Modeling of the thermo-mechanical efficiency of the bimetal strip heat engines

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Abstract. This paper presents a theoretical demonstration of the bimetal strip heat engine working, based on the study of the thermo-mechanical instability of the pre-buckled bimetallic beams. Starting from the Euler buckling equation, this paper describes the bimetal strips like classical but non-linear thermodynamic systems, and gives the bistability criterion of such beams. Studying the thermodynamic potentials of these beams helps to evaluate the release of the kinetic energy happening during the beam snap-through, to give the Maxwell relations between each partial derivative of the thermodynamic potentials and to show that the thermal snap-through is a first-order transition according to the Ehrenfest theory. The model is then used to draw the temperature-entropy cycle of the bimetal heat engines and to evaluate the performances of these harvesters (available mechanical energy and thermodynamic cycle efficiency).

Introduction

The development of bimetal strip heat engines has been pursued as an alternative to the Seebeck thermoelectric generators based on the properties of semiconductors like bismuth tellurides, or to the conventional heat engines using fluids as a heat transfer medium. Proofs of concept have been reported in [1-6]. These devices are designed to exploit the sudden displacement of a bimetallic membrane switching from a critical equilibrium position to a stable one, to harvest a part of the thermal energy flowing through the bimetal and to convert it into kinetic energy. The existence of a hysteretic behavior, consequence of their bistability, enables to cycle beams between a hot source and a cold one (Fig.1a,b), causing then an astable behavior of the beams. The simplicity of this concept allows scaling down the devices using silicon manufacturing techniques ([4], [7]), which is expected to reduce the beam cycling duration, increasing then the heat flowing through the bimorph and finally multiplying the power generated by the harvester size scaling factor ([2]).

With this article, we aim at consolidating the theory of the bimetal strip heat engine: in a first part, we explain how the non-linear Euler buckling equation can be used to predict the bistability of bimetallic beams, whereas in a second part, the evolutions of the strain energy and entropy of the beams are studied along the equilibrium path. Finally, the model is used to evaluate the performances of the bimetallic strips as heat transfer media at the microscale.

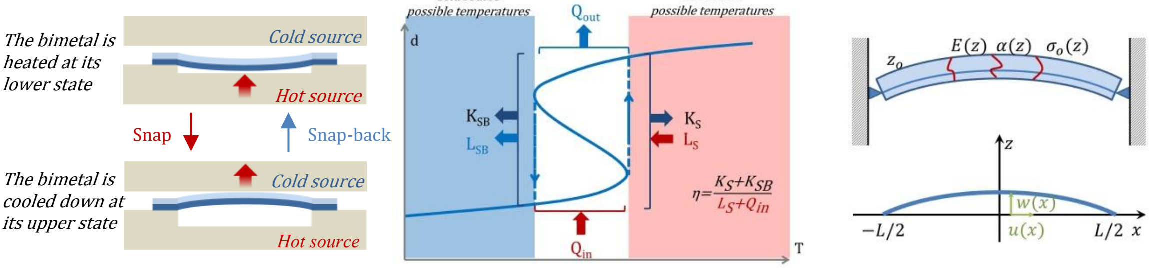

Figure 1. (a) Architecture scheme and (b) principle of the harvester. (c) Beam structure modeled

1. Snap-through modeling

The thermo-mechanical instability of the bimetallic beams is a consequence of the Euler buckling phenomenon described by equation (1), giving the dependence of the lateral displacement field on the axial thrust p_T and the beam bending stiffness i_e.

$$k^2.w_{,xx}+w_{,xxxx}=0 \tag{1a}$$

$$k^2=p_T/i_e \tag{1b}$$

Whereas the behavior of perfect and straight beam predicted by (1) looks like a pitchfork bifurcation, the behavior of an imperfect beam is sometimes a little bit more complex and can be discontinuous in some cases. This discontinuity is accompanied by a kinetic energy release and a sudden displacement of the beam called snap-through, caused by two imperfections of the beam having antagonistic effects. The bimetallic beam presented in Fig.2 is used to explain the conditions of occurrence of the thermal snap-through, by taking into account the combined effects of the residual stress and the thermal expansion. The beam's mechanical properties can be globally described through the mean of the parameters set (n_e, i_e, n_α, m_α, F_o, M_o), where n_e represents the beam stretching stiffness, n_α the mean thermal stress coefficient of the beam, and F_o the mean residual stress in the beam (measured at θ^o) which makes the beam buckle initially. The two beam's imperfections are described by m_α representing the asymmetry of the thermal expansion causing the thermal curvature of the beam in a given direction, and M_o representing the asymmetry of the residual stress in the beam's materials which orientates initially the beam buckling in the opposite direction to the thermal curvature. M_o and m_α must have the same signs to allow the thermo-mechanical instability to occur.

$$n_e=\int_{y,z} E(z).\delta z.\delta y \ ; \ i_e=\int_{y,z} E(z).z^2.\delta z.\delta y \tag{2a}$$

$$n_\alpha=\int_{y,z} E(z).\alpha(z).\delta z.\delta y \ ; \ m_\alpha=\int_{y,z} E(z).\alpha(z).z.\delta z.\delta y \tag{2b}$$

$$F_o=\int_{y,z} \sigma_o(z).\delta z.\delta y \ ; \ M_o=\int_{y,z} \sigma_o(z).z.\delta z.\delta y \tag{2c}$$

Once these beam parameters are introduced, as we demonstrated in [8,9] it is possible to show that the thermodynamic principles, applied to the beam equilibrium, can be written as

$$dK+dF=0 \tag{3a}$$

$$(T+T°).dS=\delta Q \tag{3b}$$

Where K is the kinetic energy of the beam, F its Helmholtz free energy, S its entropy and T its temperature (T° absolute temperature at which the stress is measured). Q represents the heat transferred to, or rejected from the beam during the heating and cooling periods. To understand the evolution of the internal beam efforts, one must express the constitutive equations of the beam. The Helmholtz free energy differential is given in (4a). As the volume and the temperature are the privileged variables of F in a classical thermodynamic system, we introduce here λ which is the difference between the length of the curved beam and the length of the straight beam, coming from the non-linearity of the Van Karman strain tensor (4c).

$$dF=-S.dT+(P-P_o).d\lambda \tag{4a}$$

$$dW_i=(P-P_o).d\lambda \tag{4b}$$

$$\lambda=\frac{1}{2}.\int_{-L/2}^{L/2} w_{,x}^2 .dx \tag{4c}$$

The force P_o is an invariant of the beam properties defined by (5a), which can be seen as a constant force applied by a pressure reservoir acting on the beam. The ratio of the M_o and m_α is also used to define a reference temperature T_o (5b). The force P describes the effect of the beam internal efforts (5c): the beam non-linear stretching force linking to the beam length variation λ, the mean thermal expansion, and the clamp thrust p_T which verifies the non-linear Euler equation (1).

$$P_o=n_\alpha.M_o/m_\alpha-F_o \tag{5a}$$

$$T_o=M_o/m_\alpha \tag{5b}$$

$$P=n_e.\lambda/L-n_\alpha.(T-T_o)+p_T \tag{5c}$$

Considering (4a), at a given temperature, the beam equilibrium is characterized by the equality.

$$P=P_o \tag{6}$$

As shown in [8,9], the instability appears when the external force P_o exceeds the beam Euler load P_γ (7a) which is a solution of (7b). γ, varying from 0 to ∞, models the stiffness of the clamp (0 for a simply supported beam, ∞ for a doubly clamped beam).

$$P_o>P_\gamma \Leftrightarrow n_\alpha.M_o/m_\alpha-F_o>i_e.k_\gamma^2 \tag{7a}$$

$$k_\gamma.\,\text{cotan}\left(k_\gamma.L/2\right)+\gamma=0 \tag{7b}$$

The evolution of the beam shape can then be plotted in function of the external force applied to the beam, showing the appearance of an unstable behavior, as shown on Fig.2a, when (7a) is verified. As seen, the bistable beams exhibit a thermal hysteresis characterized by two temperatures called snap temperature T_S and snap-back temperature T_{SB} at which the beam snaps from a critical position (or limit point) to a stable equilibrium position. Each switching is accompanied by a kinetic energy release which can be harvested if the bimetal is coupled with a piezoelectric transducer [2] for example.

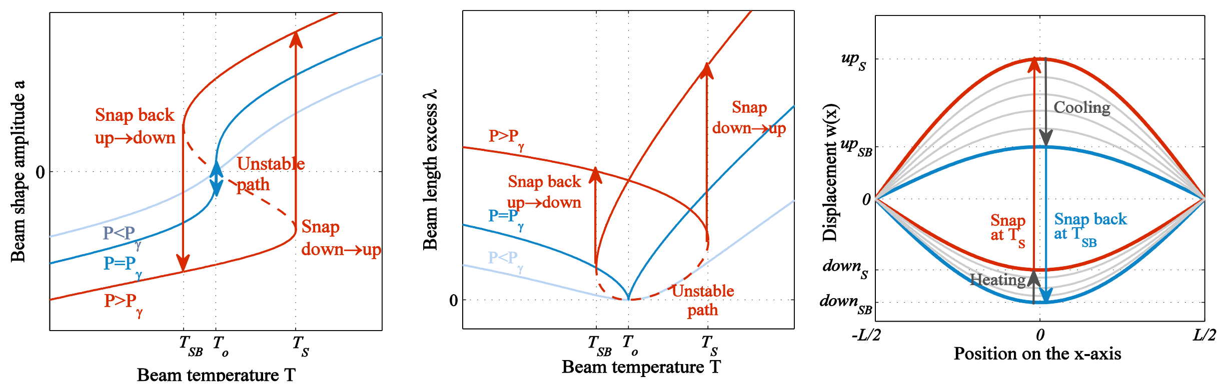

Figure 2. Evolutions of the beam deflexion (a) and length λ (b) in function of the temperature and P_o for a simply-supported beam (γ=0). (c) Evolution of the bistable beam shape in function of the temperature for a simply-supported beam (γ=0).

2. First-order transition and energy harvesting

By using (1-7), we show that the beam strain energy W_i of the harmonic functions is given by (8).

$$W_i=\frac{n_e}{2.L}.\lambda^2+i_e\left(k^2-2.\frac{k^3.\sin(k.L)}{k^2.L-k.\sin(k.L)}\right).\lambda-(n_\alpha.T-F_o).\lambda \tag{8}$$

Drawing the potential W_i helps understanding the energetic meaning of the thermal snap-through. As seen on Fig. 3a, for a bistable beam, W_i can exhibit two equilibrium wells separated by a potential barrier formed by an unstable equilibrium position. The beam energy evolves with the temperature and the stable equilibrium positions can become critical when reaching T_S or T_{SB}, making the beam snap to the other stable position and release its energy kinetically.

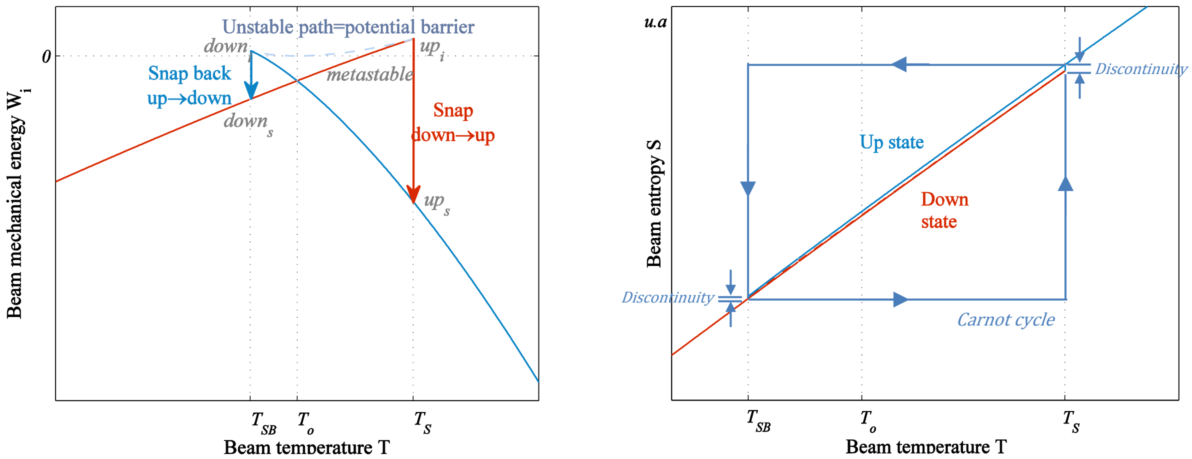

Figure 3. (a) Evolution of the strain energy and (b) entropy for a bistable beam in function of the temperature. (b) shows the thermodynamic cycle of the bimetal strip heat engine.

The Schwartz theorem is used to give the Maxwell relations between the partial derivatives of the entropy and beam force P. The bimetallic strip, as a thermodynamic system, can then be described with the matrix (9) where the partial derivative of the entropy with respect to λ expresses the thermo-mechanical coupling inside the beam materials.

$$d\begin{pmatrix}S\\P\end{pmatrix}=\begin{pmatrix}\frac{C_\lambda}{\theta} & -P^{\lambda}_{,T}\\ P^{\lambda}_{,T} & P^{T}_{,\lambda}\end{pmatrix}.d\begin{pmatrix}T\\ \lambda\end{pmatrix} \tag{9}$$

The expression of the entropy (10) is found by using (9) and its evolution in function of the beam temperature is drawn on Fig.3b.

$$S-S°=\left(4.i_e.\frac{k^3.\sin(k.L)}{k^2.L-k.\sin(k.L)}+n_\alpha.(T-T_o)\right).\frac{\lambda}{T-T_o}+C_v.\ln(T+T°) \tag{10}$$

Two discontinuities of the entropy happen at T_S and T_{SB}, meaning that the thermal snap-through is a first-order transition according to the Ehrenfest and Landau theories [10]. Similarly to any other first-order phase change, a latent heat must be supplied to the beam to switch at T_S and rejected from the beam to switch back at T_{SB}. The order parameter associated to the transition is the deflection of the beam, which is either negative when the beam is at its lower state and in contact with the hot source, or positive when it is cooled at its upper state. The existence of a hysteresis on the temperature-entropy representation of the bimetals thermodynamic cycle shows the possibility to harvest energy thanks to a bimetal-based heat engines. This cycle can be compared to the classical Carnot cycle made of two isentropic and two isothermal curves and representing the maximal efficiency of a theoretically reversible heat engine.

3. Applications to micro-scale bimetals heat engine

The model can be used to evaluate the efficiency of a bimetal strip heat engine both at the macroscale and the microscale. In this paper, we study only the micro-harvesters, and take the example of a bimorph of aluminium (Al) and silicon oxide (SiO_2). This bimorph owns naturally a high difference of thermal expansion coefficients (α_{Al}=23.10^{-6}K^{-1}, α_{SiO2}=2,9.10^{-6}K^{-1}). Fig.4a gives the evolution of the beam deflexion in function of the thermal hysteresis of a 200µm-long beam (with 2µm-thick layer for each material). Fig.4b gives the energy density which can be harvested when the beam snaps and snaps back. Using (10) to evaluate the thermal exchange of the beam with its surrounding, it can be shown (Fig.4c), that a Carnot-relative efficiency of around 1.7% (up to 3.5% with a W-Al beam [9]) and an energy density of about 300µJ.cm^{-3} can be obtained with two-degree hysteresis SiO_2-Al beam.

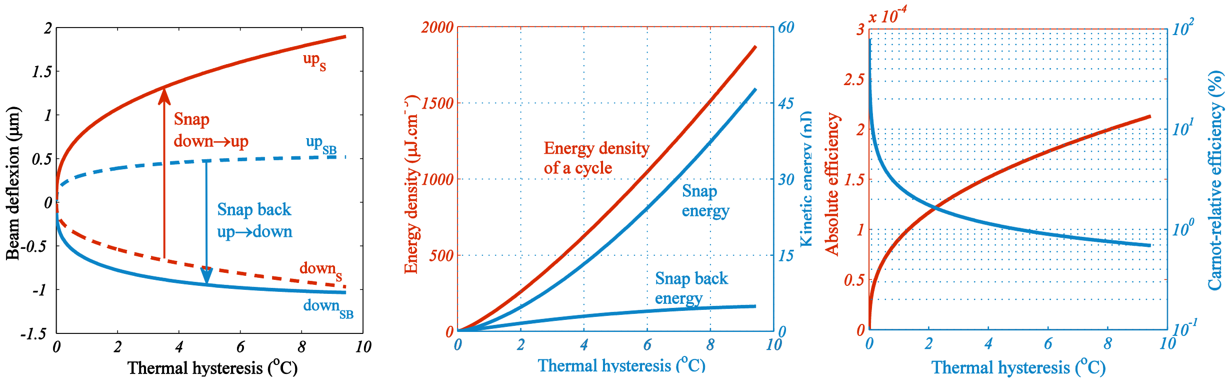

Figure 4. (a) Evolution of the beam deflection in function of the thermal hysteresis (mean temperature of the hysteresis: 75°C) (b) Evolution of the kinetic energy density of the bimetallic beam (Normalization by the volume occupied by the beam during a cycle (Fig 4.a)) (c) Absolute efficiency and Carnot-relative efficiency of the beam's thermodynamic cycle

4. Conclusion

By proposing a thermodynamic representation of the bimetallic beams, we found the necessary conditions to observe the occurrence of the thermo-mechanical bistability, and demonstrated that the thermal snap-through is a first-order transition according to the Ehrenfest theory. The hysteretic behavior of these bimorphs can then be used to harvest a part of the heat flowing through them by converting it into strain energy. Thanks to our model, we evaluated the thermo-mechanical efficiency of such heat engines both at the macro and micro-scales. A theoretical Carnot-relative efficiency of around 2~4% and a theoretical strain energy density of 300µJ/cm^3 per snap can be obtained with a two-degree-hysteresis beam, which would be sufficient to power Wireless Sensor Nodes according to the scaling laws developed in [2].

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5. References

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