

# On Channeling Potential for Relativistic Electrons in Crossed Laser Field

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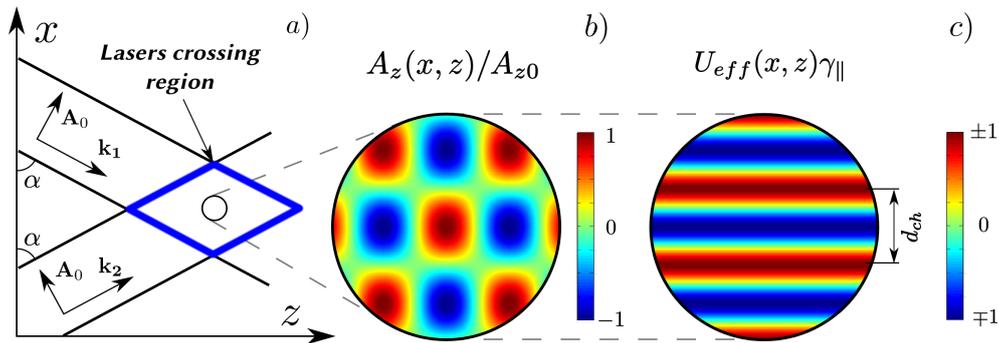
**Abstract.** In this work the expression for the ponderomotive potential responsible for electron channeling in the field of crossed laser beams is analyzed. The influence of lasers crossing angle and electron velocity on the ponderomotive force is described in two plane waves approximation revealing “*nonrelativistic inversion*” phenomenon. Some estimations for the beam density of electrons channeled in such system are given.

## 1. Introduction

Channeling is a phenomenon which takes place when a relativistic projectile propagates in a periodic structure of some sort (e.g. crystals) and its interaction with the structure (crystal lattice nodes) could be effectively averaged forming well defined potential channels (either planar or axial). Particles with transverse energy less than the channel barriers become trapped in a channel potential, i.e. channeled, and perform oscillations transverse to the channel direction. While one can find more information on crystal channeling elsewhere [1, 2] we only note here that the term “*channeling*” is widely used nowadays describing a variety of phenomena such as neutral particles beams channeling [3, 4], particles channeling in crystals, capillaries and nanotubes [5, 6, 7] and electrons channeling in plasma channels [8]. Moreover, motion of charged particles in crossed electromagnetic waves [9, 10, 11, 12, 13, 14] can be described with channeling phenomenology, which is the main topic of this paper.

In general, both laser-electron and laser-plasma interactions are very important research areas being nowadays under active development. Hence, clear understanding of electron dynamics in an external laser field is no doubt crucial for many applications. Some of them rely on electron beam motion in the field of a standing electromagnetic wave or crossed laser beams. Being first to consider the problem, Kapiza and Dirak have statistically derived the possibility of electron deflection by standing electromagnetic field [15]. Later with the development of laser technologies this problem was considered in terms of channeling in series of works by Bertolotti *et al.* [9, 10]. In those works the detailed dynamics of an electron channeled in the field of standing electromagnetic wave and crossed laser beams were discussed. And also the proposal for a new type radiation source based on particle channeling oscillations in such systems was made. These ideas could be found in other separate studies (see [11, 16, 17] and Refs. therein).





**Figure 1.** (a) Schematic geometry of two crossed p-polarized laser beams. (b) The longitudinal component of summarized vector potential (arb. un.) in the central area of lasers crossing region, in two plane waves approximation. Here  $\alpha \in (0, \pi/2)$ . The peaks of  $A_z$  migrate in the direction of  $Oz$ -axis. (c) Normalized effective potential distribution for the same area as in Fig.1.b. The potential absolute value depends on lasers parameters, angle  $\alpha$  and velocity of an electron propagating in such a field. Sign ambiguity characterizes possible “inversion” described below.

And finally, in the last decade this topic gains additional attention with the latest advance of achievable laser intensities, growing interest to laser wakefield acceleration techniques and demand of new beam diagnostic tools.

Kaplan *et al.* [18, 19] reconsider a well known ponderomotive force introducing its “inversion” for particular lasers polarization and projectiles energy. Following their work, Smorenburg *et al.* [20] has derived generalized formula for the ponderomotive potential in the region of two counterpropagating laser beams of arbitrary polarization. According to the article, ponderomotive force sign inversion for the p-polarization case is possible for particular electron energy.

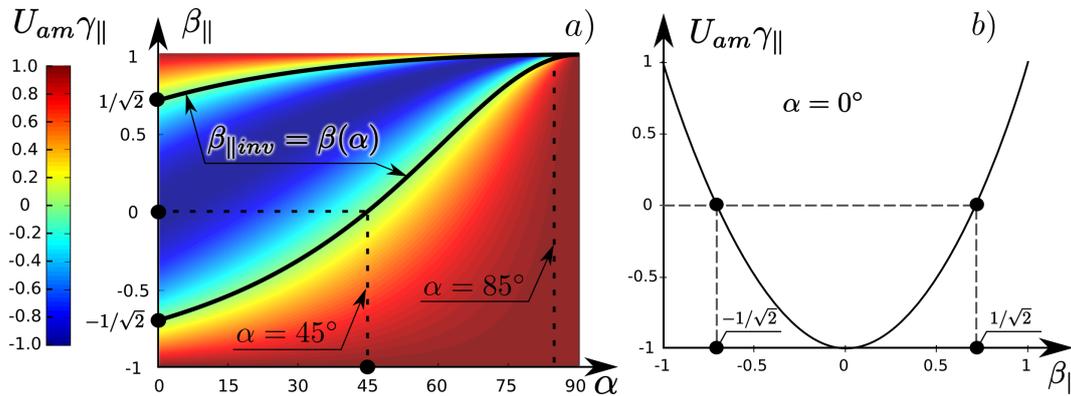
Lately Balcou, Andriyash *et al.* (see [21, 22, 23] and Refs. therein) in a series of articles have again made a proposal for a radiation source utilizing channeling of electrons in crossed laser beams basing it on both numerical and analytical results of charged particles dynamics and radiation in such system. They also consider the phenomenon to be a promising tool for electron beam diagnostics allowing emittance measurements [22].

## 2. Electron dynamics in the field of two laser beams

The case of electron dynamics in standing electromagnetic wave created by two counterpropagating laser beams of arbitrary polarization is thoroughly covered in [18, 19, 20]. But directing lasers so that they become crossed at some angle (see Fig. 1.a) causes the potential to change dramatically and differ from the expressions presented in mentioned papers. The case of crossed plane electromagnetic p-polarized waves is covered here.

### 2.1. General information

Let us consider two p-polarized plane electromagnetic waves of equal frequency  $\omega$  and amplitude  $A_0$ ,  $\mathbf{k}_1 = w/c\{-\cos\alpha; 0; \sin\alpha\}$  and  $\mathbf{k}_2 = w/c\{\cos\alpha; 0; \sin\alpha\}$ . Therefore, their superposition gives  $A_x = A_{x0} \sin(\omega t - zk \sin\alpha) \cos(xk \cos\alpha)$  for transverse vector potential and  $A_z = A_{z0} \cos(\omega t - zk \sin\alpha) \sin(xk \cos\alpha)$  for longitudinal vector potential components as shown in Fig. 1.b, where  $k = \omega/c$ . Let us call planes with  $xk \cos\alpha = \pi(n + 1/2)$  the “antinodes” and planes with  $xk \cos\alpha = \pi n$  the “nodes”, meaning those are nodes and antinodes of  $A_z$ . Obviously, for  $\alpha = \pi/2$  the system is similar with the one described in [18, 19, 20]. Electrons with sufficiently small transverse momentum and longitudinal momentum  $|p_{\parallel}| < m_e c$  (where  $m_e$  is the electron rest mass and  $c$  is the speed of light) will oscillate



**Figure 2.** (a) The amplitude of  $U_{am}\gamma_{\parallel}$  normalized by  $(2m_e\omega^2)/(e^2A_0^2k^2)$  as a function of the angle  $\alpha$  is shown in color. The black curves show the electron speed  $\beta_{\parallel inv}$  for which no ponderomotive force affects its motion. (b) The same but for a fixed  $\alpha = 0^\circ$ , which corresponds to counterpropagating electromagnetic waves.

near the nodes due to ponderomotive force. On the other hand, for electrons with the same transverse momentum and longitudinal momentum  $|p_{\parallel}| > m_e c$  the nodes will be scattering, while the antinodes — attracting. Finally, electrons with  $|p_{\parallel}| = p_{\parallel inv} \equiv m_e c$  will “feel” no ponderomotive potential and perform no averaged oscillations. Thus, according to [18], “this potential inversion is considered to pin down a line between relativistic and nonrelativistic motions”.

## 2.2. Crossed electromagnetic waves

Notable, the description given in [18, 19, 20] is not applicable to the case of arbitrary  $\alpha$ . As shown in [24], the expression for the averaged effective potential in laboratory frame is  $U_{eff} = U_{am} \cos(2kx \cos \alpha)$  with its amplitude defined by

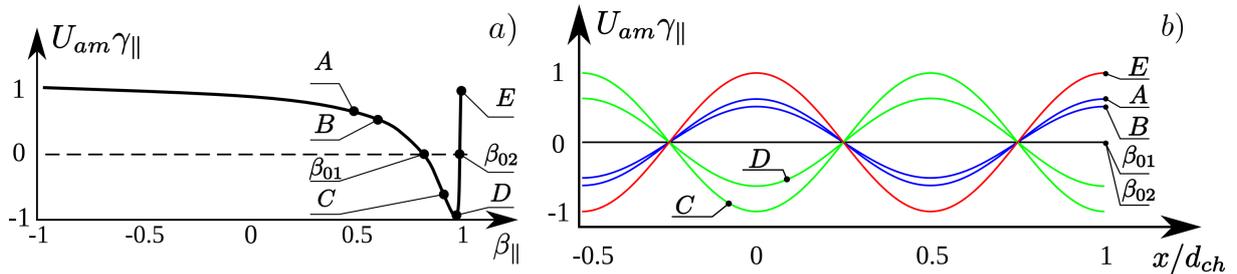
$$U_{am} = \frac{e^2 A_0^2 k^2 [(1 + \cos^2 \alpha) \beta_{\parallel}^2 - \cos(2\alpha) - 2\beta_{\parallel} \sin \alpha]}{2\gamma_{\parallel} m_e \omega^2 (1 - \beta_{\parallel} \sin \alpha)^2}, \quad (1)$$

where  $e$  is the electron charge,  $\beta_{\parallel}$  is the electron velocity projection on  $Oz$ -axis normalized by  $c$ ,  $\gamma_{\parallel} = 1/\sqrt{1 - \beta_{\parallel}^2}$ . The averaged effective potential has a spatial period of  $d_{ch} = \pi/(k \cos \alpha)$  equal to the distance between two nodes planes of  $A_z$  depicted in Fig. 1.c.

Finding roots of the quadratic equation on  $\beta_{\parallel}$  in the numerator of Eq. (1) one can see that for the case of counterpropagating electromagnetic waves ( $\alpha = 0$ ) they are  $\beta_{\parallel inv} = \pm 1/\sqrt{2}$ , which gives  $p_{\parallel} = m_e c$ . But, in general, the value of longitudinal electron velocity, for which  $U_{am} = 0$  and no channeling oscillations occur, is a function of  $\alpha$

$$\beta_{\parallel inv}(\alpha) = \frac{\sin \alpha \pm \cos \alpha \sqrt{[1 + \cos(2\alpha)]}}{1 + \cos^2 \alpha} \quad (2)$$

This dependence for the *inversion speed* is shown in Fig. 2.a (black curves) together with the normalized effective potential amplitude  $U_{am}\gamma_{\parallel}$  (in color). In Fig. 2.b one can find the values of  $\beta_{\parallel inv}$  for the case considered in the papers mentioned above ( $\alpha = 0$ ) and they are similar to those reported by other authors. However, they change significantly if *crossed* lasers are considered, e.g. an electron with zero longitudinal velocity (“*nonrelativistic inversion*”) is affected by *no* ponderomotive force in the field of normally aligned lasers (in Fig. 2.a  $U_{am}\gamma_{\parallel} = 0$  for  $\alpha = 45^\circ$  and  $\beta = 0$ ). The dependence of normalized



**Figure 3.** (a) The averaged potential amplitude normalized by  $(e^2 A_0^2 k^2)/(2m_e \omega^2)$  versus the particle longitudinal velocity  $\beta_{\parallel}$ . The potential calculated for  $\alpha = 85^\circ$  is shown.  $\beta_{01}$  and  $\beta_{02}$  are the electron longitudinal velocities, for which  $U_{am}(\beta_{01}) = U_{am}(\beta_{02}) = 0$ . Negative  $\beta_{\parallel}$  corresponds to the electron motion in opposite to the  $\mathbf{k}_s = \mathbf{k}_1 + \mathbf{k}_2$  direction, while for positive  $\beta_{\parallel}$  the electron moves being co-directional to  $\mathbf{k}_s$ . (b) The normalized potential distribution as a function of transverse coordinate normalized by  $d_{ch}$  for electrons with different longitudinal velocities. Curves A, B, C, D, E correspond, respectively, to the particle with  $\beta_A = 0.5$ ,  $\beta_B = 0.6$ ,  $\beta_C = 0.92$ ,  $\beta_D = 0.967$ ,  $\beta_E = 0.99986$ .

channeling potential amplitude as a function of electron longitudinal velocity  $U_{am}\gamma_{\parallel}(\beta_{\parallel})$  is shown in Fig. 3.a for  $\alpha = 85^\circ$ . One can notice that the potential amplitude there strongly depends not only on the value of longitudinal velocity but also on its direction. The spatial distribution of the potential across  $Ox$ -axis is shown in Fig. 3.b where its inversion could be easily seen for different  $\beta_{\parallel}$  values. Indeed, as aforementioned, some electrons (with velocities  $\beta_A$ ,  $\beta_B$  and  $\beta_E$ ) are scattered on the nodes of  $A_z$  and oscillate around the antinodes of  $A_z$ , whereas for  $\beta_C$  and  $\beta_D$  the potential is inverted. And  $\beta_{01}$  and  $\beta_{02}$  are the inversion velocities here.

We also note here that all the reported results were checked with numerical simulations of electrons dynamics in crossed electromagnetic waves. Electron trajectories in the region of waves overlapping were calculated with the help of forth-order Runge-Kutta method validating the analytically derived expressions.

### 2.3. Beam confinement

As regards to the question of electron beam dynamics in the field of crossed electromagnetic waves, we provide some estimations for the electron density value, for which channeling of the majority of electrons is still possible in the considered system. For simplicity, the cylindrical beam (uniform distribution) of diameter  $d$  stretched along  $Oz$ -axis is considered. An electron on the beam surface will be affected by the space charge force normal to  $Oz$   $F_{\perp} = 2\pi e^2 n_0 (1 - \beta_{\parallel}^2) d$ . For particles of the beam to remain trapped in the channel the ponderomotive force in a channel  $F_{ch} \approx U_{am}/d_{ch}$  needs to be greater then the space charge force

$$\frac{U_{am}}{d_{ch}} \geq 2\pi e^2 n_0 \frac{d_{ch}}{\gamma_{\parallel}^2} \rightarrow n_0 \leq \frac{\gamma_{\parallel}^2 U_{am}}{2\pi e^2 d_{ch}^2}, \quad (3)$$

where  $n_0$  is the electron density of a channeled beam. Expressing  $U_{am}$  with external electromagnetic field intensity, we get

$$n_0 \leq \frac{2\gamma_{\parallel} I}{m_e c \omega^2 d_{ch}^2} \quad (4)$$

Thus, for the intensities  $I = 10^{14} - 10^{22} \text{ Wcm}^{-2}$  the limiting density is  $n_0 \sim 10^{19} - 10^{27} \text{ cm}^{-3}$  ( $\gamma_{\parallel} \sim 10^3$ ) that could make electron channeling in the field of crossed laser beams a promising technique for future applications (such as mentioned in [13, 21, 22, 24, 25]) when the feasible laser intensities will overcome  $10^{20} \text{ Wcm}^{-2}$ . Also we have to highlight that laser beam divergence and an electron beam divergence will

both toughen the channeling condition since the provided expression is applicable to the plane waves approximation.

### 3. Conclusion

Though the considered problem has been a subject of detailed research for a long time, some new peculiarities of ponderomotive potential for the field of crossed electromagnetic waves are presented. This becomes more and more important today because of many applications of laser-electron interaction. The most notable of them are laser wakefield acceleration, new radiation sources and diagnostic tools (e.g. one can find a description of beam diagnostics tool known as Shintake Monitor in [26]) based on electron dynamics and radiation in a strong external field. An interesting example is recently reported in the paper [25] on the “*anomalous radiation trapping*” of electrons in crossed laser beams with emission of high-energy photons. Ponderomotive potential inversion should be taken into account in all of those cases. And *nonrelativistic potential inversion* requires magnetic component of a Lorenz force (usually neglected) to be taken into account even for very slow electrons. Since the potential changes drastically depending on laser crossing angle it can open new possibilities (or possibly restrictions) for some applications (such as reported in [25]), which will be the subject of our future research.

### Acknowledgments

This work was supported by the Ministry of Education and Science of RF in the frames of Competitiveness Growth Program of National Research Nuclear University MEPhI, Agreement 02.A03.21.0005 and the Dept. for Education and Science of LPI RAS.

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