

# Feasibility of energy harvesting from a rotating tire based on the theory of stochastic resonance

Y. Zhang<sup>1</sup>, R. Zheng<sup>2</sup>, K. Nakano<sup>3</sup>

Institute of Industrial Science, The University of Tokyo, 4-6-1, Komaba, Meguro, Tokyo, 153-8505, Japan

Email: zysgluck@iis.u-tokyo.ac.jp, topzrc@iis.u-tokyo.ac.jp, knakano@iis.u-tokyo.ac.jp

**Abstract.** Recently the use of nonlinear bi-stable micro-electro mechanical systems (MEMS) to achieve automobile tire vibration power generation has made some progress. However, the theory of stochastic resonance has not been successfully applied to automobile tires, which can produce a larger vibrational response than for a typical resonance while inputting a weak periodic force and noise excitation into a nonlinear bi-stable system. Hence, in this paper, in view of the principle of stochastic resonance, a new model is derived by positioning a magnetic end mass attached to a cantilever beam and another permanent magnet with the same polarity on the frame. Due to the road noise excitation along with the periodic force inputted to the mechanism, whether the phenomenon of stochastic resonance can happen will be discussed. Meanwhile, on the basis of Kramers rate and duffing equations the preliminary experimental device is also designed.

## 1. Introduction

The ability to harvest or scavenge energy from a vehicle operating environment can realize many sensing applications in which regularly replacing or recharging batteries is either difficult or expensive. Many potential sources of energy have been studied, including vibrations, temperature gradients, and human motion. Harvesting energy from a vehicle tire rotating environment may be considered a beneficial method for a tire pressure monitoring system (TPMS) [1].

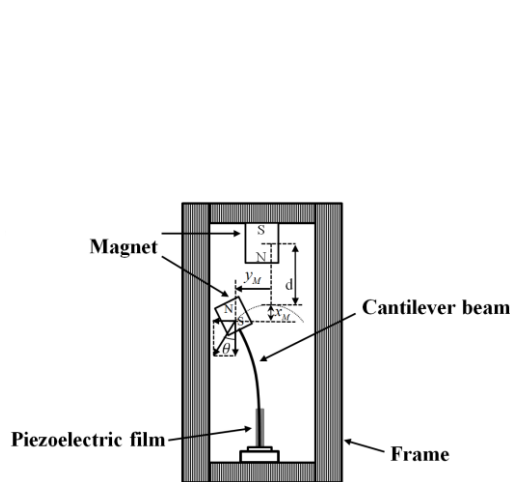
Most vibration energy harvesting systems operate effectively only at resonance [2]. Thus, for rotational environments to enlarge the narrow operating bandwidth is the key. In order to improve the bandwidth three general approaches are as follows: tuning the resonance frequency of the harvester during operation, implementing multi-mode oscillators, and employing non-linear oscillators with bistable systems to widen the bandwidth. These methods are equally applicable to rotational systems. Among that the use of nonlinear bi-stable MEMS to achieve automobile tire vibration power generation has made some progress. However, the theory of stochastic resonance has not been successfully applied to automobile tires, which can produce a larger vibrational response than for a typical resonance while inputting a weak periodic force and a noise excitation into a nonlinear bi-stable system [3].

Therefore, in view of the principle of stochastic resonance, a new model is derived to demonstrate that tire rotation can make the phenomenon of stochastic resonance occur. A bi-stable mechanism is designed by positioning a magnetic end mass attached to a cantilever beam and another permanent magnet with the same polarity on the frame shown in Figure 1. The cantilever beam is excited by

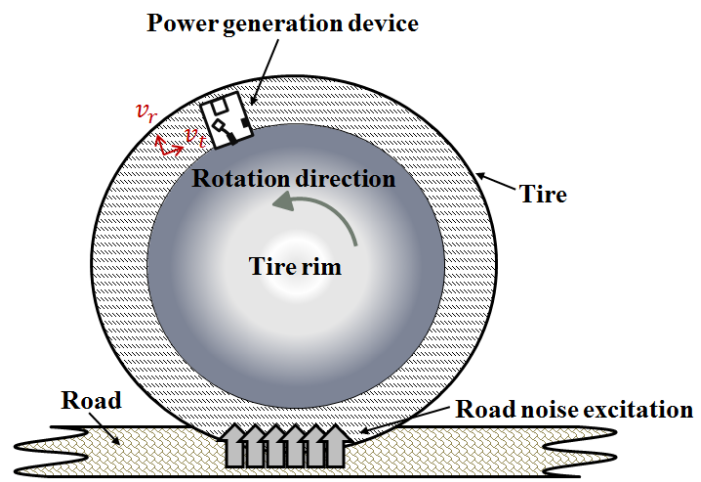


noise excitation and there is an interaction between elastic force and magnetic force of the beam, generating a nonlinear response by adjusting the distance  $d$  between the two magnets, then the system can show bi-stability. Due to the configuration of two magnets, a repulsive magnetic force will act between the two magnets. When the mechanism is mounted on a rotational tire shown in Figure 2, the road noise excitation coupled with the periodic force caused by the tire rotation and gravity of the magnetic end mass provides the external condition of the occurrence of stochastic resonance.

Different from the previous study [4], [5] where a weak external periodic force was manually added, this system can be self-provided from the gravity and rotation of the tire, thereby without consuming extra energy. In this paper, a simulation study was processed to validate this proposal, and by theoretical calculations the stochastic resonance of this system can be generated referring to Kramers rate equation. It indicates that the phenomenon of stochastic resonance occurs when the tire rotates at the frequency of 98 rad/s with the speed of 103 km/h. Based on the mathematic analysis and simulation results the design of a small-scale experimental device is proposed simultaneously.



**Figure 1.** Structure of the power generation device using stochastic resonance.



**Figure 2.** Illustration of the power generation device mounted to a rim.

## 2. Mathematic model and simulation results

If the magnetic force and gravity of the magnet end mass are taken into consideration, which may cause the angle of the beam to be  $\theta$  and the tangential displacement of the end mass to be  $y_M$ , the centripetal displacement to be  $x_M$ . when the tire rotates at a constant speed. By adjusting the distance  $d$  between two magnets, a repulsive magnetic force  $F_M$  and a tangential component  $F_H$  are also changing to make the mechanism vibrate between two stable states.  $F_H$  can be written in terms of  $F_M$  and expanded into a Taylor series, calculated around  $y_M = 0$  and truncated as follows,

$$F_H = F_M \sin \theta = \frac{F_M}{d} \frac{y_M}{\left[1 + \left(\frac{y_M^2}{d^2}\right)\right]^{1/2}} \approx \frac{F_M}{d} y_M - \frac{F_M}{2d^3} y_M^3 \quad (1)$$

When considering the tire rotation, due to the centripetal force, it is necessary to take into account the effect of gravity. Therefore, the total force of the tangential direction  $f_H$  is expressed as

$$f_H = \frac{F_M}{d} y_M - \frac{F_M}{2d^3} y_M^3 + G \sin \omega t \quad (2)$$

When the tire with a radius of  $r$  rotates at the frequency of  $\omega$  with the speed of  $v_s$ , the total kinetic energy of the end mass will be defined by  $E_K$  as

$$E_K = \frac{1}{2} m \left[ (\dot{y}_M + v_s)^2 + \dot{x}_M^2 \right] \quad (3)$$

Here, according to the thin beam theory, the potential energy of the beam with the Young's modulus and the moment of inertia of  $E$  and  $I$ , respectively, is given as follows

$$U = \frac{1}{2} EI \int_0^l (w'')^2 dx \quad (4)$$

where  $w$  is the transverse displacement of the beam and  $w'' = d^2 w / dx^2$ . Assuming  $y_M(t) = w(l, t)$ , where  $l$  is the length of the beam, the following equation can be derived as

$$w(x, t) = v(x)w(l, t) = \left( \frac{3x^2}{2l^2} - \frac{x^3}{2l^3} \right) y_M(t) \quad (5)$$

where  $v(x)$  is a static linear shape function of the cantilever beam. Assuming the transverse deflection is tiny,  $x_M$  is given as

$$x_M = \int_0^l \left[ \sqrt{1 + \left( \frac{dw}{dx} \right)^2} - 1 \right] dx = \int_0^l \frac{1}{2} \left( \frac{dw}{dx} \right)^2 dx = \frac{1}{2} \left[ \int_0^l (v')^2 dx \right] y_M^2 \quad (6)$$

According to the following generalized Lagrange's equation and equations (2) to (6), it gives

$$\frac{d}{dt} \left( \frac{\partial E_K}{\partial \dot{y}_M} \right) - \frac{\partial E_K}{\partial y_M} + \frac{\partial U}{\partial y_M} = f_H \quad (7)$$

The motion equation is derived as follows

$$m\ddot{y}_M + c\dot{y}_M + \left[ EI \int_0^l (v'')^2 dx - \frac{F_M}{d} \right] y_M + \frac{F_M}{2d^3} y_M^3 = N(t) \sin \omega t + G \sin \omega t \quad (8)$$

where  $c$  is the viscous damping coefficient,  $N(t) = \sqrt{2D}\xi(t)$  is the noise excitation from road,  $G$  as the periodic force from gravity of the end magnet. Since the rotation of tire, the magnitude and direction of these two parts will become to be  $N(t) \sin \omega t + G \sin \omega t$ .  $EI \int_0^l (v'')^2 dx$  is the spring

coefficient,  $\frac{F_M}{d}$  is the linear coefficient of the two magnets,  $\frac{F_M}{2d^3}$  is the nonlinear coefficient. When

$EI \int_0^l (v'')^2 dx - \frac{F_M}{d} = -a$ ,  $\frac{F_M}{2d^3} = b$ , the equation (8) will be transformed into:

$$m\ddot{y}_M + c\dot{y}_M - ay_M + by_M^3 = N(t) \sin \omega t + G \sin \omega t \quad (9)$$

Base on this mathematic model, the numerical simulation is implemented, where  $a = 157$ ,  $b = 5 \times 10^7$ ,  $D = 0.001$ ,  $c = 0.27 N / m / s$ ,  $G = 0.1 N$ .

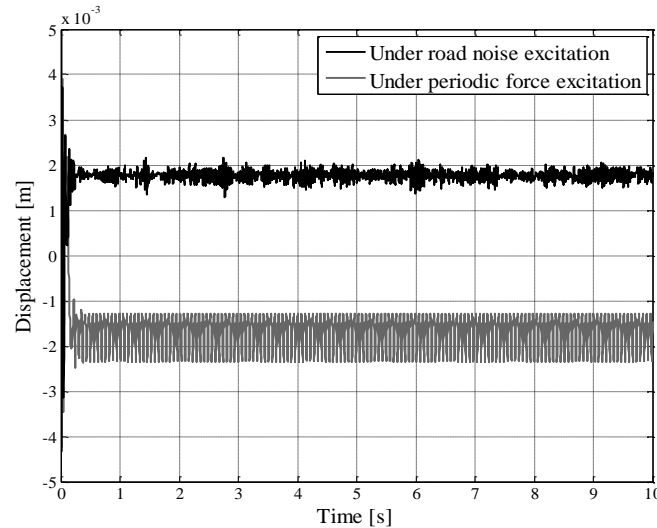
On the basis of Kramers rate equation, the stochastic resonance frequency can be calculated by the following equation as

$$f = \frac{a}{\sqrt{2}} \exp \left( -\frac{a^2}{4bD} \right) \quad (10)$$

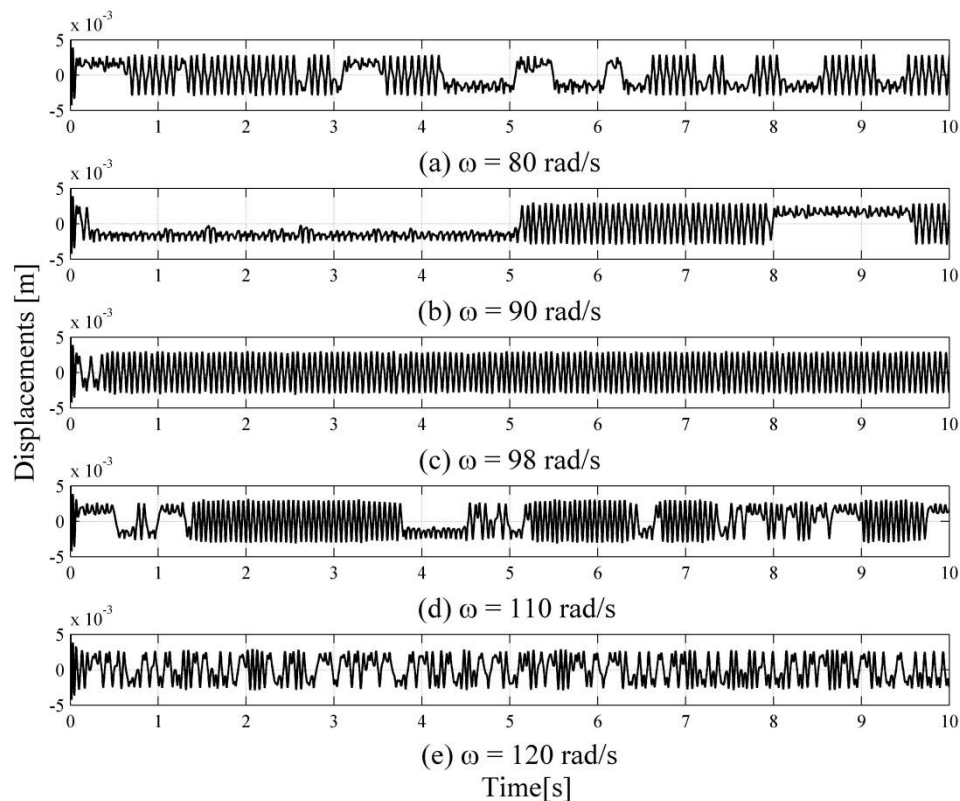
where  $D$  is the intensity of the noise excitation. By calculating the frequency of stochastic resonance according to equation (10), it can be indicated that stochastic resonance can occur at the frequency of 98 rad/s.

In order to confirm whether the stochastic resonance can occur and distinguish the stochastic resonance and normal resonance, numerical simulations are carried out under three conditions, i.e. only the noise excitation, the periodic force and both of them are applied on the device, respectively. In the case of only the noise excitation is applied or the condition that the frequency of the periodic force is 98 rad/s, the displacement responses show that the inter-well dynamics cannot be activated as shown in figure 3.

From figure 4, it can be validated that the cantilever beam can be brokenly excited to vibrate between two potential wells when the noise and periodic excitations are applied at the same time. And as predicted by the calculated parametric excitation frequency of 98 rad/s, the cantilever beam can keep the inter-well motion all the time, which indicates the phenomenon of the stochastic resonance.



**Figure 3.** Responses under road noise excitation or periodic force excitation: the black line fluctuating at  $1.8 \times 10^{-3}$  m and the grey line fluctuating at  $-1.8 \times 10^{-3}$  m represent the responses under road noise and periodic force excitation respectively.



**Figure 4.** Responses under road noise excitation and periodic force at different frequencies of tire rotation: (a) 80, (b) 90, (c) 98, (d) 110, and (e) 120 rad/s.

### 3. Design for experimental device

For the mathematic mode, the potential energy can be expressed as:

$$U(y_M, t) = -\frac{1}{2}ay_M^2 + \frac{1}{4}by_M^4 \quad (11)$$

Differentiating equation (11) with respect to time lead to:

$$F_M' = -ay_M + by_M^3 \quad (12)$$

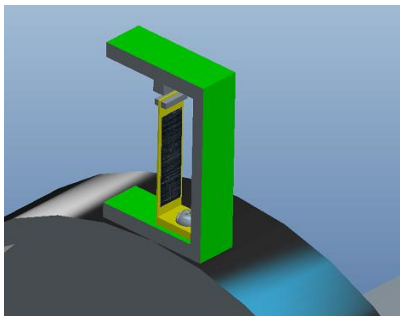
where  $F_M'$  is the nonlinear spring force. After differentiating  $F_M'$  the nonlinear spring coefficient  $k'$  can be given as follows

$$k' = -a + 3by_M^2 \quad (13)$$

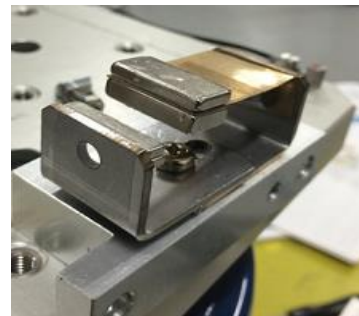
Also, the equilibrium points of two potential wells can be expressed by

$$y_0 = \pm \sqrt{\frac{a}{b}} \quad (14)$$

Substituting equation (13) into (14), the nonlinear spring coefficient  $k'$  can be obtained. Therefore, the size of the brassiness cantilever beam is designed to be  $5cm \times 2cm \times 0.5mm$ . The conceived experimental setup is shown as figure 5. Finally, the photo of the fabricated power generation device is shown in figure 6 and it is currently under testing.



**Figure 5.** The conceived experimental setup



**Figure 6.** The generation power device

### 4. Conclusion

In this study, the feasibility of energy harvesting from a rotating tire based on the theory of stochastic resonance was discussed. From the mathematic mode and numerical simulation results, it can be shown that the phenomenon of stochastic resonance occurs when the tire rotating at the frequency of 98 rad/s. Meanwhile, using the assigned and fabricated device the experiment is conducting well now.

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