H_0(x,q)\mathrm{d}q=\left|h_0(x)\right|^2, \tag{2}$$

where $h_0(x)$ is the transmission function of the first grating at $z=0$, and $H_0(x, q)$ is the convolution kernel

$$H_0(x,q)=\frac{1}{(2\pi\hbar)^2}\int h_0^*(x-\Delta/2)h_0(x+\Delta/2)\exp[ip\Delta/\hbar]\,\mathrm{d}\Delta. \tag{3}$$

where Δ is a two-point spatial separation. After a free evolution by a distance L_1, followed by a passage through the second grating with a convolution kernel H_1 and another free evolution by a distance L_2, the Wigner function of the matter wave at the Interference plane, $z=L_1+L_2$, is [12]

$$W_d(x,p)=\int\left|h_0\left(x-\frac{p}{p_z}(L_1+L_2)+\frac{q}{p_z}L_1\right)\right|^2 H_1(x-\frac{p}{p_z}L_2,q)\mathrm{d}q. \tag{4}$$

We assume that the first grating has a period d_0, and its transmission function is $h_0(x)=\sum_m a_m\exp[2\pi imx/d_0]$. Likewise, the second grating has the Fourier coefficients b_m and the period d_1,

$$|h_0(x)|^2=\sum_{l\in Z}A_l\exp[2\pi il\frac{x}{d_0}],\ \text{with}\ A_l=\sum_j a_j a_{j-l}^*, \tag{5}$$

$$H_1(x,p)=\sum_{l,j\in Z}b_j b^*{}_{j-l}\exp[2\pi il\frac{x}{d_1}]\delta(p-\hbar\pi\frac{2j-l}{d_1}). \tag{6}$$

In the case that the two gratings are set at an equal distance $L_1=L_2=L$ and an equal period $d_1=d_2=d$, the Wigner function at the interference plane, $z=L_1+L_2$, is (see Eq.(17) in Ref. [12])

$$W_T(x,p)=\frac{1}{\hbar}\sum_{l,j,m\in Z}A_l b_j b_{j-m}\exp[2\pi i(l+m)\frac{x}{d}-2\pi i(2l+m)\frac{L}{d}\frac{p}{p_z}]\exp[\pi il(2j-m)\frac{L}{L_\lambda}] \tag{7}$$

where $L_\lambda=d^2/\lambda_d$ is the Talbot length and $\lambda_d=2\pi\hbar/p_z$ is the deBrogie wavelength.

Since the Wigner function is the Fourier transformation of the first order correlation function, $G^{(1)}(x_1,x_2)$, there is

$$G^{(1)}(x_1,x_2)=\int W_T(x,p)\exp[-ip\frac{\Delta}{\hbar}]\,\mathrm{d}p$$
$$=\frac{2\pi}{\hbar}\sum_{l,j,m\in Z}A_l b_j b_{j-m}^*\exp[2\pi i(l+m)\frac{x}{d}]\exp[\pi il(2j-m)\frac{L}{L_\lambda}]\delta\left(x_2-x_1+(2l+m)\frac{\lambda L}{d}\right), \tag{8}$$

where $x=(x_1+x_2)/2$ is the mid-point position of x_1 and x_2, $\Delta=x_2-x_1$ is the two-point separation, and $G^{(1)}(x_1,x_2)\equiv\left\langle U_1^*(x_1)U_2(x_2)\right\rangle$ is the first order correlation function with $U_j(x_j)$ being the matter wave field at the position x_j ($j=1,2$).

When we pick up the $m=-2l$ components, the matter wave density at the interference plane is

$$D(x) = \left\langle U^*(x)U(x) \right\rangle = \frac{2\pi}{\hbar} \sum_{l,j,m\in Z} A_l b_j b^*_{j-m} \exp[-2\pi il \frac{x}{d} + 2\pi il(j+l)\frac{L}{L_\lambda}]. \quad (9)$$

The first term of the above *exp* function, $\exp[-2\pi ilx/d]$, shows a periodic density pattern along the transverse direction with the period *d*. This density pattern is similar to the original grating and can be recognized as the image of the grating. The second term of the *exp* function, $\exp[2\pi il(j+l)L/L_\lambda]$, shows that the periodic density pattern appears at distances that are multiples of the Talbot length L_λ, which is the so called Talbot-Lau effect.

3. Matter wave ghost imaging

According to the Talbot-Lau effect, we can obtain periodic density patterns of the matter wave at the interference plane $z=L_1+L_2$. These density patterns possess the property of point-to-point correlation and can be used for ghost imaging. As shown in Fig.1, the object is placed behind one density pattern of the matter wave. The matter wave transmission of the object is collected by the bucket detector D_2, which is a single-pixel detector and can not retrieve the spatial resolution of the object. A parallel high-resolution detector D_1 is placed behind another density pattern at the reference plane. D_1 is spatial sensitive and can collect the density distribution of the matter wave field to be detected. However, no object is placed before D_1. The signal from D_1 is for the purpose of reference.

Our starting point is the first order correlation function obtained from the Talbot-Lau effect at the interference plane $z=L_1+L_2$, which has been theoretically expressed in Eq(8). When we pick up the $m=-2l$ components, the first order correlation function at the interference plane is

$$G_T^{(1)}(x_1,x_2) = \frac{2\pi}{\hbar} \sum_{l,j,m\in Z} A_l b_j b^*{}_{j-m} \exp[-2\pi il\frac{x}{d}]\exp[2\pi il(j+l)\frac{L}{L_\lambda}]\ \delta(x_2-x_1). \quad (10)$$

After a free evolution, the diffraction field-function collected by detector D_1. is $U_1(x_1)$, and there is

$$U_1(x_1) = -i\frac{M}{2\pi\hbar t_1}\int U_T(x_{01})\exp\left\{i\frac{M}{\hbar}\frac{(x_{01}-x_1)^2+(z_{01}-z_1)^2}{2t_1}\right\}\mathrm{d}x_{01}. \quad (11)$$

The field-function collected by the detector D_2. is U_2. Since D_2. is a bucket detector, U_2 does not has the spatial resolution

$$U_2 = -i\frac{M}{2\pi\hbar t_2}\iint U_T(x_{02})h(x_2)\exp\left\{i\frac{M}{\hbar}\frac{(x_{02}-x_2)^2+(z_{02}-z_2)^2}{2t_2}\right\}\mathrm{d}x_{02}\mathrm{d}x_2 \quad (12)$$

where $h(x_2)$ is the transmission function of the object, t_1 and t_2 are the evolution time of the matter wave from the interference plane ($z=L_1+L_2$) to the detector D_1. and D_2., respectively. x_{0j} and x_j ($j=1,2$) are coordinates in the interference plane and detection plane, respectively. z_{0j} and z_j ($j=1,2$) are the coordinates in the matter wave propagation direction. In the above expression, *M* is the atom mass and $\hbar$ is the reduced Planck constant.

For an incoherent matter wave, the second order correlation function can be expressed as [13]

$$G^{(2)}(x_1,x_2) = \left\langle U_1^*(x_1)U_1(x_1)\right\rangle\left\langle U_2^*(x_2)U_2(x_2)\right\rangle + \left|G^{(1)}(x_1,x_2)\right|^2. \quad (13)$$

We set the grating functions as Eq.(5) and Eq.(6), and choose $z_{01}=z_{02}=z_0, z_1=z_2=z$, $t_1=t_2=t$. By substituting $U_1(x_1)$ and $U_2(x_2)$ into the expression of the first order correlation function, the correlation function at the detection plane is

$$G^{(1)}(x_1,x_2)$$
$$= \frac{M^2}{(2\pi\hbar)^2t^2}\iiint \Gamma_T(x_{01},x_{02})h(x_2)\exp\{-i\frac{M}{\hbar}[\frac{(x_{01}-x_1)^2}{2t}-\frac{(x_{02}-x_2)^2}{2t}]\}\mathrm{d}x_{01}\mathrm{d}x_{02}\mathrm{d}x_2$$

$$\propto \sum_{l,j,m} A_l b_j b^*_{j-m} \exp[2\pi il(j+l)\frac{L}{L_\lambda}]\exp\left[\frac{iM}{\hbar t}\left(x_1\text{-}l\frac{\pi\hbar t}{Md}\right)\right]h(x_1 - l\frac{2\pi\hbar t}{Md}) \tag{14}$$

where the integral formula $1/2\pi\int_{-\infty}^{\infty}\exp[iqx]\mathrm{d}x = \delta(q)$ has been used. According to Eq.(14) we find that the first order correlation function between the detector D_1 and D_2, contains the image information of the object according to $G^{(1)}(x_1,x_2)\propto h(x_1 - l2\pi\hbar t/Md)$, and has a transverse periodicity according to $G^{(1)}(x_1,x_2)\propto \exp\left[iM\left(x_1\text{-}l\pi\hbar t/Md\right)/\hbar t\right]$. The Talbot-Lau effect along the propagation direction is also shown in Eq.(14) according to $G^{(1)}(x_1,x_2)\propto \exp[2\pi il(j+l)L/L_\lambda]$. The above theoretical derivation shows that by scanning the reference detector D_2 we can retrieve the image of the object at the bucket detector D_1.

4. Conclusion

In this manuscript, we propose a mechanism for the correlation imaging with partially coherent matter waves. According to our theoretical derivation, it is shown that a Talbot-Lau interferometer can be used to produce a point-to-point correlation field, and the correlation imaging (ghost imaging) can be realized afterwards. The advantage of this mechanism is that it can realize the matter wave correlation imaging without a beam-splitter which simplifies the experiment system and saves the setup cost.

Acknowledgments

This work is financially supported by the National Natural Science Foundation of China (No.61108005).

References

[1] B. I. Erkmen and J. H. Shapiro, "Ghost imaging: from quantum to classical to computational," Adv. Opt. Photon. **2** (4), 405-450 (2010).
[2] Y. J. Cai and S. Y. Zhu, "Ghost imaging with incoherent and partially coherent light radiation," Phys. Rev. E **71**, 056607 (2005).
[3] R. Meyers, K. S. Deacon and Y. Shih, "Ghost-imaging experiment by measuring reflected photons," Phys. Rev. A **77** (4), 041801 (2008).
[4] M. Schellekens, R. Hoppeler, A. Perrin, J. V. Gomes, D. Boiron, A. Aspect and C. I. Westbrook, "Hanbury Brown Twiss Effect for Ultracold Quantum Gases " Science **310**, 648-651 (2005).
[5] L.-G. Wang, S. Qamar, S.-Y. Zhu and M. S. Zubairy, "Hanbury Brown-Twiss effect and thermal light ghost imaging: A unified approach," Phys. Rev. A **79** (3), 033835 (2009).
[6] R. Meyers, K. S. Deacon and Y. Shih, "Ghost-imaging experiment by measuring reflected photons," Phys. Rev. A **77** (4), 041801 (2008).
[7] J. H. Shapiro, "Computational ghost imaging," Phys. Rev. A **78** (6), 061802 (2008)
[8] H. Li, Z. Chen, J. Xiong, and G. Zeng, "Periodic diffraction correlation imaging without a beam-splitter," Opt. Express **20**, 2956-2966 (2012).
[9] D. Cassettari, B. Hessmo, R. Folman, T. Maier and J. Schmiedmayer, "Beam Splitter for Guided Atoms," Phys. Rev. Lett. **85**, 5483-5487 (2000).
[10] O. Houde, D. Kadio and L. Pruvost, "Cold Atom Beam Splitter Realized with Two Crossing Dipole Guides," Phys. Rev. Lett. **85** (26), 5543 (2000).
[11] A. Turlapov, A. Tonyushkin and T. Sleator, "Talbot-Lau effect for atomic de Broglie waves manipulated with light," Phys. Rev. A **71**, 043612 (2005).
[12] K. Hornberger, J. E. Sipe, and M. Arndt, "Theory of decoherence in a matter wave Talbot-Lau interferometer," Phys. Rev. A **70**, 053608 (2004).
[13] M. Naraschewski and R. J. Glauber, "Spatial coherence and density correlations of trapped Bose gases," Phys. Rev. A **59** (6), 4595 (1999).