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Energy Properties of rotational bands of $^{170-174}Yb$ isotopes

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Abstract.

Accounting for Coriolis mixing of experimentally known rotational bands non-adiabatic effects in energy of low-lying excited states are investigated, within phenomenological model. The Calculations for isotopes $^{170,172,174,176}Yb$, are carried out.

The energy and wave function structure of excited states are calculated. The finding reveals that the bands mixing has been found to have considerable impact on the wave function of low-lying states 0^+ and 2^+ bands.

1. Introduction

The medium and heavy mass isotopes of Ytterbium located in the rear-earth mass region are well deformed nuclei that can be populated to very high spin. Mach experimental information on even-odd mass of Yb isotopes has became more abundant [1]-[6].

A series of methods and nuclear models for describing the properties of the ground and excited positive - parity of even - even deformed nuclei are described and tested on practical problems. The adiabatic model of the nucleus proposed by Bohr and Mottelson, which is a phenomenological model, has played an important role in the study of the properties of deformed nuclei [7].

The nuclei $^{170,172,174}Yb$ have been well studied. It is important to note that these are investigated in a number of ways such as radioactive decay of $^{170,172,174}Lu$, and different nuclear reactions. In these isotopes, many 1^+ states and $K^\pi = 0^+, 2^+$ bands have been observed. For instance, the excited energy $K^\pi = 0^+_\ell(0^+_\ell)$ and $2^+_m(2^+_m)$ it rises with the increase in number of neutrons, except ^{176}Yb.

In the present paper we are investigated properties of the low - lying states of the isotopes Yb, within the phenomenological model [8, 9]. The calculation performed for the $^{170,172,174,176}Yb$ isotopes. The energy spectra and wave function of the excited states of ground band, $\beta_\ell-$, γ_m- bands are calculated.

2. The Model

To analyze the properties of low-lying positive parity states in Yb isotopes, the phenomenological model of [8] is exploited. This model takes into account the mixing of states of the $K^\pi = 0^+, 2^+$ and 1^+ bands. The Hamiltonian model is

$$H = H_{rot}(I^2) + H_{K,K'} \tag{1}$$

$$H^\sigma_{K'K}(I) = \omega_{_K}\delta_{_{K,K'}} - \omega_{rot}(I)(j_x)_{_{K,K'}}\zeta(I,K)\delta_{_{K,K'\pm 1}} \tag{2}$$

where $\omega_{_K}$– bandhead energy of rotational band, $\omega_{rot}(I)$– an angular frequency of rotational nucleus, $(j_x)_{_{K,K'}}$– matrix elements which describe Coriolis mixture between rotational bands and

$$\zeta(I,0) = 1 \qquad \zeta(I,2) = \sqrt{1 - \frac{2}{I(I+1)}}.$$

The rotational part of Hamiltonian (1) $H_{rot}(I)$ is diagonal by wave functions. Note that $H_{rot}(I)$ is determined by exploiting Harris parameterization for energy and angular momentum [10]

$$E_{rot}(I) = \frac{1}{2}\Im_0\omega^2_{rot}(I) + \frac{3}{4}\Im_1\omega^4_{rot}(I) \tag{3}$$

$$[I(I+1)]^{1/2} = \Im_0\omega_{rot}(I) + \Im_1\omega^3_{rot}(I) \tag{4}$$

where $\Im_0$ and $\Im_1$– are the inertia parameters of the rotational core.

The rotational frequency of the core $\omega_{rot}(I)$ is found by solving cubic equation (4). This equation has two imaginary roots and one real root. The real root is as follows

$$\omega_{rot}(I) = \left\{\frac{\widetilde{I}}{2\Im_1} + \left[\left(\frac{\widetilde{I}}{2\Im_1}\right)^2 + \left(\frac{\Im_0}{3\Im_1}\right)^3\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}} + \left\{\frac{\widetilde{I}}{2\Im_1} - \left[\left(\frac{\widetilde{I}}{2\Im_1}\right)^2 + \left(\frac{\Im_0}{3\Im_1}\right)^3\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}} \tag{5}$$

where $\widetilde{I} = \sqrt{I(I+1)}$. Equation (5) gives $\omega_{rot}(I)$ at the given spin I of the core.

Solving the Shrödinger equation

$$H^\sigma_{K,K'}\Psi^I_{K,K'} = \mathcal{E}_K(I)\Psi^I_{K,K'}. \tag{6}$$

we define eigne function and energy of a Hamiltonian. The total energy of state is defined by

$$E_K(I) = E_{rot}(I) + \mathcal{E}_K(I) \tag{7}$$

3. Energy Spectra and Structure of States

The calculations have been carried out for the isotopes $^{170,172,174,176}Yb$. All experimentally known rotational bands of positive parity have been included in basis of Model Hamiltonian.

The experiment suggests that $m = 5$ band with $K^\pi = 0^+_m$, one band $\ell = 1$ with $K^\pi = 2^+_\ell$, and $\nu = 19$ with $K^\pi = 1^+_\nu$ states in ^{170}Yb [11]. These all $n = m + \ell + \nu = 25$ rotational bands have been included in the basis states of Hamiltonian (1). For the isotopes $^{172,174}Yb$, basis states of Hamiltonian include $n = 15$ ($m = 5$, $\ell = 2$ and $\nu = 8$) and $n = 22$ ($m = 5$, $\ell = 2$ and $\nu = 15$), correspondingly [12, 13, 14].

The parameters of inertia $\Im_0$ and $\Im_1$ are estimated by exploiting Harris parameterization (4), and using the experimental data for energy up to spin $I \leq 8\hbar$ for ground band [15].

The Hamiltonian (2) has transformational properties, that the state (3) can be classified as quantum number — $\sigma = \pm 1$ signature, which imposes restrictions on angular momentum values.

$$(-1)^I \sigma = 1.$$

For the states with negative signature $\sigma = -1$, Hamiltonian (2) has dimension $n = \ell + \nu$, as in bands with $K^\pi = 0^+_m$ the are no condition states with odd spins I. For the states with positive signature $\sigma = +1$, Hamiltonian (2) has dimension $n = m + \ell + \nu$.

The model parameters are described as follows:

a) the bandhead energy ground ω_{0_1} and $K^\pi = 0^+_m$ bands has taken from experiment, as they are not revolted by Coriolis force. Bandhead energy of 1^+_ν bands are also defined from an experiment [11, 12]

$$\omega_{1_\nu} = E^{exp.}_{1_\nu}(I = 1) - E_{rot}(I = 1);$$

b) matrix elements $(j_x)_{2_\ell 1} = (j_x)_{2_\ell 1_\nu}$ and bandhead energy of $2^+_\ell-$ bands ω_{2_ℓ} are determined from the most favored experimental and theoretical spectrum of energy states with a negative signature $\sigma = -1$, e.a. for energy state for even spins I;
c) the matrix elements $(j_x)_{0_m 1} = (j_x)_{0_m 1_\nu}$ defined by the least square method from the best fitted of theoretical energy spectra state with positive signature $\sigma = +1$ with experimental data.

The obtained values of model parameters are presented in Table 1.

Table 1. The used in the calculations for the isotopes $^{170-176}Yb$.

A	$\jmath_0$	$\jmath_1$	ω_{2_1}	ω_{2_2}	$(j_x)_{gr,1^+}$	$(j_x)_{\beta_1,1^+}$	$(j_x)_{\gamma_1,1^+}$
170	35.064	87.814	1.075	–	0.186	0.393	0.727
172	37.422	90.176	1.389	1.540	0.275	0.977	0.325
174	38.556	84.951	1.556	2.652	0.185	0.400	0.085
176	35.945	66.448	1.2109	–	0.200	0.540	0.090

Whre: $\jmath_0, \jmath_1$ are inertial parameters of rotational core $\left(\frac{\hbar^2}{MeV}, \frac{\hbar^4}{MeV^3}\right)$, $\omega_{_K}-$ are parameters of bandhead energy (MeV), and $(j_x)_{_{K',K}}-$ are matrix elements of Coriolis mixture.

In the figures 1-4 the comparison of calculated values of energy of low-lying states for isotopes of $^{170-176}Yb$ with experimental data [11, 13, 14, 17] and [18] and with taken results by $IBM-1$ model are presented. From the comparison we can, that the model qualitatively reproduces experimental energy of rotational states up to energy $3MeV$. However, in high spin values $I \geq 10\hbar$ noticeable deviation has been observed in calculated values of energy and that obtained from experiment. Note that this deviation increases with the growth of angular momentum I. This is probably due to the fact that the influence of rotation on internal nuclei structure has not been considered in this model. In future, we will study electromagnetic properties of low-lying states $I < 10\hbar$.

Furthermore the model predicts the energies, spin and parity of β_2-, β_3-, β_4-, γ_2- and 1^+ band states and as it shown in the Tables 2 – 6 for isotopes $^{170-176}Yb$, respectively. In these tables our calculations are in good agreement with the experimental data.

The structure of the wave functions of states $\Psi_{K',K}$ for $K^\pi = 2^+_1-$ and 0^+_2- bands for $^{170,172,174,176}Yb$, are provided in Fig. 5–8, respectively. The components which have small values are not illustrated in Figures.

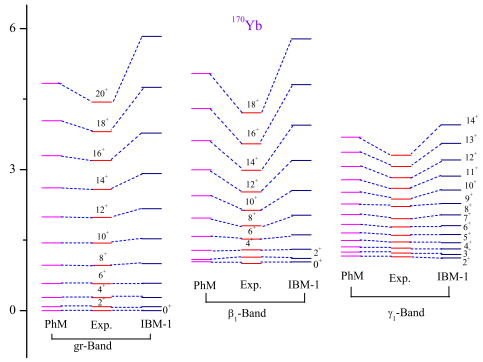

Figure 1. The comparison of calculation energy values by PhM and $IBM-1$ with experimental data for ^{170}Yb.

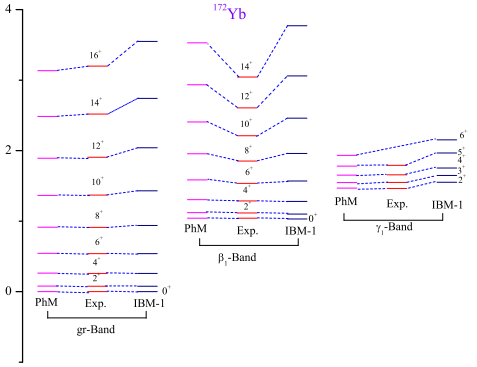

Figure 2. The comparison of calculation energy values by PhM and $IBM-1$ with experimental data for ^{172}Yb.

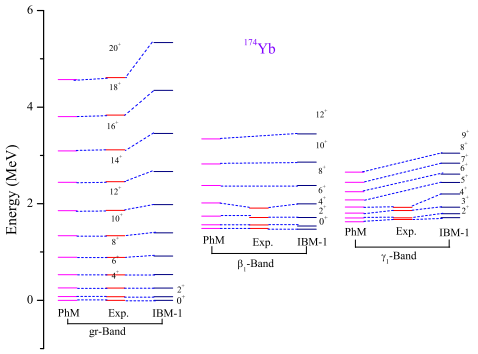

Figure 3. The comparison of calculation energy values by PhM and $IBM-1$ with experimental data for ^{174}Yb.

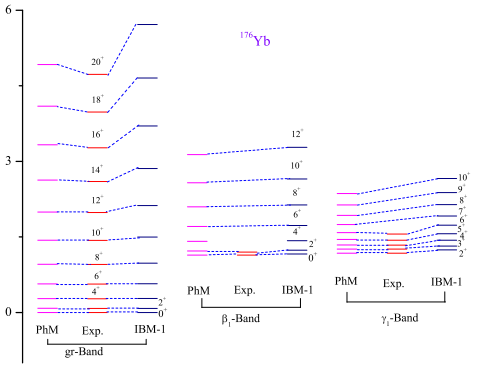

Figure 4. The comparison of calculation energy values by PhM and $IBM-1$ with experimental data for ^{176}Yb.

Table 2. The energy levels of β_2- band of Yb isotopes (in MeV).

	^{170}Yb		^{172}Yb		^{174}Yb		^{176}Yb	
I	Exp.[17, 11]	PhM	Exp.[17, 13]	PhM	Exp.[17, 14]	PhM	Exp.[17, 18]	PhM
0^+	1.228	1.228	1.404	1.404	1.885	1.885	1.518	1.518
2^+	1.306	1.313	1.476	1.483	1.958	1.962	1.610	1.601
4^+	–	1.507	1.632	1.666	2.123	2.140	–	1.792
6^+	–	1.804	–	1.947	–	2.414	–	2.086
8^+	–	2.195	–	2.317	–	2.770	–	2.476
10^+	–	2.669	–	2.769	–	3.221	–	2.954
12^+	–	3.220	–	3.295	–	3.740	–	3.512

Table 3. The energy levels of β_3– band of Yb isotopes (in MeV).

	^{170}Yb		^{172}Yb		^{174}Yb		^{176}Yb	
I	Exp.[17, 11]	PhM	Exp.[17, 13]	PhM	Exp.[17, 14]	PhM	Exp.[17, 18]	PhM
0^+	1.479	1.479	1.794	1.794	2.113	2.110	1.779	1.779
2^+	1.534	1.564	1.849	1.873	2.172	2.178	–	1.862
4^+	1.667	1.758	1.975	2.056	2.336	2.356	–	2.053
6^+	–	2.055	2.156	2.156	–	2.630	–	2.347
8^+	–	2.446	–	2.707	–	2.993	–	2.737
10^+	–	2.920	–	3.159	–	3.437	–	3.215
12^+	–	3.471	–	3.685	–	3.956	–	3.773

Table 4. The energy levels of β_4– band of Yb isotopes (in MeV).

	^{172}Yb		^{174}Yb	
I	Exp.[17, 13]	PhM	Exp.[17, 14]	PhM
0^+	1.894	1.894	2.821	2.821
2^+	1.956	1.973	–	2.898
4^+	2.100	2.156	–	3.076
6^+	–	2.437	–	3.350
8^+	–	2.807	–	3.713
10^+	–	3.259	–	4.157
12^+	–	3.785	–	4.676

Table 5. The energy levels of γ_2– band of Yb isotopes (in MeV).

	^{172}Yb		^{174}Yb	
I	Exp.[17, 13]	PhM	Exp.[17, 14]	PhM
2^+	1.608	1.619	2.728	2.727
3^+	1.700	1.698	2.793	2.804
4^+	1.803	1.802	2.882	2.905
5^+	1.926	1.931	–	3.031
6^+	2.075	2.083	–	3.179
7^+	–	2.257	–	3.350
8^+	–	2.453	–	3.542
9^+	–	2.669	–	3.754
10^+	–	2.905	–	3.986
11^+	–	3.159	–	4.236
12^+	–	3.431	–	4.505
13^+	–	3.720	–	4.791
14^+	–	4.026	–	5.093

Table 6. The energy levels of 1^+- band of Yb isotopes (in MeV).

	^{170}Yb		^{172}Yb		^{174}Yb		^{176}Yb	
I	Exp.[17, 11]	PhM	Exp.[17, 13]	PhM	Exp.[17, 14]	PhM	Exp.[17, 18]	PhM
1^+	1.606	1.605	2.009	2.006	1.624	1.605	1.819	1.818
2^+	1.832	1.662	2.047	2.059	1.674	1.657	1.867	1.874
3^+	–	1.746	2.109	2.138	1.733	1.734	–	1.956
4^+	–	1.856	2.193	2.242	1.859	1.835	–	2.065
5^+	–	1.993	–	2.371	–	1.961	–	2.200
6^+	–	2.153	–	2.523	–	2.109	–	2.359
7^+	–	2.337	–	2.697	–	2.280	–	2.542
8^+	–	2.544	–	2.893	–	2.472	–	2.749
9^+	–	2.771	–	3.109	–	2.684	–	2.977
10^+	–	3.018	–	3.345	–	2.916	–	3.227
11^+	–	3.285	–	3.599	–	3.166	–	3.496
12^+	–	3.569	–	3.871	–	3.435	–	3.785
13^+	–	3.871	–	4.160	–	3.721	–	4.093
14^+	–	4.190	–	4.466	–	4.023	–	4.419

From Fig. 5–8, we can see that $K^\pi = 0_2^+$ and $K^\pi = 2_1^+$ bands states in ^{170}Yb, $K^\pi = 0_3^+$ and $K^\pi = 2_1^+$ bands in ^{172}Yb are mixed strongly even in low spin values I. In case ^{174}Yb and ^{176}Yb, $K^\pi = 0_2^+$ and $K^\pi = 2_1^+$ bands states are very close, but the mixture effect is not so strong. In isotopes $^{170,172}Yb$, considerable deviation in signature of the states $K^\pi = 2_1^+$ band can be observed. This reflects in the values of probability electromagnetic transitions. In this case, description of quantum number K is difficult for these states. Thus, in ^{170}Yb, a number of research works [17] in this context note that the states with $I = 2^+$ (1.1386 MeV) and $I = 2^+$ (1.1454 MeV) $K = 2$ and $K = 0$, respectively. On the other hand, some works [19] document that $K = 0$ and $K = 2$, correspondingly.

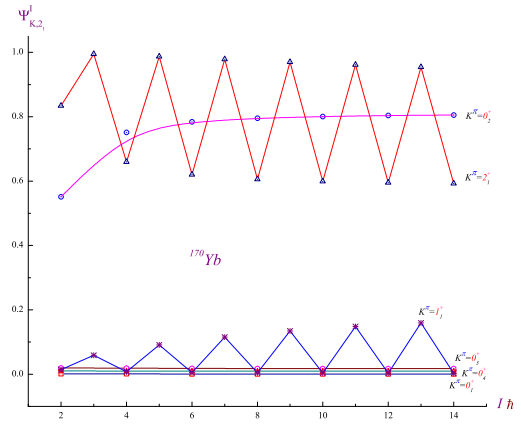

Figure 5. The structure of the wave function of γ_1 band for ^{170}Yb.

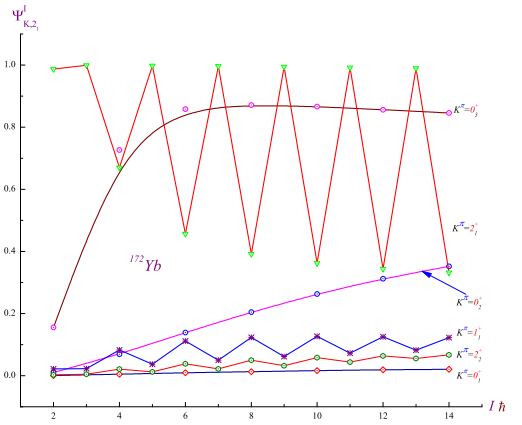

Figure 6. The structure of the wave function of γ_1 band for ^{172}Yb.

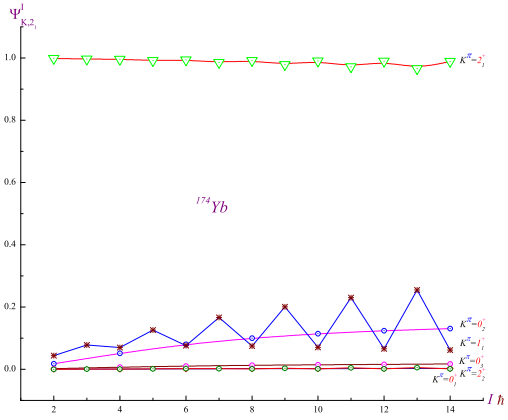

Figure 7. The structure of the wave function of γ_1 band for ^{174}Yb.

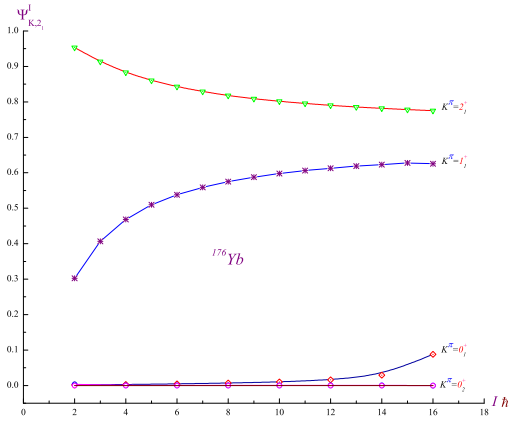

Figure 8. The structure of the wave function of γ_1 band for ^{176}Yb.

4. Conclusion

In the present work, non-adiabatic effects in energies and wave function structure of low-lying excited states are studied within the phenomenological model which taking into account Coriolis mixing of all experimentally known rotational bands for isotopes $^{170,172,174,176}Yb$.

The energy and structure of wave functions of low-lying excited states of $^{170-176}Yb$ isotopes are calculated. The finding reveals that the bands mixing has been found to have considerable impact on the wave function of low-lying states 0^+ and 2^+ bands.

The predicted low-lying levels ($gr-, \beta_1-$ and γ_1- band) by PhM are in good agreement with the experimental data as compared with those by $IBM-1$ for all nuclei of interest. In addition, the PhM is successful in predicting the β_2-, β_3-, β_4-, γ_2- and 1^+- band while $IBM-1$ fails.

Acknowledgments

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References

[1] Mo J N and et al. 1987 *Nucl. Phys. A* **472** 295
[2] Jonsson S, Roy N, Ryde H, Walus W and et al. 1986 *Nucl. Phys. A* **449** 537
[3] Beck E M, Bcelar J C, Deleplanque M A, Diamond R M and et al. 1987 *Nucl. Phys. A* **464** 472
[4] Byrski T and et al. 1987 *Nucl. Phys. A* **474** 193
[5] Fewell M P and et al. 1988 *Phys. Rev. C* **37** 101
[6] Granja C, Posíšil S, Chrien R E and Telezhnikovc S A 2005 *Nucl. Phys. A* **757** 287
[7] Bohr A and Mottelson B R 1997 *Nuclear Structure Vol.* **1.2** Benjamin New York
[8] Usmanov Ph N, Okhunov A A, Salikhbaev U S and Vdovin A 2010 *Phys. Part. Nucl. Lett.* **7** 185
[9] Usmanov Ph N, and Mikhailov I N 1997 *Phys. Part. Nucl. Lett.* **28** 348
[10] Harris S M 1965 *Phys. Rev. B* **138** 509
[11] Baglin C M 2002 *Nucl. Data Sheets* **96** 611
[12] Zilges A, von Brentano P, Wesselborg C and et al. 1990 *Nuc.Phys. A* **507** 399
[13] Singh B 1995 *Nucl. Data Sheets* **75** 199

[14] Browne E and Junde H 1999 *Nucl. Data Sheets* **87** 15
[15] Okhunov A A, Hasan Abu Kassim and Usmanov Ph N 2011 *Sains Malaysiana* **40** No.1 1
[16] Reich C W, Greenwood R C, Lokken R A 1974 *Nucle. Phys. A* **228** 365
[17] Begzhanov R B,Belinkiy V M, Zalyubovskiy I I and Kuznichenko A B 1989 *Handbook on Nuclear Physics* Vol.**1,2** Tashkent FAN
[18] Basunia M S 2006 *Nucl. Data Sheets* **107** 791
[19] Baglin C M 1996 *Nucl. Data Sheets* **77** No.2 125