

# Theory of oscillations and sterile neutrinos

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**Abstract.** We present a concise review of the theoretical status of neutrino oscillations within the (standard) 3-flavor framework and the (non-standard) 4-flavor scheme endowed with one additional sterile species (the so-called 3+1 scheme). We emphasize the slight overall preference that recently emerged for CP-violation in the 3-flavor analysis and highlight the unique role of the global data combination in the near future. After a brief introduction of the motivations for light (eV) sterile neutrinos, we discuss the bounds on their mixing with the electron neutrinos, attainable from the solar sector. The upper limit so obtained is independent of the reactor neutrino fluxes, whose calculations are affected by systematic uncertainties not completely under control. Finally, we highlight the possibility to explore sub-eV “super-light” sterile neutrinos exploiting the  $\theta_{13}$ -dedicated reactor experiments also commenting on the robustness of the 3-flavor results within the enlarged 3+1 scheme.

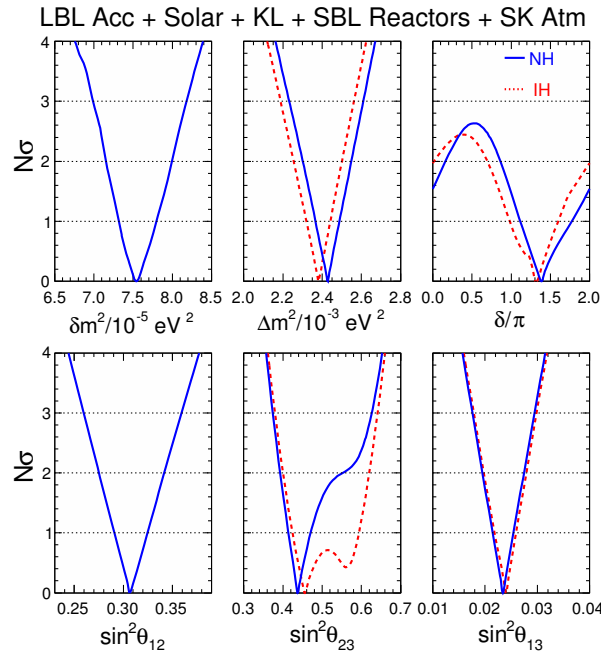
## 1. Introduction

Neutrino physics is living one of its most exciting epochs due to the recent observation of a relatively large value of the third mixing angle  $\theta_{13}$ . This discovery provides concrete hopes of completing the description of the 3-flavor framework with the determination of the leptonic CP-violation (CPV) and the neutrino-mass-hierarchy (NMH). But the field of neutrino physics may even open the way to unexpected avenues. The neutrino sector may be richer than commonly believed and not confined to the 3-flavor framework. Several anomalies have recently emerged both in laboratory and cosmological data analyses, which already indicate some cracks in the standard 3-flavor picture, pointing towards new physics. New light sterile neutrinos may provide the solution of these puzzling results. In what follows, we present a concise review of the theoretical status of neutrino oscillations within the (standard) 3-flavor framework and the (non-standard) 4-flavor scheme endowed with one new additional sterile species (the so-called 3+1 scheme).

## 2. The 3-flavor paradigm

After many years of observations performed with atmospheric, solar, accelerator and reactor experiments, neutrino oscillations have been firmly established and the measurement of the mass-mixing properties is now entering a precision era. Except for a few results obtained in short-baseline (SBL) setups — on which we will return later — all the existing phenomenology is consistent with a simple 3-flavor scheme. In such a scenario, the three known flavor fields ( $\nu_e, \nu_\mu, \nu_\tau$ ) are mixed with the three mass ones ( $\nu_1, \nu_2, \nu_3$ ), through a unitary matrix  $U$  (the so-called PMNS matrix). A common parameterization (adopted by the Particle Data Group [1])





**Figure 1.** Bounds on the oscillation parameters in terms of standard deviations  $N\sigma$  from the best fit. Solid (dashed) lines refer to NH (IH). The horizontal dotted lines mark the  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  levels for each parameter.

for such a matrix is  $U = R_{23}\Gamma_\delta R_{13}\Gamma_\delta^\dagger R_{12}$ , where the  $R_{ij}$ 's are real Euler rotations with angles  $\theta_{ij} \in [0, \pi/2]$ , while  $\Gamma_\delta = \text{diag}(1, 1, e^{i\delta})$  embeds a Dirac CP-violating phase  $\delta \in [0, 2\pi]$ .

### 2.1. Status of the 3-flavor parameters

The current neutrino phenomenology implies that the three-neutrino mass spectrum  $\{m_i\}_{i=1,2,3}$  is formed by a “doublet” of relatively close states and by a third “lone” neutrino state. In the most frequently adopted labeling convention, the lightest (heaviest) neutrino in the doublet is called  $\nu_1$  ( $\nu_2$ ) and the corresponding squared-mass difference is defined as  $\delta m^2 = m_2^2 - m_1^2 > 0$ . Thanks to solar neutrinos, and more precisely to the matter (MSW) effects involved in the flavor conversion inside the sun, we know that the lightest eigenstate  $\nu_1$  is the one having the larger superposition with the electron neutrino ( $|U_{e1}|^2 \simeq 0.674$ ,  $|U_{e2}|^2 \simeq 0.303$ ). The lone state is labeled as  $\nu_3$  and the sign of  $m_3^2 - m_{1,2}^2$  distinguishes two physically different cases. In one case, the mass spectrum is said to obey a “normal” ordering or hierarchy (NH) since, in this configuration, the lone state  $\nu_3$ , which has a very small component of the electron neutrino flavor ( $|U_{e3}|^2 \simeq 0.023$ ) is the heaviest mass eigenstate. Conversely, in the other case, the lone state  $\nu_3$  is the lightest mass eigenstate and the mass hierarchy is said to be inverted (IH). While the first case (NH) may naively seem more natural, both orderings are perfectly admissible and the one effectively realized in Nature must be determined through experiments. It is convenient to define the second independent larger squared-mass splitting as  $\Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$ , so that the two hierarchies (NH and IH) are simply related by the transformation  $+\Delta m^2 \rightarrow -\Delta m^2$ . For phenomenological and historical reasons it is common practice to dub the  $\delta m^2$ -driven ( $\Delta m^2$ -driven) oscillations as the “solar” (“atmospheric”) flavor transitions.

Figure 1 displays the estimates of the mass-mixing parameters obtained by the global data analysis [2]. The plot shows the number of standard deviations  $N\sigma$  from the best fit values. We have a good determination of the “solar” ( $\delta m^2, \theta_{12}$ ) parameters (at the level of

2.6% and 5.4% respectively) and “atmospheric” ( $|\Delta m^2|, \theta_{23}$ ) parameters (at the level of 2.9% and 11% respectively). It is important to notice that while the (more precise) estimate of the two mass-splittings is currently dominated by the measurement of the frequency of the oscillation pattern(s) performed through “artificial” (reactor and accelerator) neutrino source experiments, the (less precise) estimate of the two large mixing angles ( $\theta_{12}, \theta_{23}$ ) — directly related to the amplitude of the oscillations — is determined by an interplay of “natural” (solar and atmospheric) and “artificial” (reactor and accelerator) neutrino source measurements. The third mixing angle  $\theta_{13}$  (often called “reactor angle”) has been determined to be non-zero in 2012 by the three dedicated reactor experiments Daya Bay, Double-Chooz and RENO and by the accelerator experiment T2K, which have confirmed earlier indications coming from global analyses [3] (see also [4, 5, 6]). The current estimate of  $\theta_{13}$  is dominated by Daya-Bay and the uncertainty on this parameter is  $\sim 8\%$ . Paradoxically, among the three mixing angles, the atmospheric one ( $\theta_{23}$ ), which was the first to be measured, is currently the least well-known. In particular, it is not known if it is maximal or not and at the moment there is no robust indication in favor of any of two octants. Indeed, in our preliminary analysis (not shown) including the results presented at the Neutrino 2014 conference the slight preference for the first octant evident in Fig. 1 tends to decrease. In contrast, the same preliminary analysis confirms the weak preference (slightly below the 90% C.L.) for a non-zero value of the CP-violating phase  $\delta \sim -\pi/2$  apparent in Fig. 1. The current analysis is insensitive to the neutrino mass hierarchy.

## 2.2. Importance of the leptonic CP-violating phase $\delta$

The determination of the CP-violating phase  $\delta$  is important *per se* as it would complete our knowledge of the electroweak interactions, complementing the relevant information coming from the quark sector. But the discovery of the leptonic CPV may have other and far reaching implications. In particular, it may have some role in leptogenesis, and therefore it may be linked to the origin of the matter-antimatter asymmetry of our Universe (see below).

In the PMNS matrix, the imaginary parts of all the quartic products  $U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}$  are equal up to a sign and related by antisymmetric (in both flavor and mass indexes) coefficients

$$\Im[U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j}] \equiv J \sum_{\gamma=e,\mu,\tau} \sum_{k=1,2,3} \epsilon_{\alpha\beta\gamma} \epsilon_{ijk} . \quad (1)$$

where  $J$  is the so-called Jarlskog invariant, which gives a parameterization-independent measure of CP violation [7]. In the usual parametrization, one finds the expression

$$J = c_{12}s_{12}c_{23}s_{23}c_{13}^2s_{13} \sin \delta \equiv J^{\max} \sin \delta , \quad (2)$$

where  $c_{ij} \equiv \cos \theta_{ij}$  and  $s_{ij} \equiv \sin \theta_{ij}$ . From (2) one realizes that CP-violation can occur only if all the three mixing angles are different from zero. Using the current best fit values of the three mixing angles one derives

$$J^{\max} \simeq 3.0 \times 10^{-2} , \quad (3)$$

which is not far from the maximum value theoretically attainable  $1/6\sqrt{3} \simeq 0.1$  (see [8]). The value of  $\delta \sim -\pi/2$  currently preferred by data implies a best fit  $J^{\text{best}} \simeq -3 \times 10^{-2}$ . It is interesting to make a comparison with the quark sector, in which case the Jarlskog invariant has been precisely determined and is  $J^{\text{CKM}} \simeq 3 \times 10^{-5}$  [1]. Therefore, we see that in the eventuality of confirmation of the current preference for  $\delta \simeq -\pi/2$ , the lepton sector would be a source of CP-violation three orders of magnitude larger than that provided by the quark sector. Interestingly, the current best estimates of  $\theta_{13}$  and  $\delta$  satisfy the inequality

$$|\sin \theta_{13} \sin \delta| \simeq 0.15 > 0.11 , \quad (4)$$

which is the condition for a successful leptogenesis having as the sole source of CP-violation the Dirac phase  $\delta$  [9].

### 2.3. Role of the global analyses in the near term

In the next years the role of global analyses will become more important than ever. Whilst for the third mixing angle  $\theta_{13}$  we had at our disposal dedicated experiments capable to transform the first weak indications into a solid observation ( $\theta_{13}$  is now known to be non-zero at the incredible statistical significance of twelve standard deviations), the situation appears to be completely different for the remaining neutrino properties. From all the recent sensitivity studies it has clearly emerged that, at least for the next five years, it will be extremely difficult (if not impossible) for a single experiment to provide definitive information (say an evidence at the 3 sigma level) for any of the two searched properties CPV and NMH. Rather, each experiment will (in the best case) provide some hint with low statistical significance. In this situation, it is clear that the combination of all the available data will be a powerful tool enabling us to get the most from the neutrino data. Important synergies can manifest only at the level of the global combination, where parameter degeneracies affecting the single datasets may be reduced. Last but not least, the global analyses will test the mutual consistency of different pieces of data possibly evidencing discrepancies, which may be the harbingers of new physics beyond the Standard Model.

### 3. Light (eV) and super-light (sub-eV) sterile neutrinos

Several anomalies have recently emerged in short baseline (SBL) oscillation experiments, which indicate some “cracks” in the standard picture and point towards the existence of new physics. These are the LSND and MiniBooNE anomalies and the so-called “reactor and gallium anomalies”. Furthermore, the latest cosmological data suggests an excess in the radiation content of the Universe (the so-called “dark radiation anomaly”).

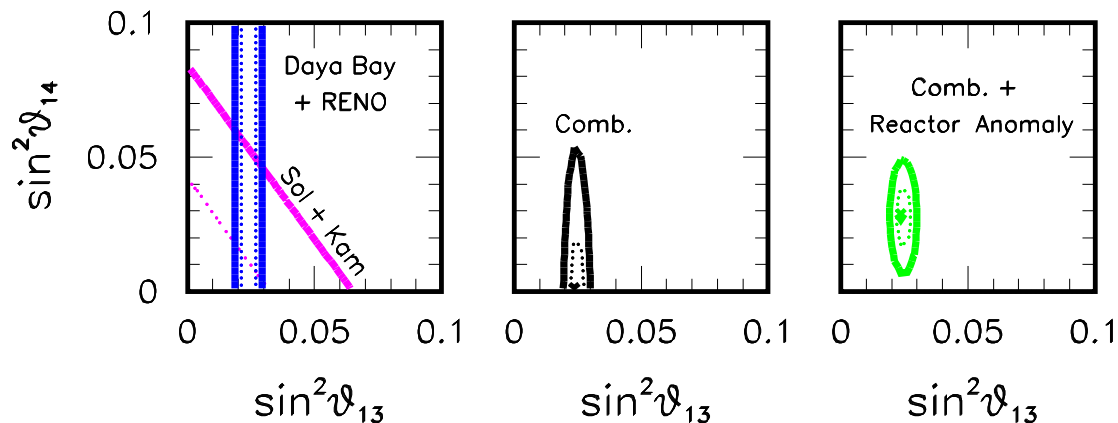
An intriguing hypothesis is that new light sterile neutrino species — Standard Model gauge singlets with mass of O(eV) — may be responsible for such inconsistencies, although they seem unable to provide a completely convincing explanation (see [10] for a critical discussion). Right-handed neutrinos appear in many extensions of the Standard Model, usually in the context of see-saw mechanisms, where their natural mass scale lies at the GUT energies. However, it is safe to affirm that the mass of these particles cannot be predicted in a robust way, and indeed it is not uncommon to find models accommodating lower masses down to the sub-eV domain.

#### 3.1. An independent test of the low-energy anomalies

As mentioned above, a consistent interpretation of the anomalous results in terms of sterile neutrino oscillations is problematic. The most important difficulty is related to the tension (arising in all schemes with extra sterile neutrinos) among the  $\nu_\mu \rightarrow \nu_e$  appearance results and the (joint  $\nu_e \rightarrow \nu_e$  and  $\nu_\mu \rightarrow \nu_\mu$ ) disappearance searches (see [10]). In this perspective, it is essential to carefully put under test any single piece of evidence, irrespective of all the others. Here, we focus on the reactor and gallium anomalies, which *taken alone* are fully consistent with a non-zero admixture of the electron neutrino with new sterile species, favoring the range  $|U_{e4}|^2 \sim [0.020, 0.060]$  at the  $2\sigma$  level [11] (see also [12]).

As we have shown in [13], where we have presented the analytical treatment of the solar MSW transitions in a general 3+s scheme, the solar sector data (Solar and KamLAND) offer a sensitive probe of such an admixture. In a subsequent paper [14], we have shown how the first evidence for non-zero  $\theta_{13}$  (coming at that time only from Double-CHOOZ and T2K) further improved the sensitivity of the solar sector to  $U_{e4}$ . In these proceedings we present an updated version of the analysis performed in [14] by incorporating the more recent (strongest) constraints on  $\theta_{13}$ , which is now known with far better precision.

In the left panel of Fig. 2, the diagonal bands indicate the region allowed by the combined solar and KamLAND data in the plane  $[\sin^2 \theta_{13}, \sin^2 \theta_{14}]$ . We recall that in the parametrization adopted in [13, 14] it is  $U_{e4}^2 \equiv \sin^2 \theta_{14}$ . We stress that the KamLAND analysis has been



**Figure 2.** Left panel: regions allowed by the solar sector data (diagonal bands) and by the dual-baseline reactor (Daya Bay and RENO) experiments (vertical bands). Middle panel: regions allowed by their combination. Right panel: combination of the constraints in the middle panel with those coming from the reactor anomaly taken from [11]. The contours refer to  $\Delta\chi^2 = 1$  (dotted line) and  $\Delta\chi^2 = 4$  (solid line).

performed using only the spectral shape information so as to render its results independent of the reactor antineutrino flux normalization. In the same panel, the vertical bands identify the range allowed for  $\theta_{13}$  by the combination of the dual-baseline reactor experiments Daya Bay and RENO. The superposition of the regions allowed by the two datasets (solar+KamLAND and Daya Bay+RENO) evidences their complementarity in constraining the two mixing angles. Their combination, shown in the central panel of Fig. 2, leads to the strong upper bound

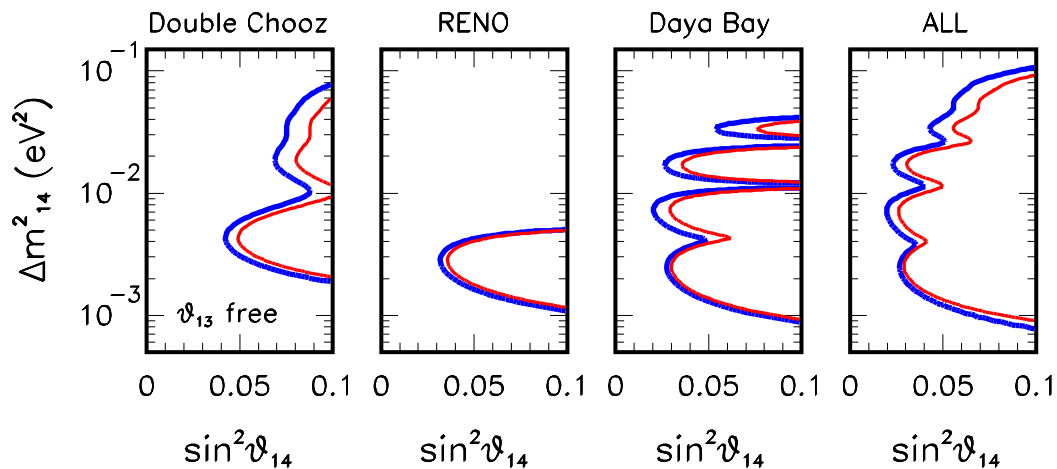
$$\sin^2 \theta_{14} \equiv |U_{e4}^2| < 0.041 \quad (90\% \text{ C.L.}). \quad (5)$$

As already stressed, this bound does not depend on the normalization of the reactor fluxes and thus represents an independent constraint on  $|U_{e4}|^2$ . Therefore, it makes sense to combine such a bound with the information coming from the reactor anomaly. The result of such an exercise is shown in the right panel of Fig. 2, where we have taken the reactor anomaly likelihood from [11]. The effect of the bound in Eq. (5) is a downshift of the  $2\sigma$  range allowed for  $|U_{e4}|^2$  from  $[0.011, 0.054]$  to  $[0.005, 0.050]$ , and a sensitive reduction of the overall statistical significance (from  $\sim 3\sigma$  to  $\sim 2.5\sigma$ ) of the indication of  $\theta_{14} > 0$ .

### 3.2. Probing super-light sterile neutrinos at reactor $\theta_{13}$ -dedicated experiments

Three dedicated reactor experiments, Daya Bay, Double Chooz and RENO, have recently performed a precision measurement of the third standard mixing angle  $\theta_{13}$  exploiting a multiple baseline comparison of  $\nu_e \rightarrow \nu_e$  disappearance driven by the atmospheric mass-squared splitting. In [15] we have shown how the same technique can be used to put stringent limits on the oscillations of the electron neutrino into a fourth “super-light” sterile species characterized by a mass-squared difference lying in the range  $[10^{-3} - 10^{-1}] \text{ eV}^2$ . Here, due to space limitations, we only present the results of the analysis obtained in [15], referring the reader to the original work for more details. Figure 3 displays the constraints on the admixture  $|U_{e4}|^2$  obtained by a general 4-flavor analysis, in which  $\theta_{13}$  is marginalized away.

The comparison of the first three panels evidences the complementarity of the three experiments induced by the different baselines involved in the three setups. Hence, in the more complicated 4-flavor scenario, in contrast to the simpler 3-flavor scheme, the fit is not dominated by the sole experiment Daya Bay. Very recently, the Daya Bay collaboration has presented the



**Figure 3.** Results obtained from the 4-flavor analysis. The contours refer to 2 d.o.f. 90% C.L. (blue thick line) and 99% (red thin line).

first results of a 4-flavor analysis [16]. Unlike our study, which could make use only of the total rate information, Daya Bay exploits the full spectral information, obtaining more stringent constraints. We close this discussion on light sterile neutrinos with a final important remark. As we have shown in [15], the estimate of  $\theta_{13}$  obtained by the combination of the three reactor experiments is substantially independent of the new mass-squared splitting provided that this is not too low ( $> 6 \times 10^{-3} \text{ eV}^2$ ). Therefore, the 3-flavor picture is robust with respect to the perturbations induced by a fourth neutrino.

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